Massive MIMO for 5G
Recent Theory

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Expectations for 5G Networks

- **5G – Next Network Generation**
  - To be introduced around 2020
  - Design objectives are currently being defined

<table>
<thead>
<tr>
<th>5G Performance Metrics</th>
<th>Expectation</th>
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<tbody>
<tr>
<td>Average rate (bit/s/active user)</td>
<td>10-100x</td>
</tr>
<tr>
<td>Average area rate (bit/s/km²)</td>
<td>1000x</td>
</tr>
<tr>
<td>Active devices (per km²)</td>
<td>10-100x</td>
</tr>
<tr>
<td>Energy efficiency (bit/Joule)</td>
<td>1000x</td>
</tr>
<tr>
<td>“Best experience follows you”</td>
<td></td>
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</tbody>
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*Source: METIS project (www.metis2020.com)*

What is the role of Massive MIMO here?
Outline, Part 2: Recent Theory

- Spectral Efficiency
  - Designing Massive MIMO for high spectral efficiency
  - What are the fundamental limits?

- Energy Efficiency
  - How is it defined?
  - Is Massive MIMO energy efficient?

- Hardware Efficiency
  - Does Massive MIMO require high-grade hardware?
  - Can it make more efficient use of hardware (lower cost, size, and power)?

- Open Problems
Massive MIMO and SPECTRAL EFFICIENCY
Evolving Networks for Higher Traffic

- Increase Network Throughput [bit/s/km²]
  - Consider a given area

- Simple Formula for Network Throughput:
  \[
  \text{Throughput} = \frac{\text{Available spectrum}}{\text{Hz}} \cdot \frac{\text{Cell density}}{\text{Cell/km}^2} \cdot \frac{\text{Spectral efficiency}}{\text{bit/s/Hz/Cell}}
  \]

- 5G goal: 1000x improvement

<table>
<thead>
<tr>
<th></th>
<th>More spectrum</th>
<th>Higher cell density</th>
<th>Higher spectral efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia (2011)</td>
<td>10x</td>
<td>10x</td>
<td>10x</td>
</tr>
<tr>
<td>SK Telecom (2012)</td>
<td>3x</td>
<td>56x</td>
<td>6x</td>
</tr>
</tbody>
</table>

- New regulations, cognitive radio, mmWave bands
- Smaller cells, heterogeneous deployments
- Massive MIMO
  - How many ??x can we expect?
Optimization of Spectral Efficiency

• How Large Spectral Efficiency can be Achieved?
  • Problem Formulation:
    $$\max_{K, \tau_p} \text{total spectral efficiency} \quad \text{[bit/s/Hz/cell]}$$
    for a given $M$ and $\tau_c$.

• Issue: Hard to find tractable expressions
  • Interference depends on all users’ positions!
  • Expressions from before: Fixed and explicit pathloss values ($\beta$)
  • We want quantitative results – averaged over user locations

• Solution: Make every user “typical”
  • Same uplink SNR: Power control inversely proportional to pathloss
  • Inter-cell interference: Code over variations in user locations in other cells
Symmetric Multi-Cell Network

- Classic Multi-Cell Network
  - Infinite grid of hexagonal cells
  - $M$ antennas at each BS
  - $K$ active users in each cell
  - Same user distribution in each cell
  - Uncorrelated Rayleigh fading
  - Statistical channel inversion: $\rho_u \eta_{lk} = \frac{p}{\beta_{lk}}$

*Every cell is “typical”*

**Propagation Parameters**
(Average interference from cell $l$ to BS $j$)

Compute $\mu_{jl}^{(1)} = \mathbb{E} \left\{ \frac{\beta_{lk}^j}{\beta_{lk}} \right\}$ and $\mu_{jl}^{(2)} = \mathbb{E} \left\{ \left( \frac{\beta_{lk}^j}{\beta_{lk}} \right)^2 \right\}$
Coordinated Pilot Allocation

- Limited Number of Pilots: $\tau_p \leq \tau_c$
  - Must use same pilot sequence in several cells
  - Base stations cannot tell some users apart: *Essence of pilot contamination*

- Coordinated Pilot Allocation
  - Allocate pilots to users to reduce contamination
  - Scalability → No signaling between BSs

- Solution: Non-universal pilot reuse
  - Pilot reuse factor $f \geq 1$
  - Users per cell: $K = \frac{\tau_p}{f}$
  - $\mathcal{P}_j$ = Cells with same pilots as BS $j$
  - Higher $f$ → Fewer users per cell, but fewer interferers in $\mathcal{P}_j$
Coordinated Precoding and Detection

- Coordinated Multi-Point (CoMP)
  - Avoid causing strong inter-cell interference
  - Scalability → No signaling between BSs

- Solution: Observe and react \((f \geq 1)\)
  - Listen to pilot signals used only in other cells
  - Utilize to suppress inter-cell interference
  - Schemes: Multi-cell ZF and multi-cell MMSE

**MMSE precoding/detection:**

\[
v_{lk} = \left( \sum_{j,m} \rho_u \eta_{jm} \hat{g}_{jm} (\hat{g}_{jm})^H + \mathbf{E}_l + \mathbf{I} \right)^{-1} \hat{g}_{lk}
\]

- All estimated channels
- Estimation error covariance matrix

\(\text{Reuse } f = 3\)
Uplink-Downlink Duality

**Duality Theorem**

The uplink SEs are achievable in the downlink using same sum transmit power

*Same precoding/detection vectors, but different power allocation*

Note: Equivalence between two lower bounds – uplink bound is looser!
Average Spectral Efficiency per Cell

- Lower Bound on Average Ergodic Capacity in Cell $j$:
  \[
  SE_j = K \left( 1 - \frac{\tau_p}{\tau_c} \right) \log_2 \left( 1 + \frac{1}{I_j} \right)
  \]
  
- Interference term depends on processing:

  \[
  I_j^{MR} = \sum_{l \in P_j(\{j\})} \left( \frac{\mu_{jl}^{(2)}}{\mu_{jl}} + \frac{\mu_{jl}^{(2)} - \left( \mu_{jl}^{(1)} \right)^2}{M} \right) + \left( \frac{\sum_{l \in \mathcal{L}} \mu_{jl}^{(1)} K + \frac{1}{\rho}}{M} \right) \left( \sum_{l \in P_j(f)} \mu_{jl}^{(1)} + \frac{1}{\rho \tau_p} \right)
  \]

  \[
  I_j^{ZF} = \sum_{l \in P_j(\{j\})} \left( \frac{\mu_{jl}^{(2)}}{\mu_{jl}} + \frac{\mu_{jl}^{(2)} - \left( \mu_{jl}^{(1)} \right)^2}{M - K} \right) + \left( \frac{\sum_{l \in \mathcal{L}} \mu_{jl}^{(1)} K + \frac{1}{\rho}}{M - K} \right) \left( \sum_{l \in P_j(f)} \mu_{jl}^{(1)} + \frac{1}{\rho \tau_p} \right) - \sum_{l \in P_j(f)} \frac{\left( \mu_{jl}^{(1)} \right)^2 K}{M - K}
  \]

  Only term that remains as $M \to \infty$: Finite limit on SE
Asymptotic Limit on Spectral Efficiency

- Lower Bound on Average Ergodic Capacity as $M \to \infty$:

$$\text{SE}_j \to K \left(1 - \frac{fK}{\tau_c}\right) \log_2 \left(1 + \frac{1}{\sum_{l \in \mathcal{P}_j(f) \setminus \{j\}} \mu_{jl}^{(2)}}\right)$$

**How Many Users to Serve?**

Pre-log factor $K \left(1 - \frac{fK}{\tau_c}\right)$ is maximized by $K^* = \frac{\tau_c}{2f}$ users

Maximal SE: $\frac{\tau_c}{4f} \log_2 \left(1 + \frac{1}{\sum_{l \in \mathcal{P}_j(f) \setminus \{j\}} \mu_{jl}^{(2)}}\right)$

Try different $f$ and $\mathcal{P}_j(f)$ to maximize the limit

**How Long Pilot Sequences?**

$$\tau_p = fK^* = \frac{\tau_c}{2} : \text{Spend half coherence interval on pilots!}$$
Numerical Results

• Problem Formulation:

\[
\begin{align*}
\text{maximize} & \quad K, \tau_p \\
\text{total spectral efficiency} & \quad [\text{bit/s/Hz/cell}]
\end{align*}
\]

for a given \( M \) and \( \tau_c \).

• Use average spectral efficiency expressions
• Compute average interference \( \mu_{jl}^{(1)} \) and \( \mu_{jl}^{(2)} \) (a few minutes)
• Compute for different \( K \) and \( f \) and pick maximum (\(< 1 \text{ minute}\))

Assumptions

Pathloss exponent: 3.7
Coherence: \( \tau_c = 400 \)
Rayleigh fading
SNR 5 dB
Asymptotic Behavior: Mean-Case Interference

**Observations**
- Uniform user distributions
- Asymptotic limits not reached
- Reuse factor $f = 3$ is desired
- $K$ is different for each scheme
- Small difference between optimized schemes
- Coordinated beamforming: Better at very large $M$
Observations

- Interferers at worst positions
- Asymptotic limits not reached
- Reuse factor $f = 4$ is desired
- $K$ is different for each scheme
- Coordinated beamforming: Brings large gains for all $M$
Flexible Number of Users

- SE w.r.t. number of users ($M = 200$ antennas)
  - Mean-case interference
  - Optimized reuse factors
  - Equal SNR (5 dB)

**Observations**

Stable SE for $K > 10$:
- Trivial scheduling:
  - Admit everyone

$M$-ZF, ZF, and MR provide similar per-cell performance

$M/K < 10$ is fine!
Spectral Efficiency per User

- User Performance for Optimized System
  - Mean-case interference
  - Optimized reuse factors
  - Equal SNR (5 dB)

**Observations**
User performance is modest: BPSK, Q-PSK, or 16-QAM

Schemes for different purposes:
M-ZF > ZF > MR
Anticipated Uplink Spectral Efficiency

**Assumptions**

- ZF processing
- Pilot reuse: $f = 3$

**Observations**

- **Baseline:** 2.25 bit/s/Hz/cell (IMT-Advanced)
- **Massive MIMO, $M = 100$:** x20 gain ($M/K \approx 6$)
- **Massive MIMO, $M = 400$:** x50 gain ($M/K \approx 9$)
- **Per scheduled user:** $\approx 2.5$ bit/s/Hz
Control Signaling

- Coherent Precoding and Detection Require CSI
  - How to initiate the transmission without array gain?

- User Initiates Transmission
  - Easy: Find an unused pilot and send a transmission request
  - Reserve some pilot sequences for such random access

- BS Initiates Transmission
  - Harder: Must contact the user without having CSI
  - Low-rate space-time coded transmission is feasible
Summary

- Massive MIMO delivers High Spectral Efficiency
  - $> 20\times$ gain over IMT-Advanced is foreseen
  - Very high spectral efficiency per cell, not per user
  - Non-universal pilot reuse ($f = 3$) is often preferred
  - MR, ZF, M-ZF prefer different values on $K$ and $f$
  - “An order of magnitude more antennas than users” is not needed

- Asymptotic limits
  - Coherence interval ($\tau_c$ symbols) limits multiplexing capability
  - Allocate up to $\tau_c/2$ symbols for pilots
  - We can handle very many users/cell – how many will there be?
Massive MIMO and ENERGY EFFICIENCY
Energy Consumption

- **Network Electricity Consumption**
  - Dominated by network infrastructure – increases continuously
  - 1000x higher data rates: Easy to achieve using 1000x more power
    Hard to achieve without using more power
  - Calls for **much higher energy efficiency**!

What is Energy Efficiency?

• Benefit-Cost Analysis of Networks
  • Systematic approach to analyze strengths and weaknesses of networks

Cost:  
Power Consumption  
[Watt = Joule/s]

Network

Benefit:  
Sum Data Rate  
[bit/s]

• Definition: Energy Efficiency (EE):

\[
EE \, [\text{bit/Joule}] = \frac{\text{Average Sum Rate} \, [\text{bit/s/cell}]}{\text{Power Consumption} \, [\text{Joule/s/cell}]}
\]

Contemporary networks:  
Very inefficient at low load

Future networks:  
Must be more efficient at any load

Future networks

Contemporary networks
**Transmit Power Scaling Law**

*Power Scaling Law*

If the transmit power $\rho$ decreases as $1/M^\alpha$ for $\alpha \leq 1/2$:

$SE$ will not go zero as $M \rightarrow \infty$

Example: Set $p = p_0/M^\alpha$ in $\text{SE}_j = K \left(1 - \frac{\tau_p}{\tau_c}\right) \log_2 \left(1 + \frac{1}{I_j}\right)$:

$$I_j^{\text{MR}} = \sum_{l \in \mathcal{P}_j(f) \setminus \{j\}} \left(\mu_{j,l}^{\text{(2)}} + \frac{\mu_{j,l}^{\text{(2)}} - (\mu_{j,l}^{\text{(1)}})^2}{M}\right) + \left(\sum_{l \in \mathcal{L}} \mu_{j,l}^{\text{(1)}} + \frac{M^{\alpha}}{p_0}\right) \left(\sum_{l \in \mathcal{P}_j(f)} \mu_{j,l}^{\text{(1)}} + \frac{M^{\alpha}}{p_0 \tau_p}\right) = \sum_{l \in \mathcal{P}_j(f) \setminus \{j\}} \mu_{j,l}^{\text{(2)}} + \mathcal{O}\left(\frac{M^{2\alpha}}{M}\right)$$

**Observations ($\alpha = 1/2$)**

*Power per antenna/user:* Decreases as $\frac{1}{\sqrt{M}}$

*Total power:* $\frac{K}{\sqrt{M}}$ increases as $\sqrt{M}$ for fixed $\frac{M}{K}$
Radiated Energy Efficiency

- Energy Efficiency with Power Scaling:

\[
EE = \frac{\text{Average Sum Rate [bit/s/cell]}}{\text{Power Consumption [Joule/s/cell]}} = \frac{B \cdot K \left(1 - \frac{\tau_p}{\tau_c}\right) \log_2 \left(1 + \frac{1}{I_j}\right)}{M^\alpha \mathbb{E} \left\{ \frac{1}{\beta_{lk}} \right\}}
\]

- Bandwidth: \(B\) Hz

- Consequence of scaling law as \(M \to \infty\):
  1. Sum rate \(\to\) constant > 0
  2. Transmit power \(\to\) 0

\(\Rightarrow\) EE \(\to\) \(\infty\)

**Is Massive MIMO Incredibly Energy Efficient?**

Yes, in terms of bringing down the radiated transmit power

But not all consumed power is radiated!
Generic Power Consumption Model

- Many Components Consume Power
  - Radiated transmit power
  - Baseband signal processing (e.g., precoding)
  - Active circuits (e.g., converters, mixers, filters)

Average Power Consumption Model:

\[
\text{APC} = \frac{Kp}{\eta} \mathbb{E}\left\{ \frac{1}{\beta_{ik}} \right\} + C_{0,0} + C_{0,1}M + C_{1,0}K + C_{1,1}MK
\]

- Power amplifier
  (\(\eta\) is efficiency)
- Circuit power per transceiver chain
- Fixed power
  (control signals, backhaul, load-independent processing)
- Cost of digital signal processing
  (e.g., channel estimation and precoding computation)

Many coefficients: \(\eta, C_{i,j}\) for different \(i, j\)

Nonlinear increasing function of \(M\) and \(K\)
Optimizing a Cellular Network for High EE

- **Clean Slate Network Design**
  - Select BS density: \( \lambda \) BSs per km\(^2\)
  - Select \( M \) and \( K \) per cell
  - Asymmetric user load \( \rightarrow \) asymmetric deployment

**Spatial Point Processes**

Tractable way to model randomness

Poisson point process (PPP):

\[ \text{Po}(\lambda A) \text{ BSs in area of size } A \text{ km}^2 \]

Random independent deployment:
Lower bound on practical performance

Source: Andrews et al.  
“A Tractable Approach to Coverage and Rate in Cellular Networks”

Real BS deployment  
Poisson point deployment
Average Uplink Spectral Efficiency

**Assumptions**

- BSs distributed as PPP: $\lambda$ BS/km$^2$
- $M$ antennas per BS, $K$ users per cell
- Random pilot allocation: $\tau_p = f K$
- Statistical channel inversion: $p / \beta_{i k}^{l}$

Pathloss over noise:

$$\beta_{i k}^{j} = \omega^{-1}(\text{distance [km]})^{-\alpha}$$

**Power per user:**

$$\mathbb{E} \left\{ \frac{p}{\beta_{i k}^{l}} \right\} = p \omega \frac{\Gamma(\alpha/2-1)}{(\pi \lambda)^{\alpha/2}}$$

**Lower Bound on Average SE with MR**

$$\overline{\text{SE}} = \left(1 - \frac{fK}{\tau_c}\right) \log_2(1 + \overline{\text{SINR}})$$

$$\overline{\text{SINR}} = \frac{M}{\left( K + \frac{1}{p} \right) \left( 1 + \frac{2}{f(\alpha-2)} + \frac{1}{p} \right) + \frac{2K}{\alpha-2} \left( 1 + \frac{1}{p} \right) + \frac{K}{f} \left( \frac{4}{(\alpha-2)^2} + \frac{1}{\alpha-1} \right) + \frac{M}{f(\alpha-1)}}$$
Maximizing Energy Efficiency

\[ \text{maximize} \quad B \cdot K \left(1 - \frac{fK}{\tau_c}\right) \log_2 (1 + \text{SINR}) \]
\[ \text{subject to} \quad \text{SINR} \geq \gamma \]

- Average SINR constraint \( \gamma \) needed to not get too low SE
- Is the solution small cells (high \( \lambda \)) or Massive MIMO (high \( M \))?

**Main Properties**

1. Can pick \( f \) to satisfy SINR constraint
2. By setting \( p = p_0 \lambda \), the EE is increasing in \( \lambda \)
3. Quasi-concave function w.r.t. \( M \) and \( K \)

\( \text{Interval} = \text{Convex set} \)

Possible to solve the problem numerically
## Simulation Parameters

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<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherence interval</td>
<td>$\tau_c$</td>
<td>400</td>
</tr>
<tr>
<td>Pathloss exponent</td>
<td>$\alpha$</td>
<td>3.76</td>
</tr>
<tr>
<td>Pathloss over noise at 1 km</td>
<td>$\omega$</td>
<td>33 dBm</td>
</tr>
<tr>
<td>Amplifier efficiency</td>
<td>$\eta$</td>
<td>0.39</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>$B$</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Static power</td>
<td>$C_{0,0}$</td>
<td>10 W</td>
</tr>
<tr>
<td>Circuit power per active user</td>
<td>$C_{1,0}$</td>
<td>0.1 W</td>
</tr>
<tr>
<td>Circuit power per BS antenna</td>
<td>$C_{0,1}$</td>
<td>1 W</td>
</tr>
<tr>
<td>Signal processing coefficient</td>
<td>$C_{1,1}$</td>
<td>3.12 mW</td>
</tr>
</tbody>
</table>

*We publish simulation code to enable simple testing of other values!*
Impact of BS Density

**Simulation**
- Different BS densities
- Other variables optimized

**Observations**
- Lower bound is tight
- Higher EE with lower $\gamma$
- EE increases with $\lambda$

**Saturation Property**

EE gain from small cells saturates at $\lambda = 10$

This is satisfied in most urban deployments (300 m between BSs)

We can safely let $\lambda \to \infty$ to simplify analysis
Optimal Number of Antennas and Users

**Real-valued Optimization**

Optimal $K \in \mathbb{R}$ found in closed-form for fixed $M / K$

Optimal $M \in \mathbb{R}$ found in closed-form for fixed $K$

Alternating optimization reaches global maximum

**Properties:** Optimal $K$ and $M$

\(\downarrow: \) Decrease as $C_{0,1}$, $C_{1,0}$, and $C_{1,1}$ increase

\(\uparrow: \) Increase as $C_{0,0}$ increases

Intuition: Activate more hardware if the relative cost is small
Impact of Number of Antennas and Users

**Simulation**

Optimized $f, \lambda, \rho$

SINR constraint: $\gamma = 3$

**Observations**

Optimal: $M = 89, K = 10$

Massive MIMO with reuse factor $f \approx 7$

Many good solutions

Why is Massive MIMO Energy Efficient?

Interference suppression: Improve SINR, not only SNR as with small cells

Sharing cost: Fixed circuit power costs are shared

Global Optimum:

$M = 89, K = 10$

EE = 10.16 Mbit/J
Optimization with Given User Density

- User Density
  - So far: $K$ and $\lambda$ design variables
  - Density: $\lambda K$ users per km$^2$
  - Heterogeneous user distribution

Can we Optimize this Density?

Increase: No, cannot “create” users
Decrease: Yes, by scheduling

Practical User Densities

- Rural: $10^2$ per km$^2$
- Urban: $10^3$ per km$^2$
- Office/Mall: $10^5$ per km$^2$

Source: METIS, “Deliverable D1.1: Scenarios, requirements and KPIs for 5G mobile and wireless system”
Impact of User Density

**Simulation**

Fixed user density $\mu$ users/km$^2$

- Rural: $\mu = 10^2$, Malls: $\mu = 10^5$
- EE maximization with constraint $K\lambda = \mu$

**Low User Density**

- Many cells with $K \approx 1$
- Most important to reduce pathloss

**High User Density**

- Massive MIMO is optimal
- Saturation for $\mu \geq 100$
- Covers both rural and shopping malls
- Share circuit power and cost over users

---

Simulation Results:

- Optimized $M$ and $K$
- MIMO: $M=89$, $K=10$
- SIMO: $M=10$, $K=1$

Graphs show energy efficiency and BS density variations with UE density.
Summary

• Transmit Power Scaling Law
  • Reduced as $1/\sqrt{M}$ per user, but total transmit power might increase
  • Reduced as $1/\sqrt{M}$ per BS antenna → Use handset technology?

• Designing Networks for Energy Efficiency
  • Large cells: First step is to reduce cell size
  • Smaller cells: Transmit power only a small part → Use Massive MIMO
  • Intuition: Suppress interference, share circuit power over many users
  • Non-universal pilot reuse is important in random deployments
  • Several Mbit/Joule achieved without coordination
Massive MIMO and HARDWARE EFFICIENCY
Many Antennas and Transceiver Chains

- Many Antenna Elements
  - LTE 4-MIMO: $3 \cdot 4 \cdot 20 = 240$ antennas
    - But only 12 transceiver chains!
  - Massive MIMO = $M$ transceiver chains

- End-to-end Channels
  - Wireless propagation channel
  - Transceiver hardware
  - Simple model:

Can We Afford $M$ High-Grade Transceiver Chains?
Can Massive MIMO utilize the hardware components more efficiently?
Orthogonal frequency-division multiplexing (OFDM)

- Transmitter

- Receiver

Main Components
Filters, I/Q mixers, DACs, ADCs, oscillators

Modeling of Hardware Impairment

- Real Transceivers have Hardware Impairments
  - Ex: Phase noise, I/Q-imbalance, quantization noise, non-linearities, etc.
  - Each impairment can be modeled (for given hardware, waveform etc.)
  - But: Impact reduced by calibration and only combined effect matters!

More impairments = Lower price, lower power, smaller size

- High-Level Hardware Model:

  Input Signal → Non-Linear System (Transmitter Hardware) → Propagation Channel → Non-Linear System (Receiver Hardware) → Output Signal

  - Bussgang’s theorem:
    \[ X \rightarrow \text{Non-Linear System} \rightarrow cX + V \]
    \[ X, V \text{ are uncorrelated Gaussian variables} \]

  - Power loss and phase rotations
  - Additive distortion noise
Classical Impact of Hardware Impairments

- Impact on Point-to-Point MIMO
  - Low SNR: Negligible impact on spectral efficiency
  - High SNR: Fundamental upper limit

**Error Vector Magnitude**

\[
EVM = \frac{\text{Distortion magnitude}}{\text{Signal magnitude}}
\]

*Distortion scales with signal power*

*LTE EVM limits: 8%-17.5%*

What about large $M$ regime?

*Large or small impact?*

*Example: 4x4 point-to-point MIMO, i.i.d. Rayleigh fading*
Distortion Noise: Definition and Interpretation

• Uplink Signal (conventional):
  \[ y = \sum_k g_k x_k + w \]

• Uplink Signal (with impairments):
  \[ y = c^{rx} \sum_k g_k (c_k^{tx} x_k + \xi_k^{tx}) + \xi^{rx} + w \]

**Distortion Noise Model**
Gaussian distributed
Independent between users and antennas

**Error Vector Magnitude**
(at transmitter)
\[ \text{EVM}^{tx} = \sqrt{\frac{\mathbb{E}\{||\xi_k^{tx}||^2\}}{\mathbb{E}\{||c_k^{tx} x_k||^2\}}} \]
What is the Impact of Distortion Noise?

**Uplink Single-User Scenario**
Rayleigh fading, SNR = 5 dB

**Observations**
*Ideal*: SE = \(\mathcal{O}(\log M)\)
*Non-ideal*: Asymptotic limits

Higher EVM → Lower limit

**Observations**
Impairments caused by user device determine the limit
Distortion noise caused by BS averages out as \(M \to \infty\)
(cf. inter-user interference)
Multi-Cell Scenario with Distortion Noise

Uplink Multi-Cell Scenario
Rayleigh fading, SNR = 5 dB
K = 8 users per cell
MR detection

Hardware Scaling Law
If BS distortion variance increases as $M^\kappa$ for $\kappa \leq 1/2$:
SE will not go zero as $M \to \infty$

Can be proved rigorously!

Observations
Small loss if law is followed
Otherwise large loss!
Utilizing the Hardware Scaling Law

- Massive MIMO can use Lower-Grade Hardware
  - Reduced cost, power consumption, and size

- Example: Analog-to-Digital Converter (ADC)
  - One $b$-bit ADC per Transceiver Chain

- Adds quantization noise roughly proportional to $2^{-2b}$:
  \[
  \sqrt{M} = c_0 \cdot 2^{-2b} \Rightarrow b = \frac{1}{2} \log_2(c_0) - \frac{1}{4} \log_2(M)
  \]
  Ex: $M = 256$ requires 2 fewer bits than $M = 1$ (even 1-bit ADCs possible)

- Circuit power roughly proportional to $2^{2b}$:
  Ex: Power of $M$ ADCs can scale as $\sqrt{M}$ rather than $M$
Interference Visibility Range

• Only Remaining Interference as $M \to \infty$:
  • Pilot contamination (reuse of pilot resources)
  • Hardware impairments (at user devices)

• Distortion Noise as Self-interference
  • Limits the visibility of inter-user interference

No reason to suppress inter-user interference below self-interference!
Summary

• Any Transceiver is Subject to Hardware Impairments
  • Massive MIMO is resilient to such imperfections
  • Distortion variance at BS may increase as $\sqrt{M}$
  • High-grade BS hardware is not required!
  • User hardware quality is the fundamental limitation

• Further Remarks
  • Analysis with more detailed hardware models show same behavior
  • Phase noise is not worse than in small MIMO systems
  • Reduced transmit power and relaxed impairment constraints
    $\rightarrow$ New compact transceiver designs?
Part 4

OPEN PROBLEMS
Open Problems and Active Research Topics

1. Channel measurements and modeling
2. Circuit and transceiver design
3. Implementation-aware algorithmic design
4. Dealing with hardware impairments and reciprocity calibration
5. Exploiting $M - K$ excess degrees of freedom
6. FDD operation for “low mobility” or “highly structured channels”
7. MAC-layer design, power control, and scheduling
8. Control signaling and BS transmission without CSI
9. New deployment scenarios (e.g., distributed arrays or cell-free)
10. Mitigation of pilot contamination
11. System-level studies and coexistence with HetNets or D2D
12. Massive MIMO in millimeter wave bands
SUMMARY
Summary

• Massive MIMO has Many Extraordinary Benefits

  • **High spectral efficiency**: >20x gains over IMT-Advanced are foreseen
    • High SE per cell, but modest per user
    • Important: Non-universal pilot reuse, pilots use large part of coherence interval

  • **High energy efficiency**: Tens of Mbit/Joule are foreseen
    • Reduced transmit power per user and antenna, maybe not per cell
    • Circuit power dominates power consumption in urban scenarios
    • Important: Interference control, sharing circuit power between users

  • **High hardware efficiency**: High-grade hardware is not needed
    • Variance of distortion noise at BS can scale with number of antennas
    • Important: Quality of user device is the limiting factor
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Bringing an Extraordinary Technology to Reality

- FP7 MAMMOET project (Massive MIMO for Efficient Transmission)
  - Bridge gap between “theoretical and conceptual” Massive MIMO
  - Develop: Flexible, effective and efficient solutions

WP1 System approach, scenarios and requirements

WP2 Efficient FE solutions (IC solutions, Comp/Calibration)

WP3 Baseband Solutions (Algorithms, Architectures & Design)

WP4 Validation and proof-of-concept
Seminal and Overview Papers


Key References (2/4)

Spectral Efficiency


Energy Efficiency


Key References (4/4)

Hardware Efficiency


QUESTIONS?

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