Optimal Coordinated Beamforming in the Multicell Downlink with Transceiver Impairments

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Introduction
Coordinated Beamforming

- Downlink Multicell Transmission
  - $N$ Base Stations
  - $K$ Users per Cell

- Universal Frequency Reuse
  - Common Narrowband Frequency Resource
  - Limiting Factor: Inter-User Interference

- $N_t$-Antenna Base Stations
  - Beamforming: Spatially Directed Signals
  - Lower Interference
Optimization of Beamforming

- Optimize System Utility
  - Many Possible Problem Formulations

- Two Main Categories of Optimization Problems

  - Convex Problems
    - Solvable in Practice (polynomial time)
    - Examples: Minimize power under rate constraints
                 Maximize (weighted) worst-user rate

  - Non-Convex Problems
    - Infeasible in Practice (exponential time)
    - Approximations Necessary
    - Examples: Weighted sum rate, Proportional fairness

Focus in this Paper: Convex Problems

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Common Unrealistic Assumptions

• Unrealistic Assumptions Enable Analysis
  - Is Convexity Lost Otherwise?

• Assumption: Perfect Channel Knowledge
  - Impractical: Estimation errors, feedback quantization, delays
  - Treated by Robust Optimization (convexity remains)

• Assumption: Centralized Optimization
  - Impractical: Limited backhaul, local computational resources
  - Handled by Primal/Dual Decomposition (convexity remains)
Other Common Unrealistic Assumptions?

- **Ideal Hardware is Commonly Assumed**
  - Physical Transceivers Suffer From Impairments
  - Examples: Non-linear amplifiers, IQ imbalance, phase noise, carrier-frequency offset, quantization noise, etc.

- **Degrading Impact on Transmission and Reception**
  - Mismatch Between Ideal and Actual Signal
  - Distortion Power is Proportional to Signal Power
Transceiver Impairments
Transceiver Hardware Impairments

• Commonly Ignored in Beamforming Optimization
  - A Few Papers on Single-User Systems
  - Minor Impact on Single-User Low-Rate Transmission
  - Major Impact on
    1) High-rate transmission
    2) Inter-user interference
    3) Low-cost transceivers

• Exact Modeling
  - Separate distortion model of each component
  - Accurate but very hardware dependent

• Simplified Modeling
  - Combined distortion model of all components
  - Accurate for residual distortion after calibration

Focus in this Paper
Generalized System Model

- Parameters for User $j$ in Cell $i$
  - Information Symbol: $x_{i,j} \sim \mathcal{CN}(0, 1)$
  - Linear Beamforming: $w_{i,j} \in \mathbb{C}^{N_t \times 1}$
  - Beamforming from Cell $i$: $W_i = [w_{i,1} \ldots w_{i,K}] \in \mathbb{C}^{N_t \times K}$
  - Channel from Cell $m$: $h_{m,i,j} \in \mathbb{C}^{N_t \times 1}$

- Received Signal at User $j$ in Cell $i$
\[
y_{i,j} = \sum_{m=1}^{N} h_{m,i,j}^H \left( \sum_{k=1}^{K} w_{m,k}x_{m,k} + z_{m}^{(t)} \right) + z_{i,j}^{(r)}
\]

Transmitter distortion  Receiver distortion

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Characterization: Receiver Distortion

- Well-Modeled as Complex Gaussian: \( z_{i,j}^{(r)} \sim \mathcal{CN}(0, \sigma_{i,j}^2) \)
  - Aggregation of Many Impairments
  - Previously Verified by Measurements and Analysis

**Example:** \( \kappa_3 \): Ratio of distortion to signal in percentage \((0 \leq \kappa_3 \leq 15)\)
- Smaller is Better

Received signal magnitude

\[
\sigma_{i,j}^2 = \sigma^2 + \nu^2 \left( \sum_{m=1}^{N} \| h_{m,i,j}^H W_m \|_F^2 \right)
\]

Increasing convex function

\[
\nu(x) = \frac{\kappa_3}{100} x
\]
Characterization: Transmitter Distortion

- Also Well-Modeled as Gaussian: \( z_m^{(t)} \sim \mathcal{CN}(0, C_m) \)
  - Linear with signal at low power
  - Faster than linear at high power

\[
C_m = \begin{bmatrix}
  c_{m,1}^2 \\
  \vdots \\
  c_{m,N_t}^2
\end{bmatrix}, \quad c_{m,n} = \eta\left(\|T_n W_m\|_F\right)
\]

- Example: \( \eta(x) = \frac{\kappa_1}{100} x \left(1 + \left(\frac{x}{\kappa_2}\right)^4\right) \)
  - \( \kappa_1 \): Base-level of distortion \((0 \leq \kappa_1 \leq 15)\)
  - \( \kappa_2 \): Dynamic range of power amplifier \((5^{th} \text{ order non-lin})\)
Optimization of Coordinated Beamforming
SINR ExpressionS

- Signal-to-interference-and-noise ratio of User $j$ in Cell $i$:

$$\text{SINR}_{i,j} = \frac{\left| h_{i,i,j}^H w_{i,j} \right|^2}{\sum_{l \neq j} |h_{i,i,j}^H w_{i,l}|^2 + \sum_{m \neq i} \| h_{m,i,j}^H W_m \|_F^2 + \sum_{m,n} (h_{m,i,j}^H T_n h_{m,i,j}) t_{m,n}^2 + r_{i,j}^2 + \sigma^2}$$

Useful signal

- Intra-cell interference
- Inter-cell interference
- Transmitter distortion
- Receiver distortion

- Extra variables:
  $$\eta(\| T_n W_m \|_F) \leq t_{m,n} \quad \forall m, n$$
  $$\nu\left(\sqrt{\sum_{m} \| h_{m,i,j}^H W_m \|_F^2}\right) \leq r_{i,j} \quad \forall i, j$$

- Should be equality
- If $t_{m,n}, r_{i,j}$ are seen as variables: Equality in optimal solution
Convexity is Retained

• Minimize Power under SINR Constraints: $\text{SINR}_{i,j} \geq \gamma_{i,j}$

• Theorem: Solvable as Convex Optimization Problem

$$\begin{align*}
\text{minimize} & \quad \beta, \mathbf{W}_i, t_{i,n}, r_{i,j}, \forall i, j, n \\
\text{subject to} & \quad t_{i,n} \geq 0, \quad r_{i,j} \geq 0, \quad \Re(h_{i,i,j}^H \mathbf{w}_{i,j}) = 0 \quad \forall i, j, n, \\
& \quad \text{tr}(\mathbf{W}_i^H \mathbf{Q}_{i,k} \mathbf{W}_i) + \sum_n \text{tr}(\delta \mathbf{Q}_{i,k} \mathbf{T}_n) t_{i,n}^2 \leq \beta q_{i,k} \quad \forall i, k, \\
& \quad \sqrt{\sum_m \|h_{m,i,j}^H \mathbf{W}_m\|_F^2 + \sum_{m,n} (h_{m,i,j}^H \mathbf{T}_n h_{m,i,j}) t_{m,n}^2 + r_{i,j}^2 + \sigma^2} \leq \sqrt{1 + \frac{1}{\gamma_{i,j}}} \Re(h_{i,i,j}^H \mathbf{w}_{i,j}) \quad \forall i, j, \\
& \quad \eta(\|\mathbf{T}_n \mathbf{W}_m\|_F) \leq t_{m,n} \quad \forall m, n, \\
& \quad \nu(\sqrt{\sum_m \|h_{m,i,j}^H \mathbf{W}_m\|_F^2}) \leq r_{i,j} \quad \forall i, j.
\end{align*}$$

Main Point: Convexity is Retained Under Transceiver Impairments
Generalization of Optimization Problems

- (P1): Minimize Power under SINR/Rate Constraints
  - Convex Optimization Problem

- (P2): Maximize Worst-User Rate
  - Solved as Sequence of (P1)-Problems
  - (Quasi-)Convex Optimization Problem
Numerical Examples
Simulation Scenario

- Maximize Worst-User Rate (Max-Min Fairness)

- Two Schemes:
  - Optimal Beamforming with Transceiver Impairments
  - Distortion-Ignoring Optimized Beamforming

- Simulation Scenario
  - 2 Base Stations
  - 3GPP LTE Case 1
Average Max-Min User Rate

• Parameters
  - $N_t = 4$ antennas/BS, $K = 2$ users/cell
  - X-axis: $\kappa_1 = \kappa_3 = \text{EVM in } \% \text{ at transmitter/receiver}$

Conclusion: Smaller loss when optimized for impairments
Impact on Multiplexing Gain

- Parameters
  - $N_t = 8$ antennas/BS, $K = 4$ users/cell
  - X-axis: Transmit Power

Conclusion: Finite High-SNR Limit (No multiplexing gain)
Summary
Summary

• Transceiver Impairments
  - Physical Transceivers are Not Perfect
  - Small Impact in the Past
  - Major Impact in the Future: High spectral efficiency
    Small inter-user interference

• Contributions
  - Tractable Mathematical Formulation
  - Minimize Power under SINR Constraints – Convex Problem
  - Maximize Worst-User Rate – Convex Problem

• Observations
  - Optimization Makes Degradations Much Smaller
  - Finite High-SNR Limit – No Multiplexing Gain
Additional Work

• Extension to General Multi-Cell Scenarios

• Analysis of Finite High-SNR Limit
  - Multiplexing is Very Useful – Although Multiplexing Gain is 0
Thank You for Listening!

Questions?

All Papers Available:
http://flexible-radio.com/emil-bjornson
Power Constraints

- Arbitrary Power Constraints in Cell $i$
  - Constraints:
    \[
    \text{tr}(W_i^H Q_{i,k} W_i) + \sum_n \text{tr}(\delta Q_{i,k} T_n) t_{i,n}^2 \le q_{i,k} \quad \forall i, k,
    \]
    Positive semi-definite shape matrix  Positive limit

- $0 \le \delta \le 1$ defines the extra power consumed by distortions

- Examples:  Per-antenna constraints
              Per-cell constraints
              Soft-shaping constraints