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Optimality Properties and Low-Complexity Solutions to Coordinated Multicell Transmission
Outline

• Introduction: Multicell Transmission

• How to measure performance?
  - Weighted sum performance
  - Simplified convex problem

• Common Optimality Properties
  - Power allocation and beamforming structure

• Low-Complexity Solution
  - Approximation suitable for distributed precoding
  - Evaluated on measured multicell channels
Intro: No Coordination

- Non-Cooperative Multicell Downlink
  - Conventional single cell processing
  - Interference at cell edge users uncontrollable
  - Can be improved by coordinating interference
Intro: Full Coordination

• Centralized Cooperative Multicell Downlink
  - Backhaul network and central station (CS)
  - Centralized processing as “one cell”
  - Impractical?
Intro: Dynamic Clusters

- Practical Coordination Structure
  - $C_k$ = Coordinate interference to terminals
  - $D_k$ = Data transmission to terminals
  - Limits sharing of data and channel knowledge (full/no coordination are special cases)
System Model

• Assumptions
  - $K_t$ base stations with $N_t$ antennas
  - $K_r$ single-antenna user terminal
  - Channel from all BSs to MS$_k$: $\mathbf{h}_k = \begin{bmatrix} h_{1k} \\ \vdots \\ h_{K_t k} \end{bmatrix}$
  - BS$_j$ knows $h_{jk}$ for $k \in C_j$

• Data Transmission
  - Signal vector to MS$_k$: $\mathbf{s}_k$
  - Received signal: $y_k = h_k^H \mathbf{c}_k \sum_{\bar{k}=1}^{K_r} \mathbf{D}_{\bar{k}} \mathbf{s}_{\bar{k}} + n_k$
    - Sorts out signals from coordinating BSs
    - Sorts out transmit antennas
    - Noise and distant interference
System Model (2)

- Transmission Strategies
  - Signal correlation matrix: \( S_k = \mathbb{E}\{s_k s_k^H\} \)
  - Arbitrary rank

- Power Constraints
  - Per base station: \( \sum_{k \in \mathcal{D}_j} \text{tr}\{D_{jk} S_k D_{jk}^H\} \leq P_j \)
    Sorts out antennas of BS\(_j\)
  - Models hardware, regulations, economy, etc.
  - Extension to arbitrary constraints: [Björnson2011]
How to Measure Performance?

- **User Performance Measure:** $R_k$
  - Increasing function of SINRs (depend on $S_k = \mathbb{E}\{s_k s_k^H\}$)
  - E.g., data rate, bit error rate, MSE, etc.

- **System Performance:** Weighted Sum
  maximize $\mu_1 R_1 + \mu_2 R_2 + ...$
  $s_1, s_2, ...$
  - Finds Pareto optimal points
  - Fairness depends on $\mu_1, \mu_2, ...$
(P1): Weighted Sum Maximization

**• Optimization Problem**

\[
\begin{align*}
\text{maximize} & \quad \sum_{k=1}^{K_r} \mu_k R_k(S_1, \ldots, S_{K_r}, \sigma_k^2) \\
\text{subject to} & \quad \sum_{\tilde{k} \in D_j} \text{tr}\{D_{j\tilde{k}} S_{\tilde{k}} D_{j\tilde{k}}^H\} \leq P_j \quad \forall j \\
& \quad S_k \succeq 0 \quad \forall k.
\end{align*}
\]

**• Difficult Problem**
- Non-convex and NP-hard
- Find structure of the optimal solution?
(P2): Quality of Service Constraints

- Given Performance/QoS Point \((\gamma_1, \ldots, \gamma_{Kr})\)
  - Find strategy: \(R_k \geq \gamma_k, \forall k\)

- What to Optimize?
  - Minimize power? \(\Rightarrow\) Solution may use too much power!
  - Instead: Best possible under power constraints

- Solution: Optimize worst noise \(\alpha^2 \sigma_k^2\) that can be handled
  - \(\alpha\)-parameter: Constraints \(R_k \geq \gamma_k\) satisfied if \(\alpha \geq 1\)

\[
\begin{align*}
\text{(P2):} & \quad \max_{S_1, \ldots, S_{Kr}, \alpha} \quad \alpha \\
\text{subject to} & \quad R_k(S_1, \ldots, S_{Kr}, \alpha^2 \sigma_k^2) \geq \gamma_k, \forall k, \\
& \quad S_k \succeq 0, \quad \sum_{k \in D_j} \text{tr}\{D_{jk} S_k D_{jk}^H\} \leq P_j, \forall j,k.
\end{align*}
\]
Connection: (P1) and (P2)

- (P2) solves “half” the original problem!

- Price for convexity
  - Need to know optimal user QoS!

- (P1) and (P2): Common Properties
  - Equal if optimal performance of (P1) are constraints in (P2)
  - Properties of (P2) that holds for any \( (\gamma_1, \ldots, \gamma_{K_T}) \)
  - These also holds for (P1)!
Optimality Property 1

Exists optimal solutions with

1. Full power usage (if $|C_k| \leq N_t$)
2. Single-stream beamforming (i.e., $S_k = w_k w_k^H$)

• Intuitive Results – Non-trivial Proofs
  - Insufficient antennas: Power should be limited
  - Multi-stream solutions exists in special cases

• Allows Simplifications
  - Use total power at all transmitters
  - No SIC-receivers or vector coding required
Optimality Property 2

Uplink-downlink duality for (P2)
- Based on Lagrange duality theory
- Transmit beamformers ⇔ Receive filters

• Motivation
- Easier to solve uplink problems
Beamforming Parametrization
- Optimal strategies \( S_k = w_k w_k^H \) satisfy

\[
    w_k = c_k \left( \sum_j a_j D_{j,k} + \sum_{\bar{k} \neq k} b_{\bar{k}} D_{\bar{k}}^H C_{\bar{k}}^H h_{\bar{k}} h_{\bar{k}}^H C_{\bar{k}} D_{\bar{k}} \right)^{-1} D_{\bar{k}}^H h_{\bar{k}}
\]

for some parameters \( a_j, b_{\bar{k}} \in [0, 1] \).

- **Optimal Strategies**
  - Depends on \( K_i + K_r \) parameters
  - Power allocation \( c_k \) also function of these

- **New Approach: Find Good Parameters**
  - Iterative search
  - Heuristic selection – Easy to find good ones!
Simple Distributed Strategy

• Motivation: Centralized Solutions Require
  - Much backhaul signaling (CSI, data, sync)
  - High Computational Resources

• Distributed Low-Complexity Solution:
  - Select parameters $a_j$, $b_k$ in Property 3 heuristically
  - Calculate independently on each BS

• Result
  - Distributed Virtual SINR (DVSINR) Beamforming
  - Tailored for weighted sum performance
Measurement-Based Evaluation

- Multicell Channel Measurements
  - Realistic urban scenario in Stockholm
  - Correlation between BSs (usually ignored)
  - Two sectorized 4-antenna BSs

BS: Rooftops
MS: Street level
(4 users)
Measurement-Based Evaluation (2)

• Weighted Sum Rate
  - Data rate: \( R_k(\cdot) = \log_2(1 + \text{SINR}_k) \)
  - Proportional fairness

• Precoding Schemes
  1. Optimal Precoding
  2. Modified Optimal Precoding: \( |\sum_j \text{inter.}_j|^2 \rightarrow \sum_j |\text{inter.}_j|^2 \)
     (No interference cancellation between BSs)
  3. DVSINR – Multicell (data from both BSs)
  4. DVSINR – Single-cell (date from one BS)
  5. Single-cell processing
• **Average Weighted Sum Rate**
  - Large gain with interference coordination
  - Small gain with joint data: Both DVSINR approaches good

![Graph showing Average Weighted Sum Rate vs. Output Power per subcarrier and BS (dBm)](image)

**Gap:**
- Interference Cancellation between BS
- Unreasonable In Practice!

**Single-cell processing:**
- Bounded performance
Measurement-Based Evaluation (4)

• Average User Rates
  - Large improvements for cell edge terminals
  - Not all terminals benefit from multicell coordination

![Graph showing average user rates vs. output power per carrier and BS in dBm]
Summary

• Interference Limits Multicell Performance
  - Managed by multicell coordination

• Optimization: Weighted Sum Performance
  - Full power usage and single-stream beamforming
  - Simple parametrization of optimal beamforming

• Distributed Approximation: DVSINR
  - Heuristic use of parameterization

• Measurement-based Multicell Evaluation
  - Interference coordination greatly improves performance
  - Important measurement observations:
    • Not all terminals benefit from multicell coordination
    • Small practical benefit with joint data transmission
References

• Journal version
  - Includes multicarrier and arbitrary power constraints


• Previous work


Thank You for Listening!

Questions?

Papers and Presentations Available:
http://www.ee.kth.se/~emilbjo