A Multi-cell MMSE Precoder for Massive MIMO Systems and New Large System Analysis

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Background & System Model
Evolving Cellular Networks

• Cellular Network Architecture
  • Area divided into cells
  • One fixed base station (BS) serves all the users
  • Demand increases by 30-40% per year!

• Network Throughput [bit/s/km²]
  • Consider a given area

• Simple Formula for Network Throughput:

\[
\text{Throughput} = \frac{\text{Available spectrum}}{\text{Cell density}} \cdot \frac{\text{Spectral efficiency}}{\text{bit/s/Hz/Cell}}
\]

\[
\text{Throughput} = \frac{\text{bit/s/km}^2}{\text{Hz}} \cdot \frac{\text{Cell/km}^2}{\text{bit/s/Hz/Cell}}
\]

 Dominant factors in the past!  

5G opportunity: Improve spectral efficiency
Aggressive Spatial Spectral Reuse

• Needed: Many active users per km$^2$
  • Interference Management is Key
  • “Small cells” are interference limited

• Massive MIMO
  • BSs with many antennas: $M$
  • Multiplexing of many users: $K$
  • Important: $M \gg K$
  • Uplink: Linear detection
  • Downlink: Linear precoding

Array gain gives signal gain without causing more interference!
Massive MIMO Transmission Protocol

- **Coherence Blocks**
  - Fixed channel responses
  - Coherence time: $T_c$ s
  - Coherence bandwidth: $W_c$ Hz
  - Depends on mobility and environment
  - Block length: $S = T_c W_c$ symbols
  - Typically: $S \in [100,10000]$ symbols

- **Time-Division Duplex (TDD)**
  - Downlink and uplink on all frequencies
  - $B$ symbols/block for uplink pilots – for channel estimation
  - $S - B$ symbols/block for uplink and/or downlink payload data
Channel Acquisition in Massive MIMO

- Limited Number of Pilots: $B \leq S$
  - Must use same pilot sequence in multiple cells
  - Base stations cannot tell some users apart: *Essence of pilot contamination*

- Coordinated Pilot Allocation
  - Allocate pilots to users to reduce contamination
  - Scalability → No signaling between BSs

- Solution: Non-universal pilot reuse
  - Pilot reuse factor $\beta \geq 1$
  - Users per cell: $K = B / \beta$
  - Higher $\beta$ → Fewer users per cell, but reuse in fewer cells

\[ \text{Reuse } \beta = 1 \quad \text{Reuse } \beta = 3 \quad \text{Reuse } \beta = 4 \]
**Single-Cell Linear Detection**

*Inactive* interference suppression

- MF: Maximize channel gain

*Active* interference suppression

- ZF: Minimize interference
- MMSE: Maximize SINR

Commonly used also for:

- Multi-cell cases
- Downlink precoding

But what is optimal?
Multi-Cell MMSE Detection

- **Optimal: Multi-Cell MMSE Detection**

\[
g_{jk} = \left( \sum_{l,m} p_{lm} \hat{h}_{jlm} \hat{h}_{jlm}^H + E_j + \sigma^2 I \right)^{-1} \hat{h}_{jjk}
\]

- **Suppress both intra- and inter-cell interference**

References:
Main Contributions:
Multi-Cell Precoding
The same rates are achievable in the uplink and the downlink if

- Detection and precoding vectors are the same
- Same sum power, but optimized power allocation
Proposal: Multi-Cell MMSE Precoding

- Multi-Cell MMSE Precoding

\[ w_{jk} = \frac{1}{\lambda_{jk}} g_{jk} = \frac{1}{\lambda_{jk}} \left( \sum_{l,m} p_{lm} \hat{h}_{jlm} \hat{h}_{jlm}^H + E_j + \sigma^2 I \right)^{-1} \hat{h}_{j,k} \]

**Features:**

Optimality property from uplink-downlink duality

Only requires “local” channels to BS \( j \): Scalable!

Supports imperfect CSI with arbitrary pilot allocation

Estimated desired channel

Uplink powers

All estimated channels

Sum of estimation error covariance matrices
Property 1: Parallel Channel Estimates

- **System Model**
  - Channel from BS $j$ to user $m$ in cell $l$
    
    \[
    h_{jlm} \sim CN(0, d_{jlm} I_M)
    \]
  - Uplink transmit power: $p_{lm}$
  - Pilot sequence: $v_{ilm} \in \{v_1, ..., v_B\}$

- **Uplink Pilot Transmission**

    \[
    Y_j = \sum_l \sum_m \sqrt{p_{lm}} h_{jlm} v_{ilm}^T + N_j
    \]

- **Estimate of $h_{jlm}$**:
  \[
  \hat{h}_{jlm} = \frac{\sqrt{p_{lm} d_{jlm}}}{\sum_l \sum_k p_{lk} d_{jlm} v_{ilk}^H v_{ilm} + \sigma^2} Y_j v_{ilm}^*
  \]

  *User-specific scalar*

  There are only $B$ channel directions to estimate!
Property 2: Simplified Precoder Expression

- **Paradigm:** Listen to all pilot sequences
  - \( B \) estimated directions:
    \[
    \hat{H}_{\nu,j} = Y_j [v_1^* \ldots v_B^*]
    \]

- **Multi-Cell MMSE Precoding**
  \[
  w_{jk} = \frac{1}{\lambda_{jk}} \left( \sum_{l,m} p_{lm} \hat{h}_{jlm} \hat{h}_{jlm}^H + E_j + \sigma^2 I \right)^{-1} \hat{h}_{jjk}
  \]
  \[
  = \frac{1}{\lambda_{jk}} \left( \hat{H}_{\nu,j} \Lambda_j \hat{H}_{\nu,j}^H + (\sigma^2 + \phi_j) I \right)^{-1} \hat{h}_{jjk}
  \]

- **Reuse** \( \beta = 3 \)

- **Diagonal matrix:**
  Uplink powers and scalars from estimates

- **Sum of estimation error variances**

**Much easier to implement and analyze!**
Property 3: Achievable Downlink Performance

- Achievable Spectral Efficiency

\[ R_{jk} = \left( 1 - \frac{B}{S} \right) \log_2(1 + \eta_{jk}) \]

with \[ \eta_{jk} = \frac{\rho_{jk} \left| \mathbb{E}\{ h_{jjk}^H w_{jk} \} \right|^2}{\sum_{l,m} \mathbb{E}\left\{ |h_{ljk}^H w_{lm}|^2 \right\} - \rho_{jk} \left| \mathbb{E}\{ h_{jjk}^H w_{jk} \} \right|^2 + \sigma^2} \]

**Large-Scale Approximation:** \( \eta_{jk} - \tilde{\eta}_{jk} \to 0 \) as \( M, K \to \infty \) with a fixed ratio,

\[ \tilde{\eta}_{jk} = \frac{\rho_{jk} \rho_{jk} d_{jjk}^2 \frac{\delta_{jk}^2}{\theta_{jk}}}{\sum_{(l,m) \neq (j,k)}: \text{same pilot} \rho_{lm} p_{jk} d_{ljk}^2 \frac{\delta_{lm}^2}{\vartheta_{lm}} + \sum_{(l,m)}: \text{diff pilot} \frac{\rho_{lm} d_{ljk} \mu_{ljk} m}{M} \frac{\sigma^2}{\vartheta_{lm}} + \sigma^2} \]

Computed using semi-closed form expressions from random matrix theory
Simulations
Simulation Setup

- Hexagonal Network: 19 cells with wrap-around
  - Random user locations: $> 0.14 \cdot \text{radius}$
  - Shadowing: 2.2 dB standard deviation
  - Coherence block: $S = 500$
  - Pilot reuse patterns: $B = fK$

- Uplink channel-inversion power control:
  $$p_{jk} = \rho_{jk} = \frac{P}{d_{jk}}$$

  Effective SNR $P/\sigma^2$, here: 0 dB

- Downlink equal power control:
  Give $-3$ dB at cell edge (ignoring shadowing)
Reuse Factors and Large-Scale Approximations

![Graph showing achievable sum SE per cell (bit/s/Hz) vs. number of antennas for different approximation and simulation cases.](image-url)
Comparison of Different Precoding Schemes

\[ \beta = 4 \]
Summary
Summary

• Proposal: Multi-Cell MMSE Precoding
  • Motivated by uplink-downlink duality
  • All channels = Only $B$ pilots to estimate channels from
  • Large-system performance analysis
  • Substantial gains over single-cell precoders

• Additional results
  • Large-scale approximation enables efficient power control optimization
Questions?

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Slides and papers available online:
http://www.commsys.isy.liu.se/en/staff/emibj29