Beamforming and Quantized Feedback in Spatially Correlated Multi-User MIMO Systems

Emil Björnson
PhD Student in Telecommunications
Royal Institute of Technology (KTH)
Sweden
Biography

- Born 1983 in Malmö, Sweden.
- Master Thesis: “*Beamforming Utilizing Channel Norm Feedback in Multiuser MIMO Systems*”
- PhD Student in Telecommunications at *Royal Institute of Technology (KTH)*, since Feb 2007.
- Two publications: SPAWC’07, ICASSP’08.
Outline

- Introduction to a Multi-user System
- How To Choose Performance Measure?
- Receive Beamforming with Subspace Cancellation
- Quantization of the Channel Norm
- Performance Evaluation
- Conclusions
INTRODUCTION
TO A MULTI-USER SYSTEM
Downlink multi-user system

- Downlink of multi-antenna system
  - Elevated base station, $n_T$ antennas.
  - Multiple users, $n_R$ antennas.
Spatial Correlation

- Some spatial directions are statistically more favorable for a given user:

- Excellent for simultaneous transmission to several users (beamforming, SDMA).
System Model

- Urban environment, elevated base station
  - Spatially correlated transmitter.
  - Independent receive antennas.

- Channel model:
  - Rayleigh fading multi-antenna channel to user $k$
    \[
    H_k = [h_{k,1}, \ldots, h_{k,n_R}]^H \in \mathbb{C}^{n_R \times n_T},
    \]
    with independent rows $h_{k,i} \in \mathcal{CN}(0,R_k)$. 

Received signal

- Transmission of $s_k$ to user $k$.

$$y_k(t) = v_k^H H_k \left( \sqrt{p_k} w_k s_k(t) + \sum_{j \neq k} \sqrt{p_j} w_j s_j(t) \right) + n_k(t)$$

- Transmit beamformer: $w_k \in \mathbb{C}^{n_T}$
- Receive beamformer: $v_k \in \mathbb{C}^{n_R}$

- Transmitter knows statistics.
- Receiver $k$ knows $H_k$ and the statistics.
HOW TO CHOOSE PERFORMANCE MEASURE?
How to optimize performance?

- We need a performance measure:
  - Maximize sum of data rates of all users?
  - Maximize the minimal rate among all users?

- What about fairness?

- The measure depend on the application:
  - Balance between throughput and fairness.
Signal-to-interference/noise ratio

- Achievable rate depends on the SINR:

$$\text{SINR}_k = \frac{p_k \| v_k^H H_k w_k \|^2}{\sum_{i \neq k} p_i \| v_k^H H_k w_i \|^2 + \sigma_k^2}.$$

- Tricky to maximize the SINR:
  - Transmit beamformer $w_k$ affects all users.
  - Transmitter only knows the channel statistics.
  - Receiver is unaware of the other users’ channels.
A suboptimal approach

- It’s not enough to maximize the SINR
  - Robust estimation necessary at transmitter
- The effect of the receive beamformer should be predictable at transmitter.
- Next, a suboptimal approach is proposed
  - Clear framework for interference suppression.
  - Based on my paper from ICASSP’08.
RECEIVE BEAMFORMING WITH SUBSPACE CANCELLATION
Subspace Partitioning

- Partitioning of covariance matrix $R_k$:

$$R_k = \begin{bmatrix} u_{k}^{(D)} & U_k^{(I)} & U_k^{(0)} \end{bmatrix} \begin{bmatrix} \lambda_1 & \cdots & \lambda_{n_T} \end{bmatrix} \begin{bmatrix} u_{k}^{(D)} & U_k^{(I)} & U_k^{(0)} \end{bmatrix}^H$$

- $u_{k}^{(D)}$ dominating eigenvector (largest eigenvalue).
- $U_k^{(I)}$ eigenvector subspace of non-negligible eigenvalues.
- $U_k^{(0)}$ eigenvector subspace with eigenvalues close to zero.
Proposed Transmission Strategy

• Signal power should be received along the dominating eigenvector:
  \[ w_k = u_k^{(D)} \]

• Interference in \( U_k^{(I)} \) can be mitigated without loss of signal power.

• Receiver can cancel out the \( n_R - 1 \) strongest eigenvalues in \( U_k^{(I)} \).
Illustration of Receive Beamforming

Observe that the eigenvectors are unaffected.
How can it be done in practice?

- We cancel out transmissions within $U_k^{(I)}$ by choosing
  
  $$v_k \in \text{null} \left( (H_k U_k^{(I)})^H \right).$$

- This will make
  
  $$v_k^H H_k U_k^{(I)} = 0.$$

- To make the nullspace one-dimensional:
  
  - Let $U_k^{(I)}$ contain exactly $n_R - 1$ eigenvectors.
With the proposed receive beamforming, the effective channel is

$$\tilde{h}_k^H = \mathbf{v}_k^H \mathbf{H}_k \in \mathcal{CN}(0, \mathbf{Q}_k)$$

- Same eigenvectors in $\mathbf{Q}_k$ as in the original $\mathbf{R}_k$.
- The eigenvalues of $\mathbf{Q}_k$ will be

$$[\lambda_1, 0, \ldots, 0, \lambda_{n_R+1}, \ldots, \lambda_{n_T}]$$

- The distribution is known at the transmitter!
Estimation of the SINR

- With the proposed receive beamforming, the SINR becomes

$$\text{SINR}_k = \frac{p_k \|\tilde{h}_k w_k\|^2}{\sum_{i \neq k} p_i \|\tilde{h}_k w_i\|^2 + \sigma_k^2}.$$ 

- Estimation of signal/interference power can be improved by feedback of $\|\tilde{h}_k\|^2$
  - For large $n_R$, the channel is almost rank one.
  - Statistics tell how $\|\tilde{h}_k\|^2$ is distributed spatially.
Illustration of norm and statistics

- Assume that $v_1$ has larger variance than $v_2$.
- Each circle represents a value of $\sqrt{v_1^2 + v_2^2}$.
- The statistics tell how the power is distributed between $v_1$ and $v_2$.
- The uncertainty reduces with increasing norm.
QUANTIZATION OF THE CHANNEL NORM
Feedback of the channel norm

- Feedback $\|\tilde{h}_k\|^2$ improves SINR estimation.
- Formulas for MMSE estimation of signal/interference powers in my ICASSP’08 paper.
- Example (power in one eigendirection):

$$E\{|v_l|^2|\rho\} = \frac{1}{g_\rho} \left[ \frac{(A_\rho + \lambda_l)e^{-\frac{A_\rho}{\lambda_l}} - (B_\rho + \lambda_l)e^{-\frac{B_\rho}{\lambda_l}}}{\prod_{i \neq l} \left(1 - \frac{\lambda_i}{\lambda_l}\right)} \right. + \sum_{k \neq l} \frac{\lambda_l \left( e^{-\frac{A_\rho}{\lambda_l}} - e^{-\frac{B_\rho}{\lambda_l}} \right) - \lambda_k \left( e^{-\frac{A_\rho}{\lambda_k}} - e^{-\frac{B_\rho}{\lambda_k}} \right)}{(1 - \frac{\lambda_k}{\lambda_l}) \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} \bigg]$$
How to quantize the norm?

- **Limited feedback capacity:**
  - Only a few bits of feedback available per user.

- **Maximum entropy quantization:**
  - Divide probability density of the norm into interval of equal probability.

- **When do we use the norm information?**
  - SINR estimation after user selection.
  - Should use the post-user-selection distribution!
Post-User-Selection Distribution

- Users with strong values of $\|\tilde{h}_k\|^2$ are more probable to be selected.
  - The behavior depends on the scheduler.
  - Hard to derive post-user-selection probability since each user is unaware of other users.

- Possible to derive analytically for:
  - Select the $M$ users (out of $N$) that have the largest CDF values.
Heuristic post-user-selection PDF

- Idea of the heuristic quantization:
  - One interval for really poor channel norms.
  - The rest of the probability mass divided equally.
PERFORMANCE EVALUATION
Simulation System

- **Base Station**
  - 8 antennas in a uniform circular array (UCA).
  - 15 degrees of angular spread.

- **Mobile users**
  - 4 antennas at each user.
  - Uniformly distributed in the cell.
Simulation Model

• Resource Allocation
  • Several users are selected in each time slot (greedily to maximize proportional fairness).
  • Transmit beamformers used to maximize the signal power, under the condition:
    • No interference allowed in other users dominating eigenmodes.
  • Equal power allocation, 10 dB at cell boundary.
Simulation Results

- CDF of the cell throughput with 8 users:

![CDF of the cell throughput with 8 users](image)
Simulation Results

- CDF of the cell throughput with 32 users:

```
<table>
<thead>
<tr>
<th>Cell throughput [bits/slot], 32 users</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Distribution Function</td>
</tr>
</tbody>
</table>
```

Cumulative Distribution Function

0  10  20  30  40  50

0.0  0.2  0.4  0.6  0.8  1.0
Observations

- 1 bit gives 50% of the performance gain, 3 bits gives 90%, 5 bits gives 99%.
- Multi-user opportunistic beamforming needs 3 bits just to choose a beamformer. It is clearly outperformed, even with exact feedback.
CONCLUSIONS
Conclusions

• When statistics are known at base station, transmission should take place along the strongest eigenvector.
• Receiver can cancel out interference in other eigenvector subspaces.
• Feedback of the channel norm makes reliable SINR estimation possible.
• Only a few bits are needed per user.
Some references

• References
  E. Björnson and B. Ottersten, “Exploiting Long-Term Statistics in Spatially Correlated Multi-User MIMO Systems with Quantized Channel Norm Feedback.”

• Even more details are given in an upcoming journal paper.