Massive MIMO for 5G
Recent Theory

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Expectations for 5G Networks

• 5G – Next Network Generation
  • To be introduced around 2020
  • Design objectives are currently being defined

<table>
<thead>
<tr>
<th>5G Performance Metrics</th>
<th>Expectation</th>
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<tbody>
<tr>
<td>Average rate (bit/s/active user)</td>
<td>10-100x</td>
</tr>
<tr>
<td>Average area rate (bit/s/km²)</td>
<td>1000x</td>
</tr>
<tr>
<td>Active devices (per km²)</td>
<td>10-100x</td>
</tr>
<tr>
<td>Energy efficiency (bit/Joule)</td>
<td>1000x</td>
</tr>
<tr>
<td>“Best experience follows you”</td>
<td></td>
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</tbody>
</table>

What is the role of Massive MIMO here?

Source: METIS project (www.metis2020.com)
Outline, Part 2: Recent Theory

- **Spectral Efficiency**
  - Designing Massive MIMO for high spectral efficiency
  - What are the fundamental limits?

- **Energy Efficiency**
  - How is it defined?
  - Is Massive MIMO energy efficient?

- **Hardware Efficiency**
  - Does Massive MIMO require high-grade hardware?
  - Can it make more efficient use of hardware (lower cost, size, and power)?

- **Open Problems**
Massive MIMO and SPECTRAL EFFICIENCY
Evolving Networks for Higher Traffic

- Increase Network Throughput [bit/s/km²]
  - Consider a given area

- Simple Formula for Network Throughput:
  \[
  \text{Throughput} = \frac{\text{Available spectrum}}{\text{Hz}} \cdot \frac{\text{Cell density}}{\text{Cell/km}^2} \cdot \frac{\text{Spectral efficiency}}{\text{bit/s/Hz/Cell}}
  \]

- 5G goal: 1000x improvement

<table>
<thead>
<tr>
<th></th>
<th>More spectrum</th>
<th>Higher cell density</th>
<th>Higher spectral efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokia (2011)</td>
<td>10x</td>
<td>10x</td>
<td>10x</td>
</tr>
<tr>
<td>SK Telecom (2012)</td>
<td>3x</td>
<td>56x</td>
<td>6x</td>
</tr>
</tbody>
</table>

New regulations, cognitive radio, mmWave bands  
Smaller cells, heterogeneous deployments  
Massive MIMO

How many ??x can we expect?
Optimization of Spectral Efficiency

• How Large Spectral Efficiency can be Achieved?
  • Problem Formulation:
    \[ \max_{K, \tau_p} \text{total spectral efficiency} \quad [\text{bit/s/Hz/cell}] \]
    for a given \( M \) and \( \tau_c \).

• Issue: Hard to find tractable expressions
  • Interference depends on all users’ positions!
  • Expressions from before: Fixed and explicit pathloss values (\( \beta \))
  • We want quantitative results – averaged over user locations

• Solution: Make every user “typical”
  • Constant SNR: Power control inversely proportional to pathloss
  • Inter-cell interference: Code over variations in user locations in other cells
Symmetric Multi-Cell Network

- Classic Multi-Cell Network
  - Infinite grid of hexagonal cells
  - $M$ antennas at each BS
  - $K$ active users in each cell
  - Same user distribution in each cell
  - Uncorrelated Rayleigh fading
  - Statistical channel inversion: $\rho_u \eta_{lk} = \frac{p}{\beta_{lk}}$

Every cell is “typical”

**Propagation Parameters**
(Average interference from cell $l$ to BS $j$)

Compute $\mu_{jl}^{(1)} = \mathbb{E} \left( \frac{\beta_{lk}^j}{\beta_{lk}^l} \right)$ and $\mu_{jl}^{(2)} = \mathbb{E} \left( \frac{\beta_{lk}^j}{\beta_{lk}^l} \right)^2$
Coordinated Pilot Allocation

- Limited Number of Pilots: \( \tau_p \leq \tau_c \)
  - Must use same pilot sequence in several cells
  - Base stations cannot tell some users apart: *Essence of pilot contamination*

- Coordinated Pilot Allocation
  - Allocate pilots to users to reduce contamination
  - Scalability → No signaling between BSs

- Solution: Fractional pilot reuse
  - Pilot reuse factor \( f \geq 1 \)
  - Users per cell: \( K = \frac{\tau_p}{f} \)
  - \( \mathcal{P}_j \) = Cells with same pilots as BS \( j \)
  - Higher \( f \) → Fewer users per cell, but fewer interferers in \( \mathcal{P}_j \)

\[ \begin{align*}
\text{Reuse } f = 1 & \quad \text{Reuse } f = 3 & \quad \text{Reuse } f = 4
\end{align*} \]
Coordinated Precoding and Detection

- Coordinated Multi-Point (CoMP)
  - Avoid causing strong inter-cell interference
  - Scalability \( \rightarrow \) No signaling between BSs
- Solution: Observe and react \( (f \geq 1) \)
  - Listen to pilot signals used only in other cells
  - Utilize to suppress inter-cell interference
  - Schemes: Multi-cell ZF and multi-cell MMSE

MMSE precoding/detection:

\[
v_{lk} = \left( \sum_{j,m} \rho_u \eta_{jm} \hat{g}_{jm} (\hat{g}_{jm}^H) + E_l + I \right)^{-1} \hat{g}_{lk}
\]

- \( \hat{g}_{lk} \) All estimated channels
- \( \hat{g}_{jm} \) Estimation error covariance matrix
Duality Theorem

The uplink SEs are achievable in the downlink using same sum transmit power

Same precoding/detection vectors, but different power allocation

Note: Equivalence between two lower bounds – uplink bound is looser!
Average Spectral Efficiency per Cell

- **Lower Bound on Average Ergodic Capacity in Cell \(j\):**
  \[
  SE_j = K \left( 1 - \frac{\tau_p}{\tau_c} \right) \log_2 \left( 1 + \frac{1}{I_j} \right)
  \]

  *Loss from pilots* \(\tau_c\) \(\tau_p\)

  *SINR* \(I_j\)

- **Interference term depends on processing:**
  \[
  I_j^{\text{MR}} = \sum_{l \in \mathcal{P}_j(f) \setminus \{j\}} \left( \frac{\mu_{jl}^{(2)}}{M} + \left( \frac{\mu_{jl}^{(2)} - (\mu_{jl}^{(1)})^2}{M} \right) \right) + \frac{\left( \sum_{l \in \mathcal{L}} \mu_{jl}^{(1)} K + \frac{1}{\rho} \right)}{M} \left( \sum_{l \in \mathcal{P}_j(f)} \mu_{jl}^{(1)} + \frac{1}{\rho \tau_p} \right)
  \]

  *Pilot contamination*

  *Interference from all cells* \(\frac{1}{\text{Estimation quality}}\)

  *Interference suppression*

  \[
  I_j^{\text{ZF}} = \sum_{l \in \mathcal{P}_j(f) \setminus \{j\}} \left( \frac{\mu_{jl}^{(2)}}{M - K} + \left( \frac{\mu_{jl}^{(2)} - (\mu_{jl}^{(1)})^2}{M - K} \right) \right) + \frac{\left( \sum_{l \in \mathcal{L}} \mu_{jl}^{(1)} K + \frac{1}{\rho} \right)}{M - K} \left( \sum_{l \in \mathcal{P}_j(f)} \mu_{jl}^{(1)} + \frac{1}{\rho \tau_p} \right) - \sum_{l \in \mathcal{P}_j(f)} \left( \frac{\mu_{jl}^{(1)}}{M - K} \right)^2 K
  \]

  *Only term that remains as* \(M \to \infty\): \(\text{Finite limit on SE}\)
Asymptotic Limit on Spectral Efficiency

- Lower Bound on Average Ergodic Capacity as $M \to \infty$:

$$SE_j \to K \left( 1 - \frac{fK}{\tau_c} \right) \log_2 \left( 1 + \frac{1}{\sum_{l \in \mathcal{P}_j(f) \backslash \{j\}} \mu_{jl}^{(2)}} \right)$$

**How Many Users to Serve?**

Pre-log factor $K \left( 1 - \frac{fK}{\tau_c} \right)$ is maximized by $K^* = \frac{\tau_c}{2f}$ users

Maximal SE: $\frac{\tau_c}{4f} \log_2 \left( 1 + \frac{1}{\sum_{l \in \mathcal{P}_j(f) \backslash \{j\}} \mu_{jl}^{(2)}} \right)$

Try different $f$ and $\mathcal{P}_j(f)$ to maximize the limit

**How Long Pilot Sequences?**

$$\tau_p = f K^* = \frac{\tau_c}{2} : \text{ Spend half coherence interval on pilots!}$$
Numerical Results

- Problem Formulation:

\[
\text{maximize } \frac{m}{K, \tau_p} \text{ total spectral efficiency [bit/s/Hz/cell]}
\]

for a given \(M\) and \(\tau_c\).

- Use average spectral efficiency expressions
- Compute average interference \(\mu_j^{(1)}\) and \(\mu_j^{(2)}\) (a few minutes)
- Compute for different \(K\) and \(f\) and pick maximum (< 1 minute)

Assumptions
- Pathloss exponent: 3.7
- Coherence: \(\tau_c = 400\)
- Rayleigh fading
- SNR 5 dB
Asymptotic Behavior: Mean-Case Interference

Observations

- Uniform user distributions
- Asymptotic limits not reached
- Reuse factor $f = 3$ is desired
- $K$ is different for each scheme
- Small difference between optimized schemes
- Coordinated beamforming: Better at very large $M$
Asymptotic Behavior: Worst-Case Interference

Observations

- Interferers at worst positions
- Asymptotic limits not reached
- Reuse factor $f = 4$ is desired
- $K$ is different for each scheme
- Coordinated beamforming: Brings large gains for all $M$
Flexible Number of Users

- SE w.r.t. number of users ($M = 200$ antennas)
- Optimized reuse factors
- Equal SNR (5 dB)

**Observations**

Stable SE for $K > 10$:
Trivial scheduling:
Admit everyone

$M$-ZF, ZF, and MR provide similar per-cell performance

$M/K < 10$ is fine!
Spectral Efficiency per User

- User Performance for Optimized System
  - Optimized reuse factors
  - Equal SNR (5 dB)

Observations
User performance is modest:
BPSK, Q-PSK, or 16-QAM

Schemes for different purposes:
P-ZF > ZF > MR
Anticipated Uplink Spectral Efficiency

Assumptions

ZF processing
Pilot reuse: $f = 3$

Observations

- Baseline: 2.25 bit/s/Hz/cell (IMT-Advanced)
- Massive MIMO, $M = 100$: x20 gain ($M/K \approx 6$)
- Massive MIMO, $M = 400$: x50 gain ($M/K \approx 9$)
- Per scheduled user: $\approx 2.5$ bit/s/Hz
Control Signaling

• Coherent Precoding and Detection Require CSI
  • How to initiate the transmission without array gain?

• User Initiates Transmission
  • Easy: Find an unused pilot and send a transmission request
  • Reserve some pilot sequences for such random access

• BS Initiates Transmission
  • Harder: Must contact the user without having CSI
  • Low-rate space-time coded transmission is feasible
Summary

- Massive MIMO delivers High Spectral Efficiency
  - > 20x gain over IMT-Advanced is foreseen
  - Very high spectral efficiency per cell, not per user
  - Fractional pilot reuse ($f = 3$) is often preferred
  - MR, ZF, M-ZF prefer different values on $K$ and $f$
  - “An order of magnitude more antennas than users” is not needed

- Asymptotic limits
  - Coherence interval ($\tau_c$ symbols) limits multiplexing capability
  - Allocate up to $\tau_c/2$ symbols for pilots
  - We can handle very many users/cell – how many will there be?
Massive MIMO and

ENERGY EFFICIENCY
Energy Consumption

- Network Electricity Consumption
  - Dominated by network infrastructure – increases continuously
  - 1000x higher data rates: Easy to achieve using 1000x more power
    Hard to achieve without using more power
  - Calls for much higher energy efficiency!

What is Energy Efficiency?

- Benefit-Cost Analysis of Networks
  - Systematic approach to analyze strengths and weaknesses of networks

Cost: Power Consumption [Watt = Joule/s]

Benefit: Sum Data Rate [bit/s]

- Definition: Energy Efficiency (EE):

$$EE \ [\text{bit/Joule}] = \frac{\text{Average Sum Rate} \ [\text{bit/s/cell}]}{\text{Power Consumption} \ [\text{Joule/s/cell}]}$$

Contemporary networks: Very inefficient at low load

Future networks: Must be more efficient at any load

User load
Transmit Power Scaling Law

**Power Scaling Law**

*If the transmit power \( p \) decreases as \( 1/M^\alpha \) for \( \alpha \leq 1/2 \):*

\[
\text{SE will not go zero as } M \to \infty
\]

Example: Set \( p = p_0/M^\alpha \) in \( \text{SE}_j = K \left( 1 - \frac{\tau_p}{\tau_c} \right) \log_2 \left( 1 + \frac{1}{l_j} \right) \):

\[
l_j^{\text{MR}} = \sum_{l \in \mathcal{P}_j(f) \setminus \{j\}} \left( \mu_{jl}^{(2)} + \frac{\mu_{jl}^{(2)} - (\mu_{jl}^{(1)})^2}{M} \right) + \left( \sum_{l \in \mathcal{L}} \mu_{jl}^{(1)} K + \frac{M^\alpha}{p_0} \right) \left( \sum_{l \in \mathcal{P}_j(f \setminus \{j\})} \mu_{jl}^{(1)} + \frac{M^\alpha}{p_0 \tau_p} \right) = \sum_{l \in \mathcal{P}_j(f \setminus \{j\})} \mu_{jl}^{(2)} + O \left( \frac{M^{2\alpha}}{M} \right)
\]

**Observations** (\( \alpha = 1/2 \))

*Power per antenna/user: Decreases as \( \frac{1}{\sqrt{M}} \)*

*Total power: \( \frac{K}{\sqrt{M}} \) increases as \( \sqrt{M} \) for fixed \( \frac{M}{K} \)
Radiated Energy Efficiency

- Energy Efficiency with Power Scaling:
  \[ EE = \frac{\text{Average Sum Rate [bit/s/cell]}}{\text{Power Consumption [Joule/s/cell]}} = \frac{B \cdot K \left(1 - \frac{\tau_p}{\tau_c}\right) \log_2 \left(1 + \frac{1}{T_j}\right)}{\frac{Kp_0}{M^\alpha} \mathbb{E} \left\{ \frac{1}{\beta_{lk}} \right\}} \]

- Bandwidth: \( B \) Hz

- Consequence of scaling law as \( M \to \infty \):
  1. Sum rate \( \to \) constant \( > 0 \)
  2. Transmit power \( \to 0 \)

Is Massive MIMO Incredibly Energy Efficient?
Yes, in terms of bringing down the radiated transmit power
But not all consumed power is radiated!
**Generic Power Consumption Model**

- Many Components Consume Power
  - Radiated transmit power
  - Baseband signal processing (e.g., precoding)
  - Active circuits (e.g., converters, mixers, filters)

- Average Power Consumption Model:
  \[
  APC = \frac{Kp}{\eta} \mathbb{E} \left\{ \frac{1}{\beta_{lk}} \right\} + C_{0,0} + C_{0,1}M + C_{1,0}K + C_{1,1}MK
  \]

  - *Power amplifier* ($\eta$ is efficiency)
  - *Circuit power per transceiver chain*
  - *Fixed power* (control signals, backhaul, load-independent processing)
  - *Cost of digital signal processing* (e.g., channel estimation and precoding computation)

**Nonlinear increasing function of** $M$ and $K$

**Many coefficients:** $\eta, C_{i,j}$ for different $i, j$
Optimizing a Cellular Network for High EE

- **Clean Slate Network Design**
  - Select BS density: $\lambda$ BSs per km$^2$
  - Select $M$ and $K$ per cell
  - Asymmetric user load $\rightarrow$ asymmetric deployment

**Spatial Point Processes**

*Tractable way to model randomness*

*Poisson point process (PPP):* $\text{Po}(\lambda A)$ BSs in area of size $A$ km$^2$

*Random independent deployment: Lower bound on practical performance*

Source: Andrews et al. “A Tractable Approach to Coverage and Rate in Cellular Networks”
Average Uplink Spectral Efficiency

Assumptions

- BSs distributed as PPP: $\lambda$ BS/km²
- $M$ antennas per BS, $K$ users per cell
- Random pilot allocation: $\tau_c = fK$
- Statistical channel inversion: $\rho / \beta_{\text{lk}}$

Pathloss over noise:

$\beta_{\text{lk}}^j = \omega^{-1} (\text{distance [km]})^{-\alpha}$

Power per user:

$$\mathbb{E} \left\{ \frac{\rho}{\beta_{\text{lk}}} \right\} = \rho \omega \frac{\Gamma(\alpha/2-1)}{\left(\pi \lambda\right)^{\alpha/2}}$$

Lower Bound on Average SE with MR

$$\overline{\text{SE}} = \left(1 - \frac{fK}{\tau_c}\right) \log_2(1 + \overline{\text{SINR}})$$

$$\overline{\text{SINR}} = \left( \frac{1}{p} \right) \left( \frac{2}{f(\alpha - 2) + \frac{1}{p}} \right) + \frac{2K}{\alpha - 2} \left( 1 + \frac{1}{p} \right) + \frac{K}{f} \left( \frac{4}{(\alpha - 2)^2 + \frac{1}{\alpha - 1}} \right) + \frac{M}{f(\alpha - 1)}$$
Maximizing Energy Efficiency

\[
\begin{align*}
\text{maximize} & \quad B \cdot K \left( 1 - \frac{fK}{\tau_c} \right) \log_2(1 + \text{SINR}) \\
\text{subject to} & \quad \text{SINR} \geq \gamma
\end{align*}
\]

- Average SINR constraint \( \gamma \) needed to not get too low SE
- Is the solution small cells (high \( \lambda \)) or Massive MIMO (high \( M \))?  

**Main Properties**

1. Can pick \( f \) to satisfy SINR constraint
2. By setting \( p = p_0\lambda \), the EE is increasing in \( \lambda \)
3. Quasi-concave function w.r.t. \( M \) and \( K \)

\[\text{Maximum}\]

\[\text{Interval} = \text{Convex set}\]
## Simulation Parameters

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<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherence interval</td>
<td>$\tau_c$</td>
<td>400</td>
</tr>
<tr>
<td>Pathloss exponent</td>
<td>$\alpha$</td>
<td>3.76</td>
</tr>
<tr>
<td>Pathloss over noise at 1 km</td>
<td>$\omega$</td>
<td>33 dBm</td>
</tr>
<tr>
<td>Amplifier efficiency</td>
<td>$\eta$</td>
<td>0.39</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>$B$</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Static power</td>
<td>$C_{0,0}$</td>
<td>10 W</td>
</tr>
<tr>
<td>Circuit power per active user</td>
<td>$C_{1,0}$</td>
<td>0.1 W</td>
</tr>
<tr>
<td>Circuit power per BS antenna</td>
<td>$C_{0,1}$</td>
<td>1 W</td>
</tr>
<tr>
<td>Signal processing coefficient</td>
<td>$C_{1,1}$</td>
<td>3.12 mW</td>
</tr>
</tbody>
</table>

We publish simulation code to enable simple testing of other values!
Impact of BS Density

**Simulation**
- Different BS densities
- Other variables optimized

**Observations**
- Lower bound is tight
- Higher EE with lower $\gamma$
- EE increases with $\lambda$

**Saturation Property**

$EE$ gain from small cells saturates at $\lambda = 10$

This is satisfied in most urban deployments (300 m between BSs)

We can safely let $\lambda \to \infty$ to simplify analysis
Optimal Number of Antennas and Users

Real-valued Optimization

Optimal $K \in \mathbb{R}$ found in closed-form for fixed $M / K$

Optimal $M \in \mathbb{R}$ found in closed-form for fixed $K$

Alternating optimization reaches global maximum

Properties: Optimal $K$ and $M$

\[?\]: Decrease as $C_{0,1}$, $C_{1,0}$ and $C_{1,1}$ increase

\[?\]: Increase as $C_{0,0}$ increases

Intuition: Activate more hardware if the relative cost is small
Impact of Number of Antennas and Users

**Simulation**

Optimized $f, \lambda, \rho$

SINR constraint: $\gamma = 3$

**Observations**

Optimal: $M = 89, K = 10$

Massive MIMO with reuse factor $f \approx 7$

Many good solutions

Why is Massive MIMO Energy Efficient?

Interference suppression: Improve SINR, not only SNR as with small cells

Sharing cost: Fixed circuit power costs are shared
Optimization with Given User Density

- User Density
  - So far: $K$ and $\lambda$ design variables
  - Density: $\lambda K$ users per km$^2$
  - Heterogeneous user distribution

**Practical User Densities**

- Rural: $10^2$ per km$^2$
- Urban: $10^3$ per km$^2$
- Office/Mall: $10^5$ per km$^2$

**Can we Optimize this Density?**

*Increase: No, cannot “create” users*

*Decrease: Yes, by scheduling*

Source: METIS, “Deliverable D1.1: Scenarios, requirements and KPIs for 5G mobile and wireless system”
Impact of User Density

Simulation

Fixed user density $\mu$ users/km$^2$

*Rural*: $\mu = 10^2$, *Malls*: $\mu = 10^5$

EE maximization with constraint $K\lambda = \mu$

Low User Density

Many cells with $K \approx 1$

Most important to reduce pathloss

High User Density

Massive MIMO is optimal

Saturation for $\mu \geq 100$

Covers both rural and shopping malls

Share circuit power and cost over users
Summary

• Transmit Power Scaling Law
  • Reduced as $1/\sqrt{M}$ per user, but total transmit power might increase
  • Reduced as $1/\sqrt{M}$ per BS antenna → Use handset technology?

• Designing Networks for Energy Efficiency
  • Large cells: First step is to reduce cell size
  • Smaller cells: Transmit power only a small part → Use Massive MIMO
  • Intuition: Suppress interference, share circuit power over many users
  • Fractional pilot reuse is important in random deployments
  • Several Mbit/Joule achieved without coordination
Massive MIMO and HARDWARE EFFICIENCY
Many Antennas and Transceiver Chains

- Many Antenna Elements
  - LTE 4-MIMO: $3 \cdot 4 \cdot 20 = 240$ antennas
    But only 12 transceiver chains!
  - Massive MIMO = $M$ transceiver chains

- End-to-end Channels
  - Wireless propagation channel
  - Transceiver hardware
  - Simple model:

**Can We Afford $M$ High-Grade Transceiver Chains?**

*Can Massive MIMO utilize the hardware components more efficiently?*

Image source: gigaom.com
Orthogonal frequency-division multiplexing (OFDM)

- **Transmitter**
  - Constellation mapping
  - Filters, I/Q mixers, DACs, ADCs, oscillators

- **Receiver**
  - Symbol detection
  - Parallel to serial

Modeling of Hardware Impairment

• Real Transceivers have Hardware Impairments
  • Ex: Phase noise, I/Q-imbalance, quantization noise, non-linearities, etc.
  • Each impairment can be modeled (for given hardware, waveform etc.)
  • But: Impact reduced by calibration and only combined effect matters!

More impairments = Lower price, lower power, smaller size

• High-Level Hardware Model:

  Input Signal  \rightarrow \text{Non-Linear System} \rightarrow \text{Propagation Channel} \rightarrow \text{Non-Linear System}  \rightarrow \text{Output Signal}

  \text{Transmitter Hardware} \hspace{1cm} \text{Propagation Channel} \hspace{1cm} \text{Receiver Hardware}

  • Bussgang’s theorem:

    \begin{align*}
    X & \rightarrow \text{Non-Linear System} & cX + V
    \end{align*}

    Power loss and phase rotations
    Additive distortion noise

  X, V are uncorrelated Gaussian variables
Classical Impact of Hardware Impairments

- Impact on Point-to-Point MIMO
  - Low SNR: Negligible impact on spectral efficiency
  - High SNR: Fundamental upper limit

**Error Vector Magnitude**

\[
EVM = \frac{\text{Distortion magnitude}}{\text{Signal magnitude}}
\]

*Distortion scales with signal power*

*LTE EVM limits: 8%-17.5%*

*What about large \( M \) regime?*

*Large or small impact?*

*Example: 4x4 point-to-point MIMO, i.i.d. Rayleigh fading*
Distortion Noise: Definition and Interpretation

- Uplink Signal (conventional):
  \[ y = \sum_k g_k x_k + w \]

- Uplink Signal (with impairments):
  \[ y = c^{rx} \sum_k g_k (c^{tx}_k x_k + \xi^{tx}_k) + \xi^{rx} + w \]

**Distortion Noise Model**
Gaussian distributed
Independent between users and antennas

**Error Vector Magnitude (at transmitter)**
\[
EVM^{tx} = \sqrt{\frac{\mathbb{E}\{|\xi^{tx}_k|^2\}}{\mathbb{E}\{|c^{tx}_k x_k|^2\}}} 
\]

**Antenna 1**
**Antenna 2**

- Desired signal
- Distortion noise (elliptical cloud)
What is the Impact of Distortion Noise?

**Uplink Single-User Scenario**
Rayleigh fading, SNR = 5 dB

**Observations**
*Ideal:* $SE = O(\log M)$
*Non-ideal:* Asymptotic limits
*Higher EVM → Lower limit*

**Observations**
*Impairments caused by user device determine the limit*
*Distortion noise caused by BS averages out as $M \to \infty$ (cf. inter-user interference)*
Multi-Cell Scenario with Distortion Noise

**Uplink Multi-Cell Scenario**
Rayleigh fading, $\text{SNR} = 5 \text{ dB}$
$K = 8$ users per cell
MR detection

**Hardware Scaling Law**
If distortion variance increases as $M^\kappa$ for $\kappa \leq 1/2$:
$\text{SE} \text{ will not go zero as } M \to \infty$

Can be proved rigorously!

**Observations**
Small loss if law is followed
Otherwise large loss!

\[ \kappa = 0 \]
\[ \kappa = 1/2 \]
\[ \kappa = 1 \]
Utilizing the Hardware Scaling Law

- Massive MIMO can use Lower-Grade Hardware
  - Reduced cost, power consumption, and size

- Example: Analog-to-Digital Converter (ADC)
  - One $b$-bit ADC per Transceiver Chain
    - Adds quantization noise roughly proportional to $2^{-2b}$:
      \[ \sqrt{M} = c_0 \cdot 2^{-2b} \implies b = \frac{1}{2} \log_2(c_0) - \frac{1}{4} \log_2(M) \]
      - Ex: $M = 256$ requires 2 fewer bits than $M = 1$ (even 1-bit ADCs possible)

- Circuit power roughly proportional to $2^{2b}$:
  - Ex: Power of $M$ ADCs can scale as $\sqrt{M}$ rather than $M$
Interference Visibility Range

• Only Remaining Interference as $M \to \infty$:
  • Pilot contamination (reuse of pilot resources)
  • Hardware impairments (at user devices)

• Distortion Noise as Self-interference
  • Limits the visibility of inter-user interference

No reason to suppress inter-user interference below self-interference!
Summary

• Any Transceiver is Subject to Hardware Impairments
  • Massive MIMO is resilient to such imperfections
  • Distortion variance at BS may increase as $\sqrt{M}$
  • High-grade BS hardware is not required!
  • User hardware quality is the fundamental limitation

• Further Remarks
  • Analysis with more detailed hardware models show same behavior
  • Phase noise is not worse than in small MIMO systems
  • Reduced transmit power and relaxed impairment constraints
    → New compact transceiver designs?
Part 4

OPEN PROBLEMS
Open Problems and Active Research Topics

1. Channel measurements and modeling
2. Circuit and transceiver design
3. Implementation-aware algorithmic design
4. Dealing with hardware impairments and reciprocity calibration
5. Exploiting $M - K$ excess degrees of freedom
6. FDD operation for “low mobility” or “highly structured channels”
7. MAC-layer design, power control, and scheduling
8. Control signaling and non-CSI@TX operation
9. New deployment scenarios (e.g., distributed or cell-free)
10. Mitigating pilot contamination
11. System-level studies and coexistence with small cells
12. Massive MIMO for millimeter wave bands
SUMMARY
Summary

- Massive MIMO has Many Extraordinary Benefits
  - **High spectral efficiency:** >20x gains over IMT-Advanced are foreseen
    - High SE per cell, but modest per user
    - Important: Fractional pilot reuse, pilots take up large part of coherence interval
  - **High energy efficiency:** Tens of Mbit/Joule are foreseen
    - Reduced transmit power per user and antenna, maybe not per cell
    - Circuit power dominates power consumption in urban scenarios
    - Important: Interference control, sharing circuit power between users
  - **High hardware efficiency:** High-grade hardware is not needed
    - Variance of distortion noise at BS can scale with number of antennas
    - Important: Quality of user device is the limiting factor
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Bringing an Extraordinary Technology to Reality

- FP7 MAMMOET project (Massive MIMO for Efficient Transmission)
  - Bridge gap between “theoretical and conceptual” Massive MIMO
  - Develop: Flexible, effective and efficient solutions

**WP4** Validation and proof-of-concept

**WP2** Efficient FE solutions (IC solutions, Comp/Calibration)

**WP3** Baseband Solutions (Algorithms, Architectures & Design)

**WP1** System approach, scenarios and requirements
Key References (1/4)

Seminal and Overview Papers


Key References (2/4)

**Spectral Efficiency**


Key References (3/4)

Energy Efficiency


Hardware Efficiency


QUESTIONS?

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