Energy-Efficient Future Wireless Networks:
A Marriage between Massive MIMO and Small Cells

Emil Björnson***, Luca Sanguinetti**†‡**, and Marios Kountouris**§
* ISY, Linköping University, Sweden † Dipartimento di Ingegneria dell'Informazione, University of Pisa, Italy
‡ LANEAS, CentraleSupélec, France § France Research Center, Huawei Technologies Co. Ltd.

Introduction: Energy Efficiency

- Benefit-Cost Analysis of Networks

Cost: Energy consumption [Watt = Joule/s] → **Network** → **Benefit:** Sum rate [bit/s]

  - **Definition:** Energy Efficiency (EE):

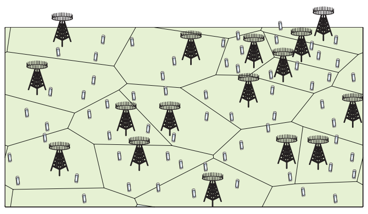
$$\text{EE [bit/Joule]} = \frac{\text{Area Spectral Efficiency [bit/symbol/km}^2]}{\text{Transmit Power + Circuit Power per Area [Joule/symbol/km}^2]}$$

  - Future networks: 1000x more data → Want 1000x higher EE

Need to Greatly Improve Energy Efficiency in Wireless Networks!

System Modeling and Preliminary Analysis

- **Methodology:**
  1. Formulate uplink EE maximization mathematically
  2. Optimize system parameters and analyze optimal solution
- **Random Network Deployment**
  - Base stations (BSs) distributed as Poisson point process (PPP), density λ BS/km²
  - M antennas per BS, K users per cell
  - Pathloss: $\omega^{-1}(\text{distance [km]})^{-\alpha}$



BSs distribution as a PPP
In any area of size A km²:
Po$(A\lambda)$ uniformly random BSs

- **System Properties**
  - Channel coherence: S transmission symbols
  - Channel estimation: Pilots of length βK
  - Power control: ρ/(pathloss from serving BS)

Proposition 1: A lower bound on the average uplink spectral efficiency (SE) is

$$\underline{\text{SE}} = \left(1 - \frac{\beta K}{S}\right)\log_2(1 + \underline{\text{SINR}}) \quad \text{[bit/symbol/user]}$$

where maximum ratio (MR) combining gives

$$\underline{\text{SINR}} = \frac{M}{\left(K+\frac{\sigma^2}{\rho}\right)\left(1+\frac{2}{\beta(\alpha-2)}+\frac{\sigma^2}{\rho}\right)+\frac{2K}{\alpha-2}\left(1+\frac{\sigma^2}{\rho}\right)+\frac{K}{\beta}\left(\frac{4}{(\alpha-2)^2}+\frac{1}{\alpha-1}\right)+M\frac{1}{\beta(\alpha-1)}}.$$

- **Energy Consumption:**
  - Depends on hardware parameters: η, $\mathcal{C}_0$, $\mathcal{C}_1$, $\mathcal{D}_0$, $\mathcal{D}_1$.

$$\text{EC} = \underbrace{\frac{S-\beta K+1}{S}\frac{\rho\omega}{\eta}\frac{\Gamma(\frac{\alpha}{2}+1)}{(\pi\lambda)^{\alpha/2}}K}_{\text{Power in amplifiers}} + \underbrace{\mathcal{C}_0}_{\text{Static power}} + \underbrace{\mathcal{C}_1 K + \mathcal{D}_0 M}_{\text{Power per chain}} + \underbrace{\mathcal{D}_1 M K}_{\text{Signal processing}} \quad \text{[Joule/symbol/cell]}$$

Problem Formulation:

Maximization of EE for given SINR constraint $\gamma \geq 0$:

$$\begin{aligned} \underset{\rho\geq 0,\,\lambda\geq 0,\,\beta\geq 1,(M,K)\in\mathbb{Z}_+}{\text{maximize}} \quad & \frac{\lambda\cdot K\cdot\underline{\text{SE}}}{\lambda\cdot\text{EC}} \\ \text{subject to} \quad & \underline{\text{SINR}} \geq \gamma \end{aligned}$$

Optimization variables:
ρ: Transmit power scaling
λ: BS density
M: BS antennas
β : Pilot reuse factor
K : Users per BS

Optimal Pilot Reuse and BS Density

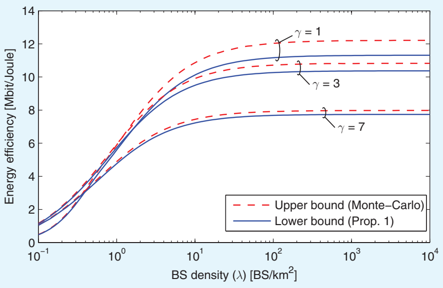
Theorem 1 (Optimal Pilot Reuse Factor): The SINR constraint is satisfied by

$$\beta^\star = \frac{\left(\frac{4K}{(\alpha-2)^2}+\frac{K+M}{\alpha-1}+\frac{2(K+\frac{\sigma^2}{\rho})}{\alpha-2}\right)\gamma}{M-\left(K+\frac{\sigma^2}{\rho}+\frac{2K}{\alpha-2}\right)\left(1+\frac{\sigma^2}{\rho}\right)\gamma}.$$

Theorem 2 (Optimal BS Density): EE increases with λ and maximized as $\lambda \to \infty$.

Saturation Property:

- Higher density λ → Less transmit power → Eventually negligible impact on EE
- Simulations show saturation for $\lambda \geq 10^2$
- 100-200 meters inter-BS distance: Saturation appears in practice!



Impact of Number of BS Antennas and Users

Theorem 3 (Optimal Number of Users per Cell): For fixed $\bar{c} = M/K > 0$, the EE is maximized by

$$K^\star = \frac{\sqrt{(G\mathcal{C}_0)^2+\mathcal{C}_0\mathcal{D}_1\bar{c}+\mathcal{C}_0 G\left(\mathcal{C}_1+\mathcal{D}_0\bar{c}\right)}-G\mathcal{C}_0}{\mathcal{D}_1\bar{c}+G\left(\mathcal{C}_1+\mathcal{D}_0\bar{c}\right)} \quad \text{with} \quad G = \frac{\frac{4\gamma}{(\alpha-2)^2}+\frac{\gamma(1+\bar{c})}{\alpha-1}+\frac{2\gamma}{\alpha-2}}{S\left(\bar{c}-\left(1+\frac{2}{\alpha-2}\right)\gamma\right)}.$$

Theorem 4 (Optimal Number of BS Antennas): For fixed K, the EE is maximized by

$$M^\star = K\frac{a_1K+a_2+\sqrt{a_1a_2K+a_1^2K^2+(1-a_0K)(a_1K+a_0a_2K)\frac{\mathcal{C}_0+\mathcal{C}_1K}{\mathcal{D}_0K+\mathcal{D}_1K^2}+a_0a_1a_2K^2+a_0a_2^2K}}{1-a_0K}$$

with $a_0 = \frac{\gamma}{S(\alpha-1)}$, $a_1 = \frac{1}{S}\left(\frac{4\gamma}{(\alpha-2)^2}+\frac{\gamma}{\alpha-1}+\frac{2\gamma}{\alpha-2}\right)$, $a_2 = \left(1+\frac{2}{\alpha-2}\right)\gamma$.

Alternating Optimization:

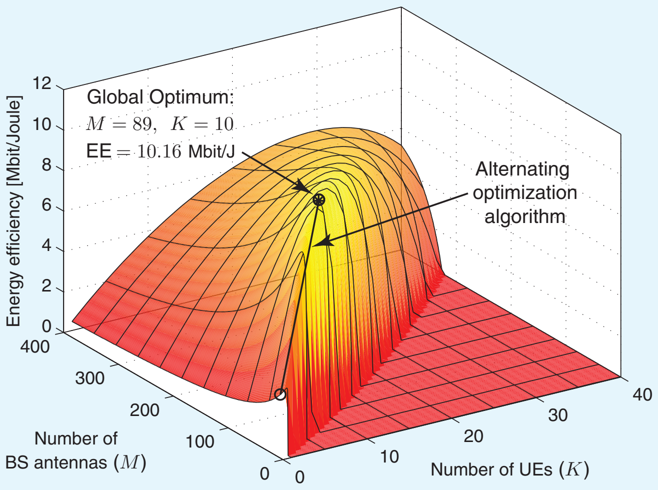
- Iterate between Theorem 3 and Theorem 4: Converge to real-valued optimum (M, K)
- EE maximization is quasi-concave problem: Integer-valued solution is in the vicinity

Insights from Analytical Expressions:

- M *increases* with K and vice versa. M and K *increase* with $\mathcal{C}_0$.
- M and K *decrease* with $\mathcal{C}_1$, $\mathcal{D}_0$, and $\mathcal{D}_1$

Simulation:

- SINR constraint $\gamma = 3$, for given values on η, $\mathcal{C}_0$, $\mathcal{C}_1$, $\mathcal{D}_0$, $\mathcal{D}_1$



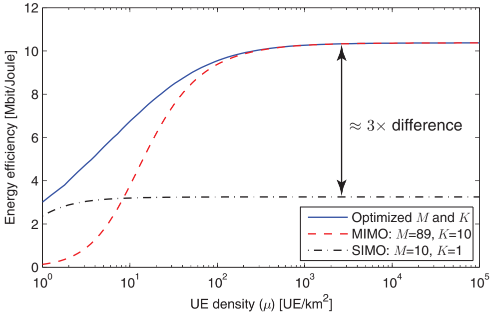
- **Notice:** $M \gg K$, a Massive MIMO (multiple input multiple output) configuration!

Impact of User Density

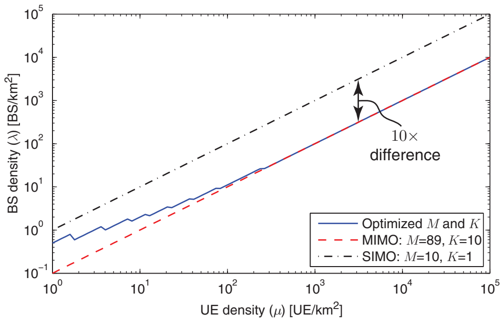
EE optimum might have too many users!

- **Simulation Setup:**
  - Fixed user density μ users/km²
  - EE maximization with $K\lambda = \mu$
  - Range: $\mu = 10^2$ (rural) to $\mu = 10^5$ (mall)
- **Low User Density**
  - Add more cells with $K \approx 1$
  - Most important: reduce pathloss
- **Medium or High User Density**
  - Small cells with Massive MIMO
  - Saturation for $\mu \geq 10^2$
  - Covers most practical scenarios: Optimal EE independent of load!

Energy Efficiency:



BS Density:



Summary: Designing Networks for Energy Efficiency

- Optimize: BS density, transmit power, pilot reuse factor, and antennas/users per cell
- Analytical contributions: EE maximizing network deployment was found!
- Optimal solution: **Small cells with Massive MIMO capability**
- Intuition: Small cells → Negligible transmit power
  Massive MIMO → Less interference, share energy costs over users

Further Results:

- E. Björnson, L. Sanguinetti, M. Kountouris, "Deploying Dense Networks for Maximal Energy Efficiency: Small Cells Meet Massive MIMO," Submitted to IEEE JSAC, April 2015. (http://arxiv.org/abs/1505.01181)