

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

$$\begin{aligned} \text{ACF:} \quad r_X(\tau) &= \text{E}\{X(t+\tau)X^*(t)\} \\ &= r_{X_1}(\tau) + r_{X_2}(\tau) + j(r_{X_1, X_2}(-\tau) - r_{X_1, X_2}(\tau)) \\ \text{PSD:} \quad R_X(f) &= R_{X_1}(f) + R_{X_2}(f) + j(R_{X_1, X_2}(f) - R_{X_1, X_2}(f)) \end{aligned}$$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c .

$$\begin{aligned} \text{Passband:} \quad s(t) &= \text{Re}\left\{\tilde{s}(t)e^{j2\pi f_c t}\right\} = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t) \\ \text{Baseband:} \quad \tilde{s}(t) &= s_I(t) + js_Q(t) \\ \text{Spectrum:} \quad S(f) &= \frac{1}{2}(\tilde{S}(f-f_c) + \tilde{S}^*(-f-f_c)), \quad \tilde{S}(f) = 2S(f+f_c)u(f+f_c) \\ \text{Filtering:} \quad y(t) &= (x * h)(t) \Leftrightarrow \tilde{y}(t) = \frac{1}{2}(\tilde{x} * \tilde{h})(t) \\ &\text{where } \tilde{x}(t), \tilde{h}(t) \text{ and } \tilde{y}(t) \text{ are the complex baseband representations of} \\ &\text{ } x(t), h(t) \text{ and } y(t), \text{ respectively.} \end{aligned}$$

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c .

$$\begin{aligned} \text{Passband:} \quad S(t) &= S_I(t) \cos(2\pi f_c t + \Psi) - S_Q(t) \sin(2\pi f_c t + \Psi) \\ &\text{with } \Psi \text{ uniformly distributed on } [0, 2\pi), \text{ independent of } \tilde{S}(t). \\ \text{Baseband:} \quad \tilde{S}(t) &= S_I(t) + jS_Q(t) \\ \text{Mean:} \quad m_S &= 0 \\ \text{ACF:} \quad r_S(\tau) &= \frac{r_{S_I}(\tau) + r_{S_Q}(\tau)}{2} \cos(2\pi f_c \tau) - \frac{r_{S_I, S_Q}(-\tau) - r_{S_I, S_Q}(\tau)}{2} \sin(2\pi f_c \tau) \\ \text{PSDs:} \quad R_S(f) &= \frac{1}{4}(R_{\tilde{S}}(f-f_c) + R_{\tilde{S}}(-f-f_c)), \quad R_{\tilde{S}}(f) = 4R_S(f+f_c)u(f+f_c) \end{aligned}$$