TSDT14 Signal Theory, Formula sheet included on exams from 2016-10-28.

Formulas on Complex Processes

Complex process: $X(t)$, where $X_1(t)$ is the real part, and $X_2(t)$ is the imaginary part.

ACF: $$r_X(\tau) = \mathrm{E}\{X(t+\tau)X^*(t)\} = r_{X_1}(\tau) + r_{X_2}(\tau) + j\big(r_{X_1,X_2}(-\tau) - r_{X_1,X_2}(\tau)\big)$$

PSD: $$R_X(f) = R_{X_1}(f) + R_{X_2}(f) + j\big(R_{X_1,X_2}(f) - R_{X_1,X_2}(f)\big)$$

Formulas on Baseband Representation - Deterministic Input

$\tilde{s}(t)$ is the message and $s(t)$ is the modulated signal, with bandwidth of $\tilde{s}(t)$ less than f_c.

Passband: $$s(t) = \mathrm{Re}\Big\{\sqrt{2}\,\tilde{s}(t)e^{j2\pi f_c t}\Big\} = s_I(t)\sqrt{2}\cos(2\pi f_c t) - s_Q(t)\sqrt{2}\sin(2\pi f_c t)$$

Baseband: $$\tilde{s}(t) = s_I(t) + js_Q(t)$$

Spectrum: $$S(f) = \tfrac{1}{\sqrt{2}}\big(\tilde{S}(f-f_c) + \tilde{S}^*(-f-f_c)\big), \qquad \tilde{S}(f) = \sqrt{2}S(f+f_c)u(f+f_c)$$

Filtering: $$y(t) = (x*h)(t) \quad \Leftrightarrow \quad \tilde{y}(t) = \tfrac{1}{\sqrt{2}}(\tilde{x}*\tilde{h})(t)$$
where $\tilde{x}(t)$, $\tilde{h}(t)$ and $\tilde{y}(t)$ are the complex baseband representations of $x(t)$, $h(t)$ and $y(t)$, respectively.

Formulas on Baseband Representation - Stochastic Input

$\tilde{S}(t)$ is the message with $m_{\tilde{S}} = 0$ and $S(t)$ is the modulated signal. The bandwidth of $\tilde{S}(t)$ must be less than f_c.

Passband: $$S(t) = S_I(t)\sqrt{2}\cos(2\pi f_c t + \Psi) - S_Q(t)\sqrt{2}\sin(2\pi f_c t + \Psi)$$
with Ψ uniformly distributed on $[0, 2\pi)$, independent of $\tilde{S}(t)$.

Baseband: $$\tilde{S}(t) = S_I(t) + jS_Q(t)$$

Mean: $$m_S = 0$$

ACF: $$r_S(\tau) = \big(r_{S_I}(\tau) + r_{S_Q}(\tau)\big)\cos(2\pi f_c\tau) - \big(r_{S_I,S_Q}(-\tau) - r_{S_I,S_Q}(\tau)\big)\sin(2\pi f_c\tau)$$

PSDs: $$R_S(f) = \tfrac{1}{2}\big(R_{\tilde{S}}(f-f_c) + R_{\tilde{S}}(-f-f_c)\big), \qquad R_{\tilde{S}}(f) = 2R_S(f+f_c)u(f+f_c)$$