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# Selfishness and Altruism on the MISO Interference Channel: The Case of Partial Transmitter CSI

Johannes Lindblom, Eleftherios Karipidis, and Erik G. Larsson

**Abstract**—We study the achievable ergodic rate region of the two-user multiple-input single-output interference channel, under the assumptions that the receivers treat interference as additive Gaussian noise and the transmitters only have statistical channel knowledge. Initially, we provide a closed-form expression for the ergodic rates and derive the Nash-equilibrium and zero-forcing transmit beamforming strategies. Then, we show that combinations of the aforementioned selfish and altruistic, respectively, strategies achieve Pareto-optimal rate pairs.

**Index Terms**—Beamforming, ergodic rate region, game theory, interference channel, Pareto optimality.

## I. INTRODUCTION

WE consider two independent closely-located wireless systems that operate concurrently in the same spectral band. System  $i$ ,  $i \in \{1, 2\}$ , consists of a base station  $BS_i$  transmitting information to a mobile station  $MS_i$ . The systems interfere with each other since each MS receives a superposition of the transmitted signals. In information theory, this spectrum sharing scenario is modeled by the interference channel (IFC) [1]. We study the multiple-input single-output (MISO) IFC [2], where each BS employs  $n > 1$  transmit antennas and each MS a single receive antenna. The BSs operate in an uncoordinated manner and the fundamental question raised is how to choose their beamforming vectors. A conflict situation is associated with this choice, since a beamforming vector which is good for one communication link may generate substantial interference to the other. Our focus is on the Pareto-optimal (PO) beamforming vectors, which correspond to operating points on the Pareto boundary of the rate region. These are points for which it is impossible to improve the rate of one link without simultaneously decreasing the rate of the other.

The capacity region for general IFCs is still an open problem, but various achievable rate regions are known [3]. When the transmitters have perfect channel state information (CSI), the achievable instantaneous rate region of the MISO IFC can be obtained as proposed in [4]. For the same scenario, a game-theoretic viewpoint was adopted in [5] to show that linear combinations of the Nash-equilibrium (NE) and zero-forcing (ZF) beamforming strategies can achieve any point on the Pareto boundary of the rate region.

Manuscript received April 27, 2009. The associate editor coordinating the review of this letter and approving it for publication was F. Jondral.

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This work was supported in part by the Swedish Research Council (VR) and the Swedish Foundation of Strategic Research (SSF). E. Larsson is a Royal Swedish Academy of Sciences (KVA) Research Fellow supported by a grant from the Knut and Alice Wallenberg Foundation.

Digital Object Identifier 10.1109/LCOMM.2009.090970

**Contributions:** In this letter, we assume that the transmitters only have *statistical* channel knowledge (hereafter, referred to as partial CSI). Therefore, we study the achievable *ergodic* rate region. First, we provide a closed-form expression for the ergodic rates. Second, we derive, for the scenario under study, the NE (selfish) and ZF (altruistic) beamforming strategies. Third, we show that the PO beamforming vectors can be interpreted as mixtures of the aforementioned strategies. This result extends the corresponding one for the perfect CSI case [5]. Furthermore, it is alternative to and provides an interpretation of the characterization in [6].

**Notation:**  $\mathcal{R}\{\mathbf{X}\}$  and  $\mathcal{N}\{\mathbf{X}\}$  denote the range-space and null-space of  $\mathbf{X}$ , respectively.  $\Pi_{\mathbf{X}} \triangleq \mathbf{X}(\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H$  is the orthogonal projection on  $\mathbf{X}$ . Note that  $\Pi_{\mathcal{R}\{\mathbf{X}\}} + \Pi_{\mathcal{N}\{\mathbf{X}\}} = \mathbf{I}$ .

## II. SYSTEM MODEL

We assume that transmission consists of scalar coding followed by beamforming<sup>1</sup> and that all propagation channels are frequency-flat. The matched-filtered symbol-sampled complex baseband data received by  $MS_i$  is modeled as

$$y_i = \mathbf{h}_{ii}^H \mathbf{w}_i s_i + \mathbf{h}_{ji}^H \mathbf{w}_j s_j + e_i, \quad j \neq i, \quad i, j \in \{1, 2\}, \quad (1)$$

where  $s_i \sim \mathcal{CN}(0, 1)$  and  $\mathbf{w}_i \in \mathbb{C}^n$  are the transmitted symbol and the employed beamforming vector by  $BS_i$ , and  $e_i \sim \mathcal{CN}(0, \sigma_i^2)$  models the receiver noise. The conjugated<sup>2</sup> channel vector between  $BS_i$  and  $MS_j$  is modeled as  $\mathbf{h}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ij})$ . Under the partial CSI scenario,  $BS_i$  has knowledge of the channel covariance matrices  $\mathbf{Q}_{ii}$  and  $\mathbf{Q}_{ij}$ . We denote  $r_{ij} \triangleq \text{rank}\{\mathbf{Q}_{ij}\}$ . Each BS can use transmit power up to  $P$ ; hereafter, we set  $P = 1$  to simplify the exposition. This gives the power constraints  $\|\mathbf{w}_i\|^2 \leq 1$ ,  $i \in \{1, 2\}$ .

## III. CLOSED-FORM ERGODIC RATE EXPRESSION

In [6], we derived a closed-form expression for the ergodic rates of the MISO IFC. Here, we present this result in a reshaped manner. For fixed channel vectors and a given pair of beamforming vectors, the following instantaneous rates (in nats/channel use) are achievable

$$R_i(\mathbf{w}_i, \mathbf{w}_j) = \log \left( 1 + \frac{|\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{|\mathbf{h}_{ji}^H \mathbf{w}_j|^2 + \sigma_i^2} \right), \quad (2)$$

<sup>1</sup>Single-stream beamforming is highly practical, but not generally optimal on MISO channels with partial CSI. In [7], we characterized the PO transmit strategies for the MISO IFC with multi-stream beamforming. The results there are weaker in that they lack the interpretation in terms of selfishness and altruism, which is one of the main results (Prop. 3) of this letter.

<sup>2</sup>We incorporate conjugation in definition to simplify subsequent notation.

for  $j \neq i$  and  $i, j \in \{1, 2\}$ . We obtain the ergodic rates averaging over the channels. From [6], we have

$$\begin{aligned} \bar{R}_i(\mathbf{w}_i, \mathbf{w}_j) &\triangleq E_{\mathbf{h}_{ii}, \mathbf{h}_{ji}} [R_i(\mathbf{w}_i, \mathbf{w}_j)] \\ &= p_{ii}(\mathbf{w}_i) \frac{f_i(p_{ii}(\mathbf{w}_i)) - f_i(p_{ji}(\mathbf{w}_j))}{p_{ii}(\mathbf{w}_i) - p_{ji}(\mathbf{w}_j)}, \end{aligned} \quad (3)$$

for  $j \neq i$  and  $i, j \in \{1, 2\}$ , where

$$f_i(x) \triangleq e^{\sigma_i^2/x} \int_{\sigma_i^2/x}^{\infty} \frac{e^{-t}}{t} dt \quad \text{and} \quad (4)$$

$$p_{ji}(\mathbf{w}_j) \triangleq \left\| \mathbf{Q}_{ji}^{1/2} \mathbf{w}_j \right\|^2 = \mathbf{w}_j^H \mathbf{Q}_{ji} \mathbf{w}_j. \quad (5)$$

In (5),  $p_{ji}(\mathbf{w}_j)$  corresponds to the average power that  $\text{MS}_i$  receives from  $\text{BS}_j$ . Lemma 1 in [6] determines that  $\bar{R}_i(\mathbf{w}_i, \mathbf{w}_j)$  is monotonously increasing with  $p_{ii}(\mathbf{w}_i)$  for fixed  $p_{ji}(\mathbf{w}_j)$  and monotonously decreasing with  $p_{ji}(\mathbf{w}_j)$  for fixed  $p_{ii}(\mathbf{w}_i)$ .

#### IV. NASH-EQUILIBRIUM STRATEGY

In absence of cooperation, each BS ‘‘selfishly’’ chooses its beamforming vector to maximize the rate towards its intended MS, disregarding the interference caused to the other. The only reasonable outcome of such a spectrum conflict is a NE. This is an operating point where none of the systems can increase its rate by unilaterally changing its beamforming vector. Namely, the NE strategy is the pair of beamforming vectors  $\{\mathbf{w}_1^{\text{NE}}, \mathbf{w}_2^{\text{NE}}\}$ , for which

$$\bar{R}_i(\mathbf{w}_i^{\text{NE}}, \mathbf{w}_j^{\text{NE}}) \geq \bar{R}_i(\mathbf{w}_i, \mathbf{w}_j^{\text{NE}}) \quad (6)$$

for  $i, j \in \{1, 2\}$ ,  $j \neq i$ , and all feasible  $\mathbf{w}_i$ .

**Proposition 1.** *A Nash equilibrium is reached when each BS employs its maximum-ratio transmission strategy, i.e., when  $\mathbf{w}_i^{\text{NE}}$  is the dominant eigenvector of  $\mathbf{Q}_{ii}$ .*

*Proof:* The BSs independently choose their beamforming vectors. Given that system  $j$  employs a beamforming vector  $\mathbf{w}_j$ , the interference power  $p_{ji}(\mathbf{w}_j)$  caused to system  $i$  is fixed. Since  $\bar{R}_i(\mathbf{w}_i, \mathbf{w}_j)$  is monotonously increasing with the useful signal power  $p_{ii}(\mathbf{w}_i)$  for fixed  $p_{ji}(\mathbf{w}_j)$ , the best response of system  $i$  is the solution of the following optimization problem

$$\max_{\mathbf{w}_i \in \mathbb{C}^n, \|\mathbf{w}_i\|^2 \leq 1} \mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i. \quad (7)$$

The optimal solution of this quadratically-constrained quadratic problem is the dominant eigenvector of  $\mathbf{Q}_{ii}$ . ■

Problem (7) has a unique solution whenever the maximum eigenvalue of  $\mathbf{Q}_{ii}$  has multiplicity 1. Otherwise, any linear combination of the corresponding eigenvectors maximizes the objective function and the equilibrium point is not unique.

#### V. ZERO-FORCING STRATEGY

The so-called ZF strategy results when each BS chooses its beamforming vector ‘‘altruistically’’, to maximize its own rate, but without causing any interference. The effect of this strategy is the decoupling of the communication links. Note that this is only possible when  $\mathcal{R}\{\mathbf{Q}_{ii}\} \not\subseteq \mathcal{R}\{\mathbf{Q}_{ij}\}$ .

**Proposition 2.** *Provided that  $\mathcal{R}\{\mathbf{Q}_{ii}\} \not\subseteq \mathcal{R}\{\mathbf{Q}_{ij}\}$ , the zero-forcing beamforming strategy  $\mathbf{w}_i^{\text{ZF}}$  is the dominant eigenvector of  $\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii} \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}}$ .*

*Proof:* Let  $\mathbf{w}_i^{\text{ZF}}$  be the solution of the optimization

$$\max_{\mathbf{w}_i \in \mathbb{C}^n, \|\mathbf{w}_i\|^2 \leq 1} \mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i \quad (8)$$

$$\text{s. t. } \mathbf{w}_i^H \mathbf{Q}_{ij} \mathbf{w}_i = 0. \quad (9)$$

Constraint (9) corresponds to finding  $\mathbf{w}_i \in \mathcal{N}\{\mathbf{Q}_{ij}\}$ , such that no interference is caused. By choosing  $\mathbf{w}_i = \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{x}_i$ , where  $\mathbf{x}_i$  is any vector in  $\mathbb{C}^n$ , constraint (9) is satisfied. Then, (8)–(9) can be equivalently reformulated as

$$\max_{\mathbf{x}_i \in \mathbb{C}^n} \mathbf{x}_i^H \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii} \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{x}_i \quad (10)$$

$$\text{s. t. } \left\| \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{x}_i \right\|^2 \leq 1. \quad (11)$$

The vector  $\mathbf{x}_i^{\text{opt}}$  which maximizes (10) is the dominant eigenvector of  $\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii} \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}}$ . Since

$$\mathcal{R}\{\mathbf{x}_i^{\text{opt}}\} \subseteq \mathcal{R}\{\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii} \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}}\} \subseteq \mathcal{N}\{\mathbf{Q}_{ij}\}, \quad (12)$$

the constraint (11) is satisfied with

$$\left\| \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{x}_i^{\text{opt}} \right\|^2 = \|\mathbf{x}_i^{\text{opt}}\|^2 = 1. \quad (13)$$

When  $\mathcal{R}\{\mathbf{Q}_{ii}\} \subseteq \mathcal{R}\{\mathbf{Q}_{ij}\}$ , then  $\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii} \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}}$  is the all-null matrix, which has no dominant eigenvector. ■

#### VI. PARETO-OPTIMAL STRATEGIES

In this section, we provide a characterization of the PO beamforming strategies for the MISO IFC. The result extends the work in [5], where the case of perfect CSI was considered. Therein, it was proven that all operating points on the Pareto boundary of the achievable instantaneous rate region are reached by beamforming vectors that are linear combinations of the NE (selfish) and ZF (altruistic) strategies.

For the partial CSI case, we showed in [6], [7] that, when  $\mathcal{R}\{\mathbf{Q}_{ii}\} \not\subseteq \mathcal{R}\{\mathbf{Q}_{ij}\}$ ,<sup>3</sup> the PO beamforming vectors satisfy

$$\mathbf{w}_i^{\text{PO}} \in \mathcal{R}\{\mathbf{Q}_{ii}, \mathbf{Q}_{ij}\} \quad \text{and} \quad \|\mathbf{w}_i^{\text{PO}}\|^2 = 1. \quad (14)$$

In the following, we give an alternative characterization of the PO beamforming vectors.

**Proposition 3.** *Provided that  $\mathcal{R}\{\mathbf{Q}_{ii}\} \supset \mathcal{R}\{\mathbf{Q}_{ij}\}$ , all Pareto-optimal beamforming vectors satisfy*

$$\mathbf{w}_i^{\text{PO}} \in \mathcal{R}\{\mathbf{Q}_{ii}, \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii}\} \quad \text{and} \quad (15a)$$

$$\|\mathbf{w}_i^{\text{PO}}\|^2 = 1. \quad (15b)$$

The characterization in (15a) is important from a game-theoretic viewpoint, since it interprets the PO beamforming strategies as combinations of the selfish and altruistic ones. The vectors in  $\mathcal{R}\{\mathbf{Q}_{ii}\}$  correspond to the selfish strategy, since  $\mathbf{w}_i^{\text{NE}} \in \mathcal{R}\{\mathbf{Q}_{ii}\}$ , and the ones in  $\mathcal{R}\{\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii}\}$  to the altruistic strategy, since  $\mathbf{w}_i^{\text{ZF}} \in \mathcal{R}\{\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii} \Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}}\} \subseteq \mathcal{R}\{\Pi_{\mathcal{N}\{\mathbf{Q}_{ij}\}} \mathbf{Q}_{ii}\}$ .

We note that (14) and (15) hold under the conditions that  $\mathcal{R}\{\mathbf{Q}_{ii}\} \not\subseteq \mathcal{R}\{\mathbf{Q}_{ij}\}$  and  $\mathcal{R}\{\mathbf{Q}_{ii}\} \supset \mathcal{R}\{\mathbf{Q}_{ij}\}$ , respectively. The former requires that  $\mathcal{R}\{\mathbf{Q}_{ii}\}$  has some components in

<sup>3</sup>This condition was missing in the formulation of Prop. 1 in [6].

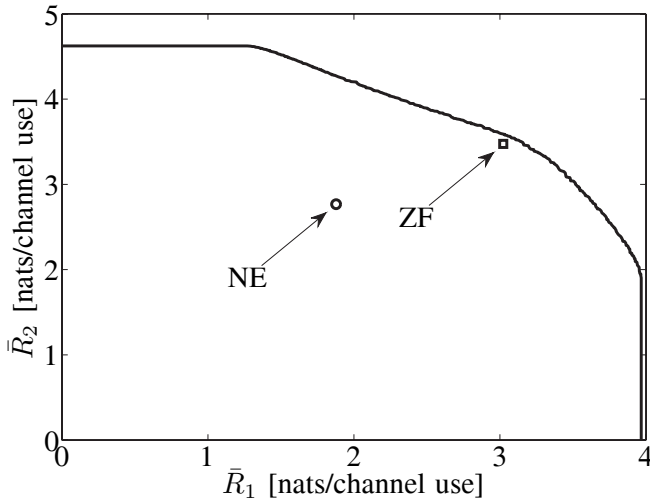


Fig. 1. Exemplary ergodic rate region; SNR = 7 dB;  $n = 5$

the  $\mathcal{N}\{Q_{ij}\}$ , so that the altruistic strategy is defined. The latter is stronger, since it says that the direct link has to offer rich enough scattering so that  $\mathcal{R}\{Q_{ii}\}$  consists of the entire  $\mathcal{R}\{Q_{ij}\}$  and some part of  $\mathcal{N}\{Q_{ij}\}$ . The reason for tightening the condition is to ensure that, in addition to the existence of the ZF strategy, the following equality holds

$$\mathcal{R}\{Q_{ij}\} = \mathcal{R}\{\Pi_{\mathcal{R}\{Q_{ij}\}}Q_{ii}\}. \quad (16)$$

This is a technical condition needed for the proof of (15a). Condition (16) means in particular that  $\mathcal{R}\{Q_{ii}\}$  must not be orthogonal to  $\mathcal{R}\{Q_{ij}\}$ , which excludes the scenario that there is no coupling among the communication links.

*Proof of (15a):* The idea is to show that when (16) holds, the characterizations in (14) and (15a) are equivalent, i.e.,

$$\mathcal{R}\{Q_{ii}, Q_{ij}\} = \mathcal{R}\{Q_{ii}, \Pi_{\mathcal{N}\{Q_{ij}\}}Q_{ii}\} \triangleq \mathcal{A}_i. \quad (17)$$

To do so, first note that the left-hand side of (17) can be written

$$\begin{aligned} \mathcal{R}\{Q_{ii}, Q_{ij}\} &= \mathcal{R}\{\Pi_{\mathcal{N}\{Q_{ij}\}}Q_{ii}, Q_{ij}\} \\ &\stackrel{(16)}{=} \mathcal{R}\{\Pi_{\mathcal{N}\{Q_{ij}\}}Q_{ii}, \Pi_{\mathcal{R}\{Q_{ij}\}}Q_{ii}\} \triangleq \mathcal{B}_i. \end{aligned}$$

In order to show  $\mathcal{A}_i = \mathcal{B}_i$ , we prove that  $\mathcal{A}_i \subseteq \mathcal{B}_i$  and  $\mathcal{B}_i \subseteq \mathcal{A}_i$ . The first part is true when all vectors  $\mathbf{x}_i \in \mathcal{A}_i$  also lie entirely in  $\mathcal{B}_i$ . Any vector  $\mathbf{x}_i \in \mathcal{A}_i$  can be written as

$$\begin{aligned} \mathbf{x}_i &= \mathbf{A}_i\boldsymbol{\alpha}_i + \mathbf{B}_i\boldsymbol{\beta}_i \\ &= \left(\Pi_{\mathcal{N}\{Q_{ij}\}} + \Pi_{\mathcal{R}\{Q_{ij}\}}\right)\mathbf{A}_i\boldsymbol{\alpha}_i + \mathbf{B}_i\boldsymbol{\beta}_i, \end{aligned}$$

where  $\boldsymbol{\alpha}_i \in \mathbb{C}^{r_{ii}}$  and  $\boldsymbol{\beta}_i \in \mathbb{C}^{\min\{r_{ii}, n-r_{ij}\}}$ , and the columns of  $\mathbf{A}_i$  and  $\mathbf{B}_i$  constitute bases of  $\mathcal{R}\{Q_{ii}\}$  and  $\mathcal{R}\{\Pi_{\mathcal{N}\{Q_{ij}\}}Q_{ii}\}$ , respectively. Clearly, we have  $\mathbf{x}_i \in \mathcal{B}_i$ , which shows  $\mathcal{A}_i \subseteq \mathcal{B}_i$ . To show  $\mathcal{B}_i \subseteq \mathcal{A}_i$  we first define

$$\mathbf{C}_i \triangleq \Pi_{\mathcal{N}\{Q_{ij}\}}\mathbf{A}_i \quad \text{and} \quad \mathbf{D}_i \triangleq \Pi_{\mathcal{R}\{Q_{ij}\}}\mathbf{A}_i.$$

The matrices  $\mathbf{C}_i$  and  $\mathbf{D}_i$  do not necessarily have full column rank, but their columns span  $\mathcal{R}\{\Pi_{\mathcal{N}\{Q_{ij}\}}Q_{ii}\}$  and

$\mathcal{R}\{\Pi_{\mathcal{R}\{Q_{ij}\}}Q_{ii}\}$ , respectively. A vector  $\mathbf{y}_i \in \mathcal{B}_i$  can now be written as

$$\begin{aligned} \mathbf{y}_i &= \mathbf{C}_i\boldsymbol{\gamma}_i + \mathbf{D}_i\boldsymbol{\delta}_i = \Pi_{\mathcal{N}\{Q_{ij}\}}\mathbf{A}_i\boldsymbol{\gamma}_i + \Pi_{\mathcal{R}\{Q_{ij}\}}\mathbf{A}_i\boldsymbol{\delta}_i \\ &= \Pi_{\mathcal{N}\{Q_{ij}\}}\mathbf{A}_i(\boldsymbol{\gamma}_i - \boldsymbol{\delta}_i) + \left(\Pi_{\mathcal{R}\{Q_{ij}\}} + \Pi_{\mathcal{N}\{Q_{ij}\}}\right)\mathbf{A}_i\boldsymbol{\delta}_i \\ &= \mathbf{C}_i(\boldsymbol{\gamma}_i - \boldsymbol{\delta}_i) + \mathbf{A}_i\boldsymbol{\delta}_i, \end{aligned}$$

for some vectors  $\boldsymbol{\gamma}_i \in \mathbb{C}^{r_{ii}}$  and  $\boldsymbol{\delta}_i \in \mathbb{C}^{r_{ii}}$ . This shows  $\mathbf{y}_i \in \mathcal{A}_i$ , completing the second part of the proof. ■

*Proof of (15b):* The proof is by contradiction. Assuming that  $\|\mathbf{w}_i^{\text{PO}}\|^2 < 1$ , we can construct  $\mathbf{w}'_i \triangleq \mathbf{w}_i^{\text{PO}} + \mathbf{u}_i$ . Choosing  $\mathbf{u}_i \in \mathcal{R}\{\Pi_{\mathcal{N}\{Q_{ij}\}}Q_{ii}\}$  and such that  $\|\mathbf{w}'_i\|^2 = 1$ , we effectively increase  $p_{ii}$  (hence,  $\bar{R}_i$ ) without affecting  $p_{ij}$  (hence,  $\bar{R}_j$ ). Thus,  $\mathbf{w}_i^{\text{PO}}$  is not PO. For details, see [6]. ■

## VII. NUMERICAL EXAMPLE

We illustrate in Fig. 1 an ergodic rate region of a MISO IFC, where the transmitters have 5 antennas and all the channel covariance matrices are rank deficient. We depict the NE and ZF operating points that correspond to the beamforming strategies defined in Prop. 1 and 2, respectively. We determine the Pareto boundary by randomly generating a large number of beamforming vectors according to Prop. 3 and selecting the uppermost resulting rate pairs. In this simulation, we see that the NE operating point is far inside the rate region, whereas the ZF point is close to the Pareto boundary. This is generally the case when interference is the major limiting factor.

## VIII. DISCUSSION

We studied the achievable ergodic rate region of the two-user MISO IFC, under the assumption that the transmitters only have partial CSI. Our main contributions are the derivations of the NE and ZF beamforming strategies, and a characterization of the PO strategies as combinations of selfishness and altruism. The results are useful for future research (especially, further game-theoretic analysis) on resource allocation problems that can be modeled by the MISO IFC.

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