

”©2008 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.”

# Primary System Detection for Cognitive Radio: Does Small-Scale Fading Help?

Erik G. Larsson and Giorgio Regnoli

**Abstract**—We consider the effect of small-scale fading on the detection of weak signals in cognitive radio systems. We formulate a model for the detection problem taking fading into account, and give the associated likelihood ratio tests. Additionally we give an expression for the asymptotic detection performance and use this to consider the tradeoff between the detection performance and the coherence time and bandwidth of the channel.

**Index Terms**—Cognitive radio, detection.

## I. INTRODUCTION

THE idea with cognitive radio is that spectrum licensed to primary users may be used in an unlicensed fashion by secondary users, provided that these secondary users do not create harmful interference for the primary users. For example, frequencies used for cellular telephony or TV broadcasting may be locally reused for a local sensor network provided that the latter transmits with low enough power. Cognitive radio has received much attention during the last few years [1], [2]. Much of this interest is sparked by recent measurements which show that radio spectrum by large is vastly under-utilized [3]. As a consequence, regulatory agencies consider policies that will let licensed spectrum be used by unlicensed users under certain conditions.

One basic challenge with cognitive radio is that before an unlicensed user can begin transmitting, she must ensure that nobody else is using the carrier in question. One way to do this is to scan the corresponding band for some time and detect whether any primary signal is present. If no signal is detected, it may be concluded “safe” to begin transmission at a small predetermined power. (The validity of the assumption, that it is actually safe to transmit if no signal is heard, is debatable. Yet some proposed rulemaking relies on a paradigm of the type “transmit-if-you-cannot-detect”.) Typically, the received primary signal is very weak [4]. The fundamental problem is then to detect whether a weak signal is present or not.

The problem of weak signal detection for cognitive radio has been previously studied in [5]. In particular it has been shown that an energy detector is unfeasible as its performance will be limited by uncertainties in the secondary receivers’ noise floor. In this paper we consider the setup of [5] but using

Manuscript received June 5, 2007. The associate editor coordinating the review of this letter and approving it for publication was Dr. Jaap van de Beek. This work was supported in part by the Swedish Research Council (VR). E. Larsson is a Royal Swedish Academy of Sciences Research Fellow supported by a grant from the Knut and Alice Wallenberg Foundation.

E. G. Larsson is with the Dept. of Electrical Engineering (ISY), Linköping University, 581 83 Linköping, Sweden (email: erik.larsson@isy.liu.se). He was previously with the Royal Institute of Technology (KTH), Stockholm, Sweden.

G. Regnoli is with the Royal Institute of Technology, School of Electrical Engineering, Osquildas väg 10, SE-100 44 Stockholm, Sweden.

Digital Object Identifier 10.1109/LCOMM.2007.070923.

a more realistic model for radio propagation, that includes small-scale fading. It turns out that this changes the problem fundamentally.

## II. MODEL

We consider a block-fading channel. We shall assume that measurements from  $K$  blocks are available, and that these blocks are subject to independent but stationary (and flat) fading. Each block has a bandwidth of  $B$  Hz and a time duration of  $T$  seconds, and thus a time-bandwidth product of  $BT$ . The value of  $T$  is effectively the coherence time of the channel and  $B$  is its coherence bandwidth. This means that each block is fully described by approximately  $2BT$  independent samples [6]. We shall set  $L = 2BT$ , and refer to  $L$  as the “coherence time-bandwidth product”. Thus the total number of samples available at the receiver is  $N = KL = 2KBT$ . Note that there is no fundamental difference between channel variations across time and across frequency. What matters is how many signal samples per channel realization ( $L$ ) that are available and that these samples are uncorrelated.

Two degenerate special cases emerge. If  $K = 1$ ,  $N = L$  then we say that we have *slow fading*. Only a single realization of the fading process is observable. At the other extreme, if  $L = 1$ ,  $N = K$ , then we talk about *fast fading*. A new channel realization is then observed for each received sample.

The objective is to discriminate between two hypotheses  $H_0$  (no signal present), and  $H_1$  (signal present), defined as follows:

$$\begin{cases} H_0 : & y_{k,l} = e_{k,l} \\ H_1 : & y_{k,l} = h_k s_{k,l} + e_{k,l} \end{cases} \quad (1)$$

with  $k = 1, \dots, K$  and  $l = 1, \dots, L$ . Here  $h_k$  is the complex channel gain for block  $k$ . The primary transmitted signal samples are  $s_{k,l}$ . We shall take the noise  $e_{k,l}$  to be i.i.d. complex Gaussian with zero mean and power  $\sigma^2$  per complex dimension.

The detection problem in (1) can be easily ill-posed under many circumstances. For example, suppose  $\sigma^2$  is unknown and one is trying to detect a Gaussian signal of unknown power over a static channel ( $K = 1$ ). This is impossible, since there is no way for the receiver to discriminate between signal and noise (they both have a Gaussian density, and so their sum is also Gaussian). On the other hand, on a non-constant channel ( $K > 1$ ) the receiver may be able to detect the time-variability of the received power and based on this infer whether a signal was transmitted or only noise is received. Hence it is clear that fading may actually help the detection of weak signals. However in the limit when  $K = N$  (fast fading), then only a single sample of the fading process is observed per realization

and one may no longer be able to discriminate between signal and noise. Thus it is also evident that if the coherence time-bandwidth product is small enough, fading may no longer be helpful. This suggests that for fixed  $N$ , there is an “optimal” coherence time-bandwidth product  $L$ .

### III. DETECTORS

In the event  $s_{k,l}$  and  $h_k$  were known to the receiver then optimal detection is accomplished by a standard matched filter. This was studied in [5] in the cognitive radio context. We shall next give statistical tests for the case that the signal and channels are unknown.

In the case that  $\{s_{k,l}\}$  are known but  $\{h_k\}$  are unknown, it is straightforward to compute the generalized likelihood-ratio test (GLRT) [7, Ch. 6]. If  $\sigma^2$  is known, we find

$$\Lambda_1(\mathbf{y}) \triangleq \ln \left( \frac{\max_{\mathbf{h}} p(\mathbf{y}|H_1, \mathbf{h})}{p(\mathbf{y}|H_0)} \right) = \frac{1}{\sigma^2} \left( \sum_{k=1}^K \|\Pi_{\mathbf{s}_k} \mathbf{y}_k\|^2 \right) \geq \eta \quad (2)$$

where  $\mathbf{y}_k = [y_{k1}, \dots, y_{kL}]^T$ ,  $\mathbf{s}_k = [s_{k1}, \dots, s_{kL}]^T$  and  $\Pi_{\mathbf{x}}$  denotes orthogonal projection, i.e.,  $\Pi_{\mathbf{x}} \mathbf{y} = \mathbf{x}(\mathbf{x}^H \mathbf{x})^{-1} \mathbf{x}^H \mathbf{y}$ . Also,  $\|\cdot\|$  is the Euclidean norm. If  $\sigma^2$  is unknown,

$$\Lambda_2(\mathbf{y}) \triangleq N \left[ \ln \sum_{k=1}^K \sum_{l=1}^L |y_{k,l}|^2 - \ln \sum_{k=1}^K \sum_{l=1}^L |y'_{k,l}|^2 \right] \geq \eta \quad (3)$$

where  $y'_{k,l}$  is the observed data with the estimated signal component projected out, i.e.,  $\mathbf{y}'_k = (\mathbf{I} - \Pi_{\mathbf{s}_k}) \mathbf{y}_k$ .

The more interesting case is when the signal  $\{s_{k,l}\}$  is also unknown. It may still have a known modulation format, but such knowledge is generally not helpful for detection, as argued in [5]. Thus to model an unknown signal we shall assume that it is Gaussian with zero mean and variance  $\sigma_s^2$  (per dimension), where  $\sigma_s^2$  generally is an unknown parameter. The GLR test can be derived with the following result. If  $\sigma^2$  is known (but  $\sigma_s^2$  unknown), the test is:

$$\Lambda_3(\mathbf{y}) \triangleq -L \left( \sum_{k=1}^K \ln \left( \sum_{l=1}^L |y_{k,l}|^2 \right) \right) + \sum_{k=1}^K \sum_{l=1}^L \frac{|y_{k,l}|^2}{\sigma^2} \geq \eta \quad (4)$$

(Note that the detector in (4) is *not* an energy detector.) If both  $\sigma^2$  and  $\sigma_s^2$  are unknown, the test is:

$$\Lambda_4(\mathbf{y}) \triangleq N \ln \left( \sum_{k=1}^K \sum_{l=1}^L |y_{k,l}|^2 - L \sum_{k=1}^K \ln \left( \sum_{l=1}^L |y_{k,l}|^2 \right) \right) \geq \eta \quad (5)$$

The last model, with  $\{s_{k,l}\}$  (including  $\sigma_s^2$ ),  $\{h_k\}$  and  $\sigma^2$  unknown is probably the most realistic one in practice<sup>1</sup> and therefore in the rest of the paper we focus on the test in (5). (The others were presented for completeness.)

### IV. ASYMPTOTIC PERFORMANCE OF (5)

The case of most interest is when both the signal  $\{s_{k,l}\}$  the channels  $\{h_k\}$ , and the noise power  $\sigma^2$  are unknown, because in this case the only hope is to be able to detect the primary signal from time-variations in the received signal strength. The

<sup>1</sup>Cf. the arguments in [5] where it was shown that the receiver's uncertainty in knowledge of  $\sigma^2$  is a major limiting factor for the performance of a standard energy detector.

associated detector is (5). We can derive its asymptotic performance by using the general theory for distributions of the GLRT test statistic [7]. There is one nuisance parameter ( $\sigma^2$ ), and  $2K$  unknown real-valued parameters which are present under  $H_1$  but not under  $H_0$ . (The variance of the signal,  $\sigma_s^2$  can be combined into the coefficients  $\{h_k\}$ .) Conditioned on  $\{h_k\}$ , we have asymptotically

$$2 \cdot \Lambda_4(\mathbf{y}) \stackrel{\text{as.}}{\sim} \begin{cases} \chi_{2K}^2 & \text{under } H_0 \\ \chi_{2K}^{\prime 2}(\lambda) & \text{under } H_1 \end{cases} \quad \text{for } N \gg 1 \quad (6)$$

where  $\chi_r^2$  and  $\chi_r^{\prime 2}(\gamma)$  are central respectively non-central  $\chi^2$  distributions with  $r$  degrees of freedom (and non-centrality parameter  $\gamma$ ). Also,

$$\lambda = KL \frac{\sigma_s^4 \sum_{k=1}^K |h_k|^4}{\sigma^4} \quad (7)$$

We have then the false alarm probability

$$P_f = P(\chi_{2K}^2 > \eta) = Q_{\chi_{2K}^2}(\eta)$$

For fixed  $\lambda$ , we have the following detection probability:

$$P_{d|\lambda} = P(\chi_{2K}^{\prime 2}(\lambda) > \eta|\lambda) = P(\chi_{2K}^{\prime 2}(\lambda) > Q_{\chi_{2K}^2}^{-1}(P_f)|\lambda)$$

The average detection probability is then

$$P_d = \int_0^\infty p(\lambda) Q_{\chi_{2K}^{\prime 2}(\lambda)}(Q_{\chi_{2K}^2}^{-1}(P_f)) d\lambda \quad (8)$$

(note that  $\lambda \geq 0$ ).

In an effort to obtain a closed-form expression we make the following calculation. In i.i.d. Rayleigh fading,  $\{h_k\}$  are complex normal with zero mean. Without loss of generality we can assume that they have unit variance (variance 1/2 per dimension) and we then find after some calculations

$$\begin{aligned} E[|h_k|^4] &= 2, & \text{Var}[|h_k|^4] &= 20 \\ E[\lambda] &= 2KL \frac{\sigma_s^4}{\sigma^4}, & \text{Var}[\lambda] &= 20KL^2 \frac{\sigma_s^8}{\sigma^8} \end{aligned}$$

This shows that in the limit when  $K \rightarrow \infty$  we have

$$\frac{1}{K} \lambda \rightarrow L \frac{\sigma_s^4}{\sigma^4} E[|h_k|^4] = 2L \frac{\sigma_s^4}{\sigma^4}$$

where the convergence speed of the limit is  $O(1/K)$ . Hence for large  $K$  we have the following approximation of (8):

$$P_{d|K \rightarrow \infty} \approx Q_{\chi_{2K}^{\prime 2}} \left( 2KL \frac{\sigma_s^4}{\sigma^4} \right) \left( Q_{\chi_{2K}^2}^{-1}(P_f) \right) \quad (9)$$

By applying the central limit theorem, one can furthermore obtain the following approximation [7]:

$$\chi_r^{\prime 2}(\gamma) \stackrel{\text{as.}}{\sim} N(r + \gamma, \sqrt{2r + 4\gamma})$$

for large  $r$ . Applying this to (9) we obtain

$$P_f = Q \left( \frac{\eta - 2K}{\sqrt{4K}} \right), \quad P_d = Q \left( \frac{\eta - 2KL \frac{\sigma_s^4}{\sigma^4} - 2K}{\sqrt{4K + 8KL \frac{\sigma_s^4}{\sigma^4}}} \right)$$

where  $Q(t) = P(N(0,1) > t) = (1/\sqrt{2\pi}) \int_t^\infty e^{-x^2/2} dx$  is the standard Gaussian Q-function. More compactly

$$P_{d|K \rightarrow \infty} = Q \left( \frac{\sqrt{K} Q^{-1}(P_f) - KL \frac{\sigma_s^4}{\sigma^4}}{\sqrt{K + 2KL \frac{\sigma_s^4}{\sigma^4}}} \right) \quad (10)$$

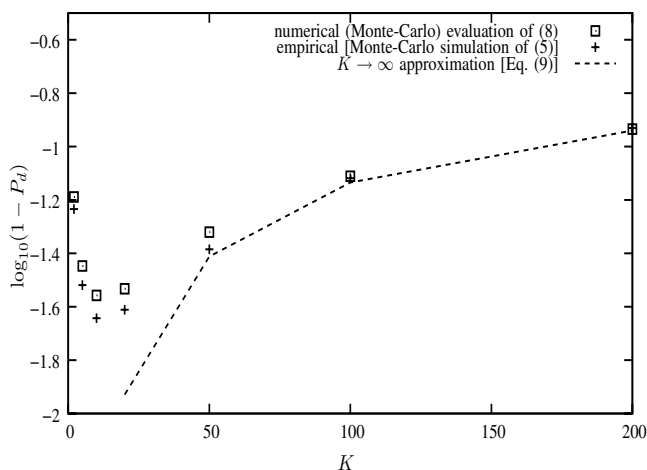


Fig. 1. Detection performance (in terms of the probability of a miss) for fixed  $N = 1000$  as a function of  $K$ . The false-alarm probability is fixed at  $P_f = 0.7$  and  $\sigma_s^2/\sigma^2 = -10$  dB.

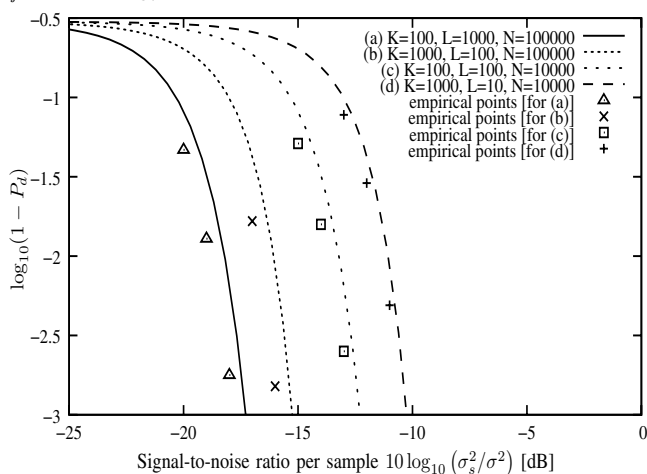


Fig. 2. Asymptotic (large  $K$ , Eq. (10)) detection performance for different  $L$ ,  $K$  and  $N = KL$  as a function of the SNR  $\sigma_s^2/\sigma^2$ , at  $P_f = 0.7$ .

Equation (10) is derived only to illustrate the fundamental hardness of the problem and it is valid only for large  $K$  (i.e., there must be fair number of independent fading realizations). It breaks down as  $K$  becomes small. We see from (10) that  $P_d$  generally increases with increasing  $K$ ,  $L$  and signal-to-noise ratio (SNR, defined as  $\sigma_s^2/\sigma^2$ ). When  $N = KL \rightarrow \infty$ ,  $P_d \rightarrow 1$ , as it should. Furthermore, for a large number of samples and fixed  $L$ , doubling  $K$  is equivalent to a 1.5 dB increase in SNR. This means that determining the presence of a primary signal by relying on detecting time-variations in its received signal strength is very expensive in terms of the number of samples required. Note that if  $K$  is large then one is essentially comparing  $\max_k |\hat{h}_k|$  (where  $\hat{h}_k$  is an estimate of the gain during block  $k$ ) to the average received signal level. Any predictions of performance for such a procedure are going to be dependent on the characteristics of the fading and especially on the tail of its distribution.

## V. ILLUSTRATIONS

In Figure 1 we illustrate the detector performance obtained via Monte-Carlo simulation and by using the asymptotic expressions developed in Section IV, for fixed  $N = 1000$  as a function of  $K$ . The false-alarm probability is fixed at

$P_f = 0.7$  and  $\sigma_s^2/\sigma^2 = -10$  dB. (This is a rather high false alarm probability, but not unrealistic if one considers that many cognitive users collaborate on the spectrum detection and obtain independent conclusions which are fused.) There exists an optimal value of  $K$  around  $K \approx 10$ , although the closed-form formula (10) is unable to predict this. (This should come as no surprise since the associated approximations were developed assuming  $K \rightarrow \infty$ .)

Figure 2 shows the asymptotic expression (10) as a function of SNR for some different  $K$ ,  $L$  and  $N = KL$  at  $P_f = 0.7$ . (The figure also shows empirical Monte-Carlo simulation results.) The predicted SNR thresholds are relatively sharp. Additionally, while we have found that detection is possible even with unknown noise variance at the receiver, this example shows explicitly how hard the detection problem really is: at fixed  $L$ , an increase in the number of samples  $N$  by a factor 10 buys only a few dB increase in SNR performance.

## VI. CONCLUSIONS

We have developed a basic model that accounts for small-scale fading effects on the detection of primary signals in a cognitive radio system. This represents an extension of what was previously done in [5], where only stationary channels were considered. In [5] the energy detector was analyzed and it was found that its performance is going to be limited by the receiver's knowledge of its own noise level. By contrast, we have explained why small-scale fading fundamentally changes the nature of the detection problem in the sense that unknown signals can be detected over an unknown channel, even if the receiver noise level is completely unknown—provided that the fading is fast enough but not too fast. The reason is that the detector can detect time-variations in the received signal strength. In theory, this phenomenon can be exploited to overcome the fundamental problems of the energy detector which were identified in [5]. However, quantitative arguments that we have presented suggest that collecting a huge, perhaps highly impractical, number of samples would be necessary even for relatively high false alarm probabilities. Possibly, correlation of the received signals could alleviate the situation somewhat, however, most information-bearing signals are relatively white.

## REFERENCES

- [1] S. Haykin, "Cognitive radio technology: brain-empowered wireless communications", *IEEE J. Sel. Areas Commun.*, vol. 23., Feb. 2005.
- [2] I. J. Mitola, "Software radios: survey, critical evaluation and future directions," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 8, pp. 25–36, Apr. 1993.
- [3] M. A. McHenry, "NSF spectrum occupancy measurements project summary," Shared Spectrum Company technical report, Aug. 2005. Available at <http://www.sharedspectrum.com/>
- [4] N. Hoven and A. Sahai, "Power scaling for cognitive radio," in *Proc. WirelessCom Symposium on Emerging Networks, Technologies and Standards*, June 2005.
- [5] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Proc. 42nd Allerton Conference on Communication Control, and Computing*, Oct. 2004.
- [6] H. L. Van Trees, *Detection, Estimation and Modulation Theory*. Wiley, 1967.
- [7] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory/Detection Theory*. Prentice-Hall, 1993.