Cooperative Transmission Based on Decode-and-Forward Relaying with Partial Repetition Coding

Majid Nasiri Khormuji and Erik G. Larsson

***Abstract*—We propose a novel half-duplex decode-and-forward relaying scheme based on *partial repetition* coding at the relay. In the proposed scheme, if the relay decodes the received message successfully, it re-encodes the message using the same channel code as the one used at the source, but retransmits only a fraction of the codeword. We analyze the proposed scheme and optimize the cooperation level (i.e., the fraction of the message that the relay should transmit). We compare our scheme with conventional repetition in which the relay retransmits the entire decoded message, with parallel coding, and additionally with dynamic decode-and-forward (DDF). We provide a finite-SNR analysis for all the collaborative schemes. The analysis reveals that the proposed partial repetition method can provide a gain of several dB over conventional repetition. It also shows that in general, power allocation is less important provided that one optimally allocates bandwidth. Surprisingly, the proposed scheme is able to achieve the same performance as that of parallel coding for some relay network configurations, but at a much lower complexity.**

***Index Terms*—Relay channel, cooperative diversity, parallel coding, repetition coding, resource allocation, power allocation, bandwidth allocation.**

I. INTRODUCTION

THE classical wireless relay setup [1], [2] consisting of a source ($\mathcal{S}$), a relay ($\mathcal{R}$), and a destination ($\mathcal{D}$) has recently received renewed attention due to its potential in wireless applications [3]–[24]. Decode-and-forward (DF) [4], [6] and amplify-and-forward (AF) [4], [5] are two well-studied relaying protocols in the literature. Some other strategies such as hybrid relaying have been studied as well [22], [23]. One advantage of DF is the possibility to vary the communication rate on the $\mathcal{S}-\mathcal{R}$ and $\mathcal{R}-\mathcal{D}$ links, which is not possible using the AF protocol in a straightforward fashion [7]. By doing so, one can allocate enough channel uses to the $\mathcal{S}-\mathcal{R}$ link such that the relay can decode the message. In this paper we confine our study to the class of decode-and-forward (DF) relaying protocols [3]–[5], [10], [13], [14], [24] in which $\mathcal{S}$ and $\mathcal{R}$ can use different channel codes. We consider only half-duplex relays, that is, the relay cannot transmit and receive simultaneously.

Most of the early work on decode-and-forward protocols for the relay channel was based on *repetition coding* at the relay (i.e., the source and the relay use the same channel code) [4], [5], [12]. Recently, it has been shown that the performance of decode-and-forward can increase by employing so-called parallel coding, i.e. letting the relay use a different channel code than the source [10], [13]. In [10], it was demonstrated that using a turbo code with different puncturing patterns at $\mathcal{S}$ and $\mathcal{R}$ can bring a few dB power gain. The main challenge of parallel coding is to design an appropriate coding structure for producing a new set of parity bits at the relay. Moreover, to decode the transmitted packet, the destination must be able to combine the received signals both from the source and from the relay. Additionally, the generalization of parallel coding to different classes of channel codes is not straightforward. In this paper, we propose that the relay uses repetition coding but repeats only *a fraction* of the message. By doing so, one can optimize the number of channel uses consumed by the relay and by the source. We obtain closed-form expressions for the outage probability of the proposed scheme and optimize the cooperation level (defined as the fraction of the coded message that is repeated by the relay) based on the geometry of the network. Moreover, we quantify the ultimate gain of the proposed partial cooperation scheme over conventional repetition coding. Our proposed *partial repetition* scheme provides a several dB power gain over conventional repetition schemes. Additionally, and somewhat surprisingly, we show that our proposed scheme performs as well as parallel coding for network configurations where the relay is close to the destination. We also compare to dynamic decode-and-forward (DDF) relaying [24] in which the relay listens until it is able to decode the message successfully. The performance of DDF is superior to that of the aforementioned schemes since it adapts to the *instantaneous* realization of the source-relay link. However, DDF relaying is not a packet-based protocol and from a practical implementation point of view, it is very complex. Additionally, DDF is a non-orthogonal scheme in which $\mathcal{S}$ and $\mathcal{R}$ may transmit simultaneously. In practice, this leads to major difficulties with time and frequency synchronization.

Our proposed partial repetition scheme has the following features:

- *Simplicity*: The proposed scheme is simpler than both

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parallel coding and DDF. In particular, it requires only maximum-ratio combining (MRC) at the destination which is computationally very inexpensive. By contrast, relaying with parallel coding or DDF requires that the destination performs code combining which is much more complex.

- *High Performance*: DF with conventional repetition is a special case of the proposed scheme. Thereby, partial repetition always outperforms conventional repetition. Moreover, and more importantly, the performance of DF with partial repetition is close to that of DF with parallel coding.
- *Flexibility*: The proposed scheme allows the user to adjust the cooperation level without changing the code structure.

We further study resource allocation for the above-mentioned DF relaying protocols. Earlier work on resource allocation for relay channels has been mostly focused on power optimization [14]–[21]. However, there are some previous papers that investigate bandwidth, or equivalently block length, optimization algorithms. The possibility of using different block lengths at the source and at the relay for decode-and-forward is considered in [7]. The work of [8], [9] treats bandwidth optimization for the relay channel and formulates a joint power-bandwidth allocation criterion for the decode-and-forward scheme. However, [9] investigates delay-limited capacity while in this paper we study outage probability. The performance results of [7]–[9], [21] heavily rely on simulation. By contrast, we derive finite-SNR analytical expressions for the outage probability of DF relaying with conventional repetition, parallel coding and with the proposed partial repetition. We formulate a joint power-bandwidth allocation problem based on the analytical results. We interestingly demonstrate that power allocation does *not* provide a considerable gain provided that optimal bandwidth allocation is used.

A. Transmission Protocol

We assume that the number of available channel uses and the total energy per packet are T and $E = PT$, respectively, where P is the average transmit power. We further assume that the relay operates in a half-duplex mode where reception and transmission occur in non-overlapping time slots. The transmission takes places in two phases. In the *first* phase, $\mathcal{S}$ transmits its data using T_s channel uses and power P_s. Both $\mathcal{R}$ and $\mathcal{D}$ listen to the transmitted signal. During the *second* phase, if the relay successfully decoded the received packet, it re-encodes the packet using a possibly different channel code and (re)transmits the re-encoded packet. Otherwise the relay remains silent. The second phase of the transmission uses power P_r and consumes $T_r = T - T_s$ channel uses. Hence $E = PT = P_sT_s + P_rT_r$. The channels used by the source and by the relay are orthogonal, with the exception of the DDF scheme (see Section II-C).

B. Channel Model

We model the channel between the nodes as quasi-static Rayleigh fading, i.e., the gain is constant during the transmission of one block. Let

$$\alpha_{ij} \triangleq \frac{|h_{ij}|^2}{N_0} \quad i \in \{s, r\},\ j \in \{r, d\}$$

for the links $\mathcal{S}-\mathcal{R}$, $\mathcal{R}-\mathcal{D}$ and $\mathcal{S}-\mathcal{D}$, where h_{ij} is the channel gain from node i to node j, and N_0 is the noise variance. Without loss of generality we can assume that $N_0 = 1$. Then the received SNR for link $i-j$ equals $P_i\alpha_{ij}$, and it is exponentially distributed with mean $P_i\gamma_{ij}$ where

$$\gamma_{ij} \triangleq \mathbb{E}|h_{ij}|^2.$$

Throughout this work we assume that the nodes know the channel gains in the direction of the information flow. That is, $\mathcal{R}$ knows h_{sr} and $\mathcal{D}$ knows h_{sd}, h_{sr}, and h_{rd}. However, we assume that there is no instantaneous forward channel state information available at $\mathcal{S}$ or $\mathcal{R}$, i.e., $\mathcal{S}$ does not know neither h_{sd}, h_{sr}, nor h_{rd} and $\mathcal{R}$ does not know neither h_{sd} nor h_{rd}.

C. Performance Measure

We use outage probability as the performance measure to compare different schemes. Assuming a radio link with received SNR $P_i\alpha_{ij}$ and spectral efficiency β [bits per channel use], the link is in outage when the instantaneously achievable spectral efficiency (assuming a Gaussian codebook and infinitely long blocks) is less than the target transmission spectral efficiency (β). Throughout this paper, we denote this outage event by

$$\mathcal{O}(P_i\alpha_{ij}, \beta) \Longleftrightarrow \log_2(1 + P_i\alpha_{ij}) < \beta.$$

II. Transmission Schemes

A. Baseline Transmission Schemes

1) Direct ($\mathcal{S}-\mathcal{D}$) Transmission: Here the relay is not used and $T_s = T$, $P_s = P$, $P_r = 0$, and $T_r = 0$. See Fig. 1(a). The transmission of the message over the direct link fails if $\mathcal{O}(P\alpha_{sd}, \beta)$. The outage probability in Rayleigh fading is given by

$$\begin{aligned} \mathrm{P_{out}} &= \Pr\left\{\alpha_{sd} < \frac{2^\beta - 1}{P}\right\} = 1 - \exp\left(\frac{1 - 2^\beta}{\gamma_{sd}P}\right) \\ &= \frac{2^\beta - 1}{\gamma_{sd}P} + O\left(\frac{1}{P^2}\right) \end{aligned} \tag{1}$$

from which it is clear that no diversity is achieved.[1]

2) Conventional DF Relaying with Repetition Coding [4]: In this baseline we consider decode-and-forward based collaborative transmission where the source and the relay use equal block lengths (i.e., $T_s = T_r = \frac{T}{2}$) but not necessarily the same power. If the relay successfully decodes the message received from the source, it re-encodes the message using the same channel code. Otherwise the relay remains silent. When the relay cooperates, the destination receives two copies of the message. Thereby, the destination may use either selection combining or maximum-ratio combining (MRC). We consider only MRC here, since it is optimal. See Fig. 1(b).

With MRC at the destination the outage event is [8]

$$\mathcal{O}(P_s\alpha_{sd}, 2\beta) \bigcap \left[\mathcal{O}(P_s\alpha_{sr}, 2\beta) \bigcup \mathcal{O}\big(P_r\alpha_{rd} + P_s\alpha_{sd}, 2\beta\big)\right] \tag{2}$$

[1]Hereafter, $f(x) = O(g(x))$ means that there exists $\Omega \in \mathbb{R}$ and $M \in \mathbb{R}$ such that $\left|\frac{f(x)}{g(x)}\right| \leq M$ whenever $x > \Omega$.

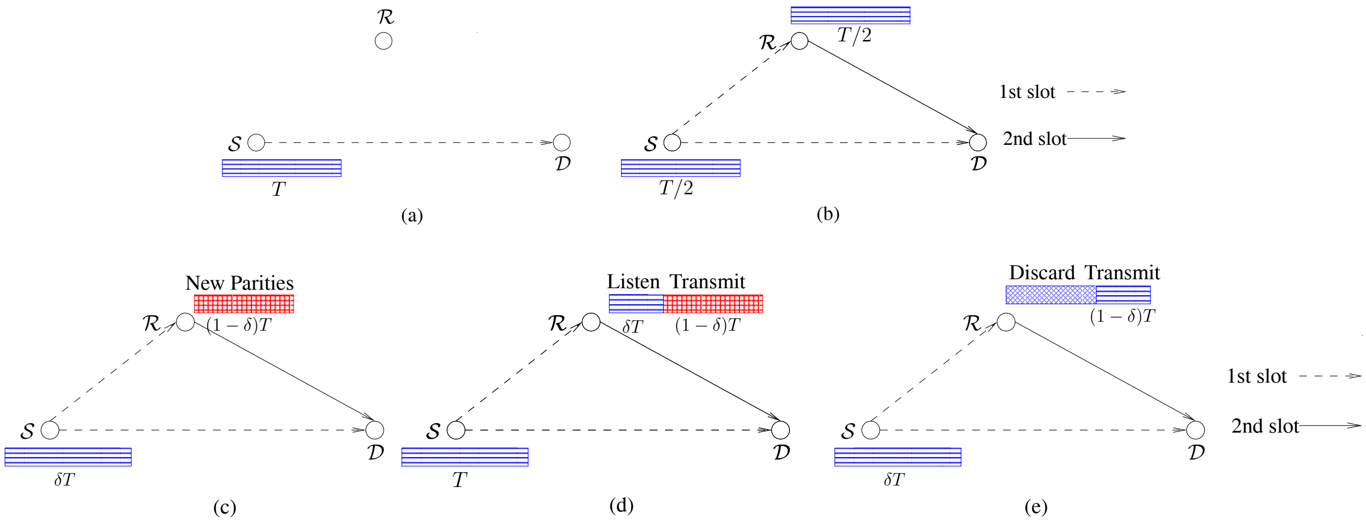

Fig. 1. Schematic of the network: (a) Direct transmission only. Here the relay does not participate in the transmission. (b) Conventional decode-and-forward relaying with repetition coding. Here the relay repeats all the regenerated data. (c) Decode-and-forward with parallel coding. Here the relay re-encodes the received data with an independent channel code to obtain new parity bits. (d) Dynamic decode-and-forward (DDF). Here the relay listens until it is able to decode the message. It then transmits during the rest of available channel uses. (e) Proposed *partial repetition* decode-and-forward scheme. Here the relay retransmits a part $1-\delta$ of the regenerated data using repetition coding, and discards the rest.

where $P_s + P_r = 2P$. Note that the events $\mathcal{O}(P_s\alpha_{sd}, 2\beta)$ and $\mathcal{O}\big(P_r\alpha_{rd} + P_s\alpha_{sd}, 2\beta\big)$ are not independent. In [12], it is shown that (2) is equivalent to the following more convenient expression:

$$\underbrace{\Big[\mathcal{O}(P_s\alpha_{sd}, 2\beta) \bigcap \mathcal{O}(P_s\alpha_{sr}, 2\beta)\Big]}_{\triangleq \mathcal{O}_1} \bigcup \underbrace{\Big[\mathcal{O}^c(P_s\alpha_{sr}, 2\beta) \bigcap \mathcal{O}\big(P_r\alpha_{rd} + P_s\alpha_{sd}, 2\beta\big)\Big]}_{\triangleq \mathcal{O}_2}, \quad (3)$$

where $\mathcal{O}^c$ denotes the complementary outage event. We therefore have

$$\begin{aligned} \mathrm{P_{out}} &= \Pr\Big(\mathcal{O}_1 \bigcup \mathcal{O}_2\Big) \\ &\overset{(a)}{=} \Pr(\mathcal{O}_1) + \Pr(\mathcal{O}_2) \\ &\overset{(b)}{=} \Pr\{\mathcal{O}(P_s\alpha_{sd}, 2\beta)\}\Pr\{\mathcal{O}(P_s\alpha_{sr}, 2\beta)\} + \\ &\quad \Pr\{\mathcal{O}^c(P_s\alpha_{sr}, 2\beta)\}\Pr\{\mathcal{O}\big(P_r\alpha_{rd} + P_s\alpha_{sd}, 2\beta\big)\} \end{aligned} \quad (4)$$

where (a) follows from the fact that the outage events $\mathcal{O}_1$ and $\mathcal{O}_2$ are disjoint and (b) follows from the fact that α_{sd}, α_{sr} and α_{rd} are mutually independent.

Using the result in Appendix A, the outage probability can be calculated to be as in Equation (5); on top of the next page.

By performing a series expansion it can be shown that

$$\mathrm{P_{out}} = \left(2^{2\beta} - 1\right)^2 \frac{1}{\gamma_{sd}P_s}\left[\frac{1}{\gamma_{sr}P_s} + \frac{1}{2\gamma_{rd}P_r}\right] + O\left(\frac{1}{P^3}\right). \quad (6)$$

We see from (6) that this scheme provides a diversity order of two, as long as P_s and P_r are nonzero.

B. DF with Parallel Coding

Next we derive the outage probability of decode-and-forward with parallel coding at the relay [7], [10], [11], [14], i.e., the relay and the source use different channel codes. If the relay decodes the transmitted message without error, it first re-encodes the message using an independent random code which is different from the channel code used at the source. It then re-transmits new information about the message in the form of a new set of parity bits. Let δ be the fraction of the channel uses that the source consumes so that $T_s = \delta T$. If the relay decodes the received message successfully, it forwards the *new parity bits* using $T_r = (1-\delta)T$ channel uses. See Fig. 1(c). The outage event is given by

$$\mathcal{O}(P_s\alpha_{sd}, \beta/\delta) \bigcap \Big[\mathcal{O}(P_s\alpha_{sr}, \beta/\delta) \bigcup \tilde{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)\Big] \quad (7)$$

where $\tilde{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)$ is defined according to

$$\tilde{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta) \iff \Big\{\delta\log_2(1 + P_s\alpha_{sd}) + (1-\delta)\log_2(1 + P_r\alpha_{rd}) < \beta\Big\}. \quad (8)$$

In (8), $\delta\log_2(1 + P_s\alpha_{sd})$ corresponds to the information flow from $\mathcal{S}$ to $\mathcal{D}$ via the direct link and $(1-\delta)\log_2(1 + P_r\alpha_{rd})$ corresponds to the information flow from $\mathcal{R}$ to $\mathcal{D}$. The probability of $\tilde{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)$ is calculated in Appendix B. Using the same approach as in (3) and (4), the probability of the outage event in (7) can be calculated to be as given by (9) (on top of the next page) where $\beta_s \triangleq \frac{\beta}{\delta}$ and $\beta_r \triangleq \frac{\beta}{1-\delta}$. It is clearly seen that this scheme provides a diversity order of two as well.

C. Dynamic Decode-and-Forward (DDF)

With dynamic decode-and-forward (DDF) [24], the relay listens until it is able to successfully decode the transmitted message from $\mathcal{S}$. Once $\mathcal{R}$ decodes the message, say after the time δT, it starts transmitting the message using a random Gaussian codebook which is independent of the one used at $\mathcal{S}$. The relayed transmission consumes $(1-\delta)T$ channel uses.

$$\mathrm{P_{out}} = \begin{cases} \left[1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}P_s}\right)\right]\left[1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{sd}P_s}\right)\right]+ \\ \exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}P_s}\right)\Bigg[1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}P_r}\right)-\frac{\gamma_{sd}P_s}{\gamma_{sd}P_s-\gamma_{rd}P_r}\exp\left(\frac{1-2^{2\beta}}{\gamma_{sd}P_s}\right)\times \\ \left[1-\exp\left(\frac{(\gamma_{rd}P_r-\gamma_{sd}P_s)}{\gamma_{sd}\gamma_{rd}P_sP_r}(2^{2\beta}-1)\right)\right]\Bigg], & \text{if } \gamma_{sd}P_s\neq\gamma_{rd}P_r \\ \left[1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}P_s}\right)\right]\left[1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{sd}P_s}\right)\right]+ \\ \exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}P_s}\right)\left[1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}P_r}\right)-\frac{2^{2\beta}-1}{\gamma_{rd}P_r}\exp\left(\frac{1-2^{2\beta}}{\gamma_{sd}P_s}\right)\right], & \text{if } \gamma_{sd}P_s=\gamma_{rd}P_r \end{cases} \tag{5}$$

$$\mathrm{P_{out}} = \begin{cases} \left(2^{\beta_s}-1\right)^2\frac{1}{\gamma_{sd}\gamma_{sr}P_s^2}+\left(1-2^{\beta_r}+\frac{\delta}{2\delta-1}2^{\beta_s}\left(2^{\beta_r\frac{2\delta-1}{\delta}}-1\right)\right)\frac{1}{\gamma_{sd}\gamma_{rd}P_sP_r}+O\left(\frac{1}{P^3}\right), & \text{if } \delta\neq\frac{1}{2} \\ \left(2^{2\beta}-1\right)^2\frac{1}{\gamma_{sd}\gamma_{sr}P_s^2}+\left(1-2^{2\beta}+2\ln(2)\beta 2^{2\beta}\right)\frac{1}{\gamma_{sd}\gamma_{rd}P_sP_r}+O\left(\frac{1}{P^3}\right), & \text{if } \delta=\frac{1}{2} \end{cases} \tag{9}$$

See Fig. 1(d). In case the relay cannot decode the message even though it has listened for the entire frame duration, it remains silent. Since $\mathcal{R}$ and $\mathcal{S}$ may transmit simultaneously, DDF is a non-orthogonal scheme. One possible solution to avoid interference from the direct link during the second phase would be to use one bit of feedback from $\mathcal{R}$ to $\mathcal{S}$ to ask $\mathcal{S}$ to stop transmitting.[2]

In what follows we analyze the outage event of DDF. If $\mathcal{R}$ is in outage even when it has listened during the entire frame, the outage event can be written as

$$\mathcal{O}(P_s\alpha_{sr},\beta)\bigcap\mathcal{O}(P_s\alpha_{sd},\beta). \tag{10}$$

Otherwise, $\mathcal{R}$ can decode the message after listening for $T_s=\delta T$ channel uses where

$$\delta=\min\left\{1,\frac{\beta}{\log_2(1+P_s\alpha_{sr})}\right\}. \tag{11}$$

The overall outage event is therefore given by

$$\left[\mathcal{O}(P_s\alpha_{sr},\beta)\bigcap\mathcal{O}(P_s\alpha_{sd},\beta)\right] \bigcup\left[\mathcal{O}^c(P_s\alpha_{sr},\beta)\bigcap\breve{\mathcal{O}}(P_s\alpha_{sd},P_r\alpha_{rd},\delta,\beta)\right] \tag{12}$$

where

$$\breve{\mathcal{O}}(P_s\alpha_{sd},P_r\alpha_{rd},\delta,\beta)\Longleftrightarrow\Big\{\delta\log(1+P_s\alpha_{sd}) +(1-\delta)\log(1+P_s\alpha_{sd}+P_r\alpha_{rd})<\beta\Big\}. \tag{13}$$

Here $\delta\log_2(1+P_s\alpha_{sd})$ represents the information in the part of the data which has been transmitted only by the source, and $(1-\delta)\log_2(1+P_s\alpha_{sd}+P_r\alpha_{rd})$ represents the information contained in the symbols simultaneously transmitted by the relay and the source. Since $\mathcal{O}(P_s\alpha_{sr},\beta)$ and $\mathcal{O}^c(P_s\alpha_{sr},\beta)$ are disjoint, the probability of the outage event in (12) when δ is chosen according to (11) is

$$\begin{aligned}\mathrm{P_{out}} &= \Pr\left\{\mathcal{O}(P_s\alpha_{sr},\beta)\bigcap\mathcal{O}(P_s\alpha_{sd},\beta)\right\} \\ &\quad+\Pr\Bigg\{\underbrace{\mathcal{O}^c(P_s\alpha_{sr},\beta)\bigcap\breve{\mathcal{O}}(P_s\alpha_{sd},P_r\alpha_{rd},\delta,\beta)}_{\triangleq\mathcal{O}_1}\Bigg\} \\ &= \frac{(2^\beta-1)^2}{\gamma_{sd}\gamma_{rd}P_sP_r}+\Pr\{\mathcal{O}_1\}+O\left(\frac{1}{P^3}\right)\end{aligned}$$

The probability of $\mathcal{O}_1$ is calculated in Appendix C. The probability of the outage event in (12) is then given by

$$\mathrm{P_{out}}=\frac{\left(2^\beta-1\right)^2+\omega(P_s\gamma_{sr},P_s\gamma_{sd},P_r\gamma_{rd})}{P_sP_r\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{P^3}\right) \tag{14}$$

where

$$\omega(P_s\gamma_{sr},P_s\gamma_{sd},P_r\gamma_{rd})\triangleq P_r\gamma_{rd}\int_0^1 g(\delta) \int_0^{2^\beta-1}\exp\left(\frac{-t}{P_s\gamma_{sd}}\right)\left[1-\exp\left(-\frac{2^{\frac{\beta}{1-\delta}}(1+t)^{\frac{\delta}{\delta-1}}-t-1}{P_r\gamma_{rd}}\right)\right]dt\,d\delta \tag{15}$$

and

$$g(\delta)\triangleq\frac{\ln(2)}{P_s\gamma_{sr}}\frac{\beta}{\delta^2}2^{\frac{\beta}{\delta}}\exp\left(\frac{1-2^{\frac{\beta}{\delta}}}{P_s\gamma_{sr}}\right). \tag{16}$$

The function $\omega(P_s\gamma_{sr},P_s\gamma_{sd},P_r\gamma_{rd})$ can be evaluated numerically. An example plot of $\omega(\cdot,\cdot,\cdot)$ is shown in Fig. 2. Since $\omega(P_s\gamma_{sr},P_s\gamma_{sd},P_r\gamma_{rd})$ is bounded for all SNR, this scheme also provides a diversity order of two.

D. Proposed Scheme: DF with Partial Repetition

We next introduce our new proposed collaborative scheme based on *partial repetition coding*. We will assume that the source uses a fraction δ of channel uses and that the relay uses a fraction $1-\delta$ of channel uses, where $\delta>0.5$. Since the relay uses repetition coding and since $1-\delta<\delta$, the relay cannot transmit all the regenerated data during the $(1-\delta)T$ channel uses allocated to it. Thereby, the relay only retransmits

[2]Generally, non-orthogonal transmission is superior to orthogonal transmission, but at the cost of higher complexity and potentially very difficult synchronization problems.

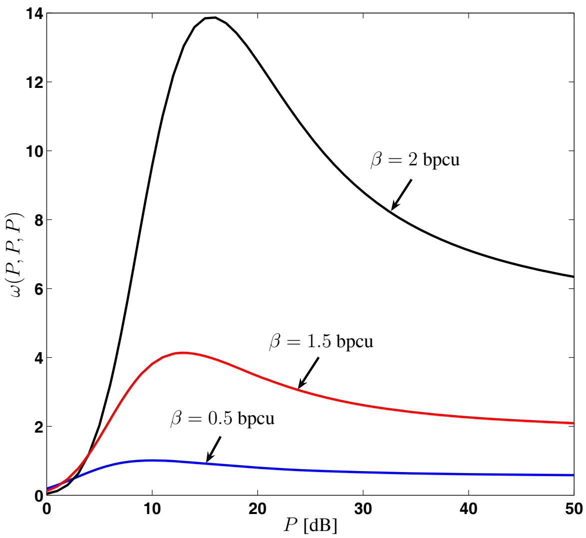

Fig. 2. Plot of $\omega(P,P,P)$ for different spectral efficiencies.

a fraction $\frac{1-\delta}{\delta}$ of the data and discards the remaining part. See Fig. 1(e). We define the "cooperation level" as $\eta \triangleq \frac{1-\delta}{\delta}$. For $\delta = 0.5$, the scheme reduces to conventional repetition coding with full cooperation at the relay. That is, the relay transmits all regenerated data and $\eta = 1$. Choosing δ close to 1 provides *marginal* cooperation (i.e., the relay transmits only a small part of the regenerated data) and $\eta \approx 0$. For $\delta = 1$ the scheme reduces to direct transmission. Thereby the proposed scheme can never be worse than direct-link-only transmission, provided that δ is properly chosen.

Having received two signals, from the source and from the relay, the destination performs MRC of the "common part of the message" transmitted by both the source and the relay, but considers the remaining part of the message separately. The outage event is thus given by

$$\mathcal{O}(P_s\alpha_{sd}, \beta/\delta) \bigcap \left[\mathcal{O}(P_s\alpha_{sr}, \beta/\delta) \bigcup \bar{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)\right] \tag{17}$$

where

$$\bar{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta) \Longleftrightarrow \Big\{(2\delta-1)\log_2(1+P_s\alpha_{sd}) + (1-\delta)\log_2(1+P_s\alpha_{sd}+P_r\alpha_{rd}) < \beta\Big\}. \tag{18}$$

Here $(1-\delta)\log_2(1+P_s\alpha_{sd}+P_r\alpha_{rd})$ represents the information contained in the bits repeated by the relay, and $(2\delta-1)\log_2(1+P_s\alpha_{sd}) = [\delta-(1-\delta)]\log_2(1+P_s\alpha_{sd})$ represents the information in the part of the data that were not repeated by $\mathcal{R}$. The probability of $\bar{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)$ is computed in Appendix D. Using the same approach as in (3) and (4), the probability of the outage event in (17), after some calculations, is found to be given by (19) (on top of the next page) where $\beta_s \triangleq \frac{\beta}{\delta}$ and $\beta_r \triangleq \frac{\beta}{1-\delta}$. This scheme also achieves a diversity order of two.

III. Resource Allocation for Collaborative Schemes

In this section we present explicit methods to allocate radio resources (i.e., choosing P_s, P_r, and when applicable, δ) for the collaborative schemes discussed in Section II. We consider the high-SNR regime where we can neglect the $O\left(\frac{1}{P^3}\right)$ terms in the outage probability expressions. All calculations in this section will be based on the assumption that the average SNRs of the links (i.e., γ_{sd}, γ_{sr} and γ_{rd}) are known, but that $\mathcal{S}$ and $\mathcal{R}$ have no instantaneous forward channel state information (see the remark at the end of Section I-B).

A. Conventional DF with Repetition Coding

Using (6) the optimal choice of (P_s, P_r) can be obtained by minimization of

$$J(P_s, P_r) = \frac{1}{\gamma_{sr}P_s^2} + \frac{1}{2\gamma_{rd}P_rP_s} \tag{20}$$

with respect to P_s and P_r, subject to $0 \le P_s \le 2P$, $0 \le P_r \le 2P$, and $P_s + P_r = 2P$.

B. DF with Parallel Coding

Using (9), the optimal (P_s, P_r, δ) can be obtained by minimization of

$$J(P_s, P_r, \delta) = \frac{\left(2^{\beta_s}-1\right)^2}{\gamma_{sr}P_s^2} + \frac{1-2^{\beta_r}+\frac{\delta}{2\delta-1}2^{\beta_s}\left(2^{\beta_r\frac{2\delta-1}{\delta}}-1\right)}{\gamma_{rd}P_sP_r} \tag{21}$$

with respect to P_s, P_r and δ, subject to $0 < \delta < 1$, $0 \le P_s \le \frac{P}{\delta}$, $0 \le P_r \le \frac{P}{1-\delta}$, and $\delta P_s + (1-\delta)P_r = P$.

C. DF with Partial Repetition (Proposed Scheme)

Using (19), the optimal (P_s, P_r, δ) can be obtained by minimization of

$$J(P_s, P_r, \delta) = \frac{\left(1-2^{\beta_s}\right)^2}{\gamma_{sr}P_s^2} + \frac{1-2^{\beta_s}}{\gamma_{rd}P_rP_s} - \frac{0.5\left(1-2^{\beta_s}\right)^2 - \frac{1-\delta}{2-3\delta}\left(2^{2\beta_s}-2^{\beta_r}\right)}{\gamma_{rd}P_rP_s} \tag{22}$$

with respect to P_s, P_r and δ, subject to $0.5 < \delta < 1$, $0 \le P_s \le \frac{P}{\delta}$, $0 \le P_r \le \frac{P}{1-\delta}$, and $\delta P_s + (1-\delta)P_r = P$.

IV. Comparisons and Simulation Results

In this section, we present some analytical and empirical results to compare the performance of the DF collaborative schemes. For these results we assume a log-distance path loss model so that $\gamma_{ij} = \frac{1}{d_{ij}^{\alpha}}$ where α is the path loss exponent and d_{ij} is the normalized distance from node i to node j. Throughout we take $\alpha = 4$.

Fig. 3 shows the optimum choice of δ (δ_{opt}) for decode-and-forward with parallel coding and with partial repetition when all nodes lie on a straight line, i.e., $d_{sd} = 1$, $d_{rd} = 1 - d_{sr}$, and $P_s = P_r = P$. The optimal value of δ is found using an exhaustive grid search over the feasible set of solutions to (21) and (22). It can be seen that the optimal δ increases as d_{sr} increases for a given spectral efficiency. In other words, the optimal cooperation level (η) decreases as d_{sr} increases. When the relay is located close to the source, the optimal δ for parallel coding is 0.5 since by symmetry the codeword

$$\mathrm{P_{out}} = \begin{cases} \left(1-2^{\beta_s}\right)^2 \frac{1}{\gamma_{sr}\gamma_{sd}P_s^2} + \left(1-2^{\beta_s}-0.5\left(1-2^{\beta_s}\right)^2 + \frac{1-\delta}{2-3\delta}\left(2^{2\beta_s}-2^{\beta_r}\right)\right)\frac{1}{\gamma_{rd}\gamma_{sd}P_rP_s} + O\left(\frac{1}{P^3}\right), \text{ if } \delta \neq \frac{2}{3} \\ \left(1-2^{1.5\beta}\right)^2 \frac{1}{\gamma_{sr}\gamma_{sd}P_s^2} + \left(1-2^{1.5\beta}-0.5\left(1-2^{1.5\beta}\right)^2 + 1.5\ln(2)\beta 2^{3\beta}\right)\frac{1}{\gamma_{rd}\gamma_{sd}P_rP_s} + O\left(\frac{1}{P^3}\right), \text{ if } \delta = \frac{2}{3} \end{cases} \quad (19)$$

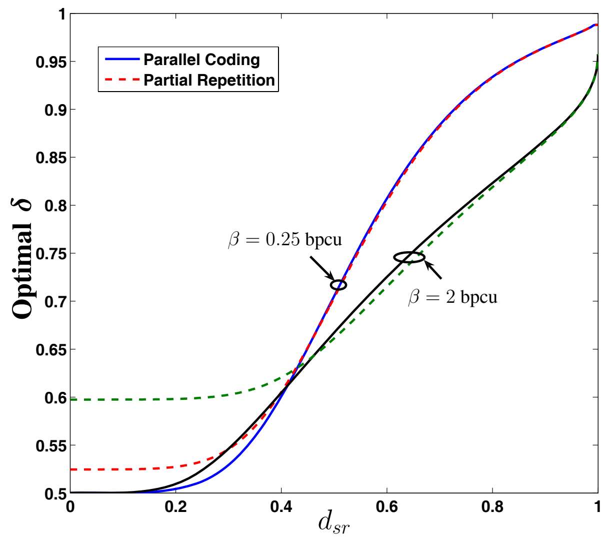

Fig. 3. Plots of the optimal δ as a function of d_{sr} for different β, when $d_{sd} = 1$, $d_{rd} = 1 - d_{sr}$, $P_s = P_r = P$, and $\alpha = 4$.

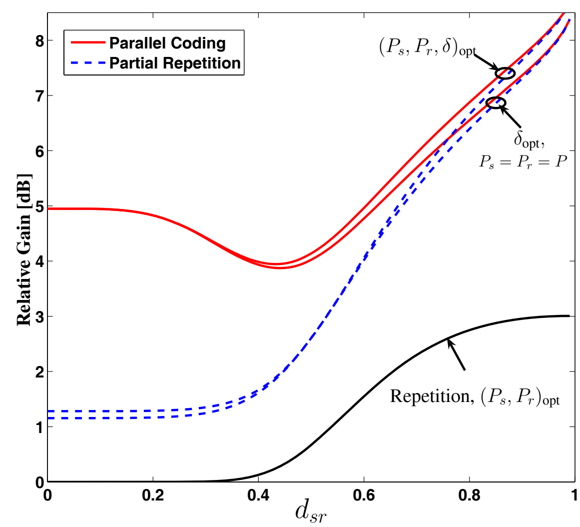

Fig. 5. Same as Fig. 4 but for $\beta = 3$ bpcu (note the different scale).

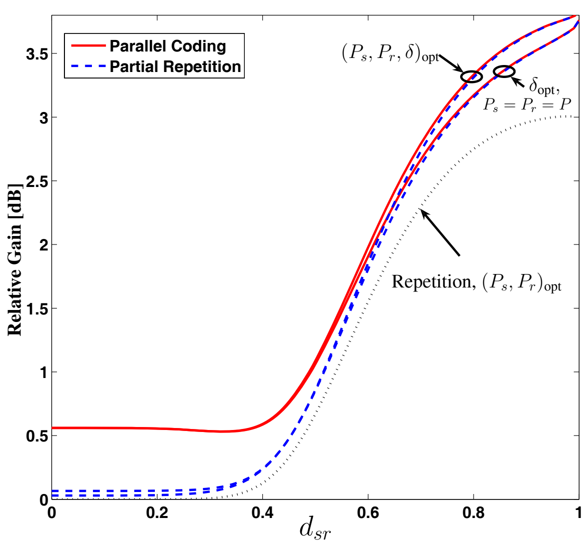

Fig. 4. The gain of DF with parallel coding and partial repetition with optimum δ over (unoptimized) repetition coding with $\delta = 0.5$ as a function of d_{sr} for $\beta = 0.5$ bpcu, when $d_{sd} = 1$, $d_{rd} = 1 - d_{sr}$, and $\alpha = 4$.

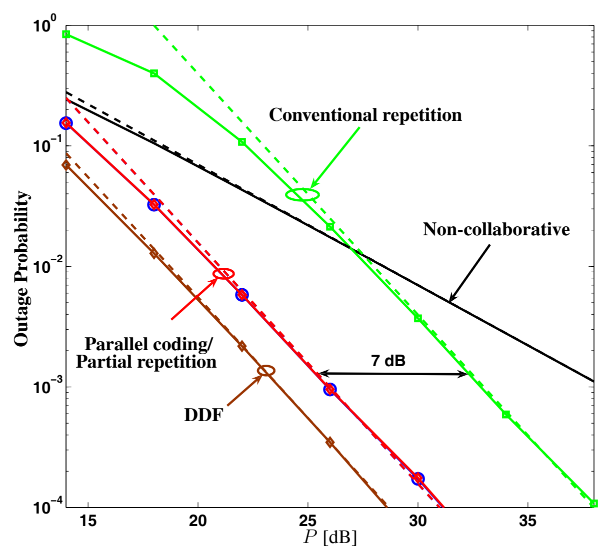

Fig. 6. Outage probability of collaborative DF schemes for $\beta = 3$ bpcu, when $d_{sd} = d_{sr} = 1$, $d_{rd} = 0.1$, and $\alpha = 4$. The solid lines are the analytical results. The dashed curves are high-SNR asymptotes obtained by dropping the $O\left(\frac{1}{P^3}\right)$ terms. The marks denote simulation results.

produced by the source and that produced by the relay should have the same "value". By contrast, for DF with partial repetition coding, $\delta_{\mathrm{opt}} > 0.5$ even when the relay is very close to the source. This is because the relay merely *repeats* what has been already sent to the destination via the direct link. Moreover, it can be seen that the optimal δ for both parallel coding and for partial repetition coding approaches the same value as d_{sr} increases.

Fig. 4 shows the gain of optimized partial repetition coding and of parallel coding, over conventional repetition coding with $\delta = 0.5$ and $P_s = P_r = P$, as a function of d_{sr}, when $d_{sd} = 1$, $d_{rd} = 1 - d_{sr}$, and $\beta = 0.5$ bpcu. The results have been obtained using (20), (21), and (22) where we have neglected the term $O\left(\frac{1}{P^3}\right)$. Thus the gains correspond to the high-SNR asymptotes. The power optimization of conventional DF with repetition coding can provide up

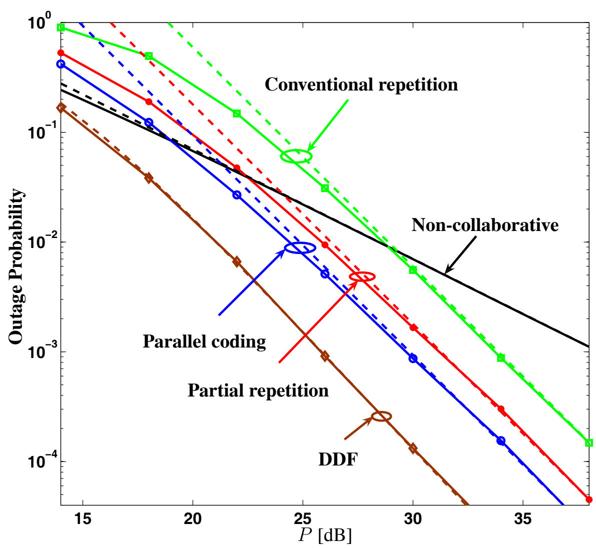

Fig. 7. Same as Fig. 6 but for $d_{rd} = 1$.

to a 3 dB gain. When the relay is located close to the source, power optimization provides a negligible gain since $(P_s, P_r)_{\text{opt}} \approx (P, P)$. For partial repetition with equal power at $\mathcal{S}$ and $\mathcal{R}$, the gain increases with d_{sr}, and somewhat surprisingly approaches that of parallel coding. This means that by forwarding only a part of the data at the relay, one can obtain a gain which is comparable to that of parallel coding for $d_{sr} > 0.5$. At low spectral efficiency and small d_{sr}, the gain of our proposed partial repetition over conventional repetition is almost negligible since $\delta_{\text{opt}} \approx 0.5$ or equivalently $\eta_{\text{opt}} \approx 1$. By joint optimization of power and bandwidth, the power gain increases when $d_{sr} > 0.5$. When $\mathcal{S}$ and $\mathcal{R}$ are close to each other, power optimization does not bring any extra gain. Fig. 5 shows the corresponding results for a higher spectral efficiency, $\beta = 3$ bpcu. The gain obtained by power optimization for DF with conventional repetition does not change when varying the spectral efficiency, which can be easily deduced from (6). However, the power gain of optimum bandwidth allocation or joint power and bandwidth allocation increases with the spectral efficiency for both parallel coding and partial repetition.

Fig. 6 shows the outage probability of the discussed schemes for $\beta = 3$ bpcu, $d_{sd} = d_{sr} = 1$, and $d_{rd} = 0.1$ as a function of the SNR. It can be seen that both partial cooperation and parallel coding provide the same performance when $P_s = P_r = P$ and $\delta = \delta_{\text{opt}}$. The power gain over conventional repetition when $\delta = 0.5$ is 7 dB at high SNR. DDF performs best with respect to other collaborative DF schemes since δ is optimized according to the *instantaneous* SNR of the $\mathcal{S}$-$\mathcal{R}$ link and since it is a non-orthogonal scheme. Fig. 7 shows the corresponding outage probabilities for $d_{rd} = 1$. Here partial repetition outperforms conventional repetition by 2.6 dB. In addition, parallel coding provides a 1.4 dB gain for this case at high SNR. For this case DDF also performs best. The dashed curves in Figs. 6 and 7 are plotted using the analytical expressions but neglecting the $O\left(\frac{1}{P^3}\right)$ term. For all schemes, the high-SNR approximation (dashed curves) and the simulation result (circles) match well at high SNR.

V. Conclusions

We have proposed a new scheme, *partial repetition* (PR), for half-duplex relaying, based on decode-and-forward. The idea is to let the relay use repetition coding, but only forward *a fraction* of the message that it receives from the source. Our method has two major advantages, which distinguishes itself from competing schemes. First, the fraction of the message that is repeated can be optimized based on either the available short-term (instantaneous) or long-term (average) channel state information. This adaptation can be made on the fly, without changing the structure or the type of the underlying channel code. Second, the receiver at the destination has very low complexity; namely, it simply consists of a maximum-ratio-combiner followed by a soft-input channel decoder for the channel code used at the source.

We have analytically quantified the finite-SNR performance of our new scheme, and presented closed-form expressions for its outage probability. For comparison purposes, we also derived analytically the finite-SNR outage performance of decode-and-forward using parallel coding (PC) [7], [10], [11], [14], and of dynamic decode-and-forward (DDF) [24]. We showed that the performance of our scheme can approach that of PC under certain circumstances (for example, when all nodes lie on a straight line and the relay is not far from the destination; see Figs. 5–6), while it maintains a performance gap to DDF. This should be understood in the light of the high implementation complexity (primarily at the destination) associated with PC and DDF. More precisely, while the optimal receiver for our PR scheme only consists of a linear combiner followed by a standard channel decoder, PC and DDF require code combining at the destination. Additionally, DDF is a non-orthogonal scheme in that the source and relay may transmit simultaneously, leading to fundamentally difficult synchronization problems. DDF also requires signaling traffic between the nodes that goes well beyond the assumptions that our new scheme makes on the network.

Appendix A
Probability of $\mathcal{O}(P_r\alpha_{rd} + P_s\alpha_{sd}, 2\beta)$

Consider

$$\begin{aligned}
P_{MRC} &\triangleq \Pr\left\{\mathcal{O}(P_r\alpha_{rd} + P_s\alpha_{sd}, 2\beta)\right\} \\
&= \int \Pr\left\{P_r\alpha_{rd} + P_s\alpha_{sd} < 2^{2\beta} - 1 | \alpha_{rd} = t\right\} f_{\alpha_{rd}}(t)dt \\
&= \int_0^{\frac{2^{2\beta}-1}{P_r}} \left[1 - \exp\left(\frac{tP_r - (2^{2\beta}-1)}{\gamma_{sd}P_s}\right)\right] \\
&\quad \times \frac{1}{\gamma_{rd}} \exp\left(\frac{-t}{\gamma_{rd}}\right) dt \\
&= 1 - \exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}P_r}\right) \\
&\quad - \underbrace{\int_0^{\frac{2^{2\beta}-1}{P_r}} \frac{1}{\gamma_{rd}} \exp\left(\frac{tP_r - (2^{2\beta}-1)}{\gamma_{sd}P_s} - \frac{t}{\gamma_{rd}}\right) dt}_{\triangleq A}
\end{aligned}$$

$$P_{MRC} = \begin{cases} 1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}P_r}\right) - \frac{\gamma_{sd}P_s}{\gamma_{sd}P_s-\gamma_{rd}P_r}\exp\left(\frac{1-2^{2\beta}}{\gamma_{sd}P_s}\right)\left[1-\exp\left(\frac{(\gamma_{rd}P_r-\gamma_{sd}P_s)}{\gamma_{sd}\gamma_{rd}P_sP_r}(2^{2\beta}-1)\right)\right], & \text{if } \gamma_{sd}P_s \neq \gamma_{rd}P_r \\ 1-\exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}P_r}\right) - \frac{2^{2\beta}-1}{\gamma_{rd}P_r}\exp\left(\frac{1-2^{2\beta}}{\gamma_{sd}P_s}\right), & \text{if } \gamma_{sd}P_s = \gamma_{rd}P_r \end{cases} \quad (23)$$

where A can be further simplified by separately considering the two cases $\gamma_{sd}P_s \neq \gamma_{rd}P_r$ and $\gamma_{sd}P_s = \gamma_{rd}P_r$. The final result is given by (23), on top of the next page.

Appendix B
Probability of $\tilde{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)$

The probability of $\tilde{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)$ in (8) can be written as follows:

$$\tilde{\mathrm{P}} \triangleq \Pr\{\delta x + (1-\delta)y < \beta\} \quad (24)$$

where $x \triangleq \log_2(1+P_s\alpha_{sr})$ and $y \triangleq \log_2(1+P_r\alpha_{rd})$. The probability density function (pdf) of the random variables x and y can be calculated as

$$f_x(t) = \frac{\ln 2}{\gamma_{sd}P_s}\exp\left(\frac{1-2^t}{\gamma_{sd}P_s}\right)2^t, \quad t \geq 0$$
$$f_y(t) = \frac{\ln 2}{\gamma_{rd}P_r}\exp\left(\frac{1-2^t}{\gamma_{rd}P_r}\right)2^t, \quad t \geq 0 \quad (25)$$

Thus,

$$\begin{aligned}\tilde{\mathrm{P}} &= \int_0^{\frac{\beta}{1-\delta}} \Pr\left\{x + \frac{1-\delta}{\delta}y < \frac{\beta}{\delta}\Big| y=t\right\} f_y(t)dt \\ &= \int_0^{\frac{\beta}{1-\delta}} \Pr\left\{x < \frac{\beta}{\delta} - \frac{1-\delta}{\delta}t\right\} f_y(t)dt \\ &= \int_0^{\frac{\beta}{1-\delta}} \left[1-\exp\left(\frac{1-2^{\frac{\beta}{\delta}-\frac{1-\delta}{\delta}t}}{\gamma_{sd}P_s}\right)\right] f_y(t)dt \\ &= \underbrace{\int_0^{\frac{\beta}{1-\delta}} f_y(t)dt}_{\triangleq A} - \underbrace{\int_0^{\frac{\beta}{1-\delta}} \exp\left(\frac{1-2^{\frac{\beta}{\delta}-\frac{1-\delta}{\delta}t}}{\gamma_{sd}P_s}\right) f_y(t)dt}_{\triangleq B}\end{aligned}$$

where A and B can be evaluated as follows:

$$A = \frac{2^{\frac{\beta}{1-\delta}}-1}{\gamma_{rd}P_r} - \frac{\left(2^{\frac{\beta}{1-\delta}}-1\right)^2}{2\gamma_{rd}^2P_r^2} + O\left(\frac{1}{P^3}\right) \quad (26)$$

$$\begin{aligned}B &= \int_0^{\frac{\beta}{1-\delta}} \exp\left(\frac{1-2^{\frac{\beta}{\delta}-\frac{1-\delta}{\delta}t}}{\gamma_{sd}P_s}\right)\frac{\ln 2}{\gamma_{rd}P_r}\exp\left(\frac{1-2^t}{\gamma_{rd}P_r}\right)2^t dt \\ &= \underbrace{\int_0^{\frac{\beta}{1-\delta}} \ln(2)\frac{2^t}{\gamma_{rd}P_r}dt}_{\triangleq B_1} - \underbrace{\int_0^{\frac{\beta}{1-\delta}} \ln(2)\frac{2^{2t}-2^t}{\gamma_{rd}^2P_r^2}dt}_{\triangleq B_2} \\ &- \underbrace{\int_0^{\frac{\beta}{1-\delta}} \ln(2)\frac{2^t\left(2^{\frac{\beta}{\delta}-\frac{1-\delta}{\delta}t}-1\right)}{\gamma_{rd}\gamma_{sd}P_rP_s}dt}_{\triangleq B_3} + O\left(\frac{1}{P^3}\right)\end{aligned} \quad (27)$$

In (27) we used the following expansion

$$e^x = 1 + x + O(x^2).$$

This yields

$$B_1 = \frac{2^{\frac{\beta}{1-\delta}}-1}{\gamma_{rd}P_r}$$
$$B_2 = \frac{1-2^{\frac{\beta}{1-\delta}}}{\gamma_{rd}^2P_r^2} + \frac{2^{\frac{2\beta}{1-\delta}}-1}{2\gamma_{rd}^2P_r^2}$$
$$B_3 = \begin{cases} \frac{1-2^{\frac{\beta}{1-\delta}}}{\gamma_{rd}\gamma_{sd}P_rP_s} + \frac{2^{\frac{\beta}{\delta}}\left(2^{\frac{(2\delta-1)\beta}{\delta(1-\delta)}}-1\right)\delta}{(2\delta-1)\gamma_{rd}\gamma_{sd}P_rP_s} & \text{if } \delta \neq \frac{1}{2} \\ \frac{1-2^{2\beta}}{\gamma_{rd}\gamma_{sd}P_rP_s} + \frac{2\ln(2)\beta 2^{2\beta}}{\gamma_{rd}\gamma_{sd}P_sP_r} & \text{if } \delta = \frac{1}{2} \end{cases}$$

Therefore, the probability of the event in (24) is given by (28) (on top of the next page) where $\beta_s \triangleq \frac{\beta}{\delta}$ and $\beta_r \triangleq \frac{\beta}{1-\delta}$.

Appendix C
Probability of $\mathcal{O}_1$

Consider

$$\begin{aligned}\Pr\{\mathcal{O}_1\} &= \Pr\left\{\mathcal{O}^c(P_s\alpha_{sr},\beta)\bigcap\breve{\mathcal{O}}(P_s\alpha_{sd},P_r\alpha_{rd},\delta,\beta)\right\} \\ &\overset{(a)}{=} \Pr\left\{\mathcal{O}^c(P_s\alpha_{sr},\beta)\right\} \\ &\quad\times \Pr\left\{\breve{\mathcal{O}}(P_s\alpha_{sd},P_r\alpha_{rd},\delta,\beta)|\mathcal{O}^c(P_s\alpha_{sr},\beta)\right\}\end{aligned} \quad (29)$$

where (a) follows from the chain rule. If $\mathcal{O}^c(P_s\alpha_{sr},\beta)$ we have

$$\delta = \frac{\beta}{\log_2(1+P_s\alpha_{sr})}. \quad (30)$$

The pdf of δ conditioned on $\mathcal{O}^c(P_s\alpha_{sr},\beta)$ can be shown to be

$$f(\delta) = \frac{\ln(2)}{P_s\gamma_{sr}\varsigma}\frac{\beta}{\delta^2}2^{\frac{\beta}{\delta}}\exp\left(\frac{1-2^{\frac{\beta}{\delta}}}{P_s\gamma_{sr}}\right) \quad (31)$$

where

$$\varsigma \triangleq \Pr\left\{\mathcal{O}^c(P_s\alpha_{sr},\beta)\right\} = \exp\left(\frac{1-2^\beta}{P_s\gamma_{sr}}\right).$$

Thus, we obtain

$$\begin{aligned}\Pr\{\mathcal{O}_1\} &= \Pr\Big\{\delta\log(1+P_s\alpha_{sd}) \\ &\qquad +(1-\delta)\log(1+P_s\alpha_{sd}+P_r\alpha_{rd}) < \beta|\delta<1\Big\} \\ &= \iiint_\psi f_{\alpha_{sd}}(t)f_{\alpha_{rd}}(r)f(\delta)\, dtdrd\delta\end{aligned} \quad (32)$$

where $f_{\alpha_{sd}}(t)$ and $f_{\alpha_{rd}}(r)$ denote the pdf of α_{sr} and α_{rd} respectively. The integration region (ψ) is given by

$$\psi \triangleq \left\{(t,r,\delta) : (1+P_st+P_rr)^{1-\delta}(1+P_st)^\delta < 2^\beta,\ \delta<1\right\}$$

$$\tilde{\mathrm{P}} = \begin{cases} \left(1 - 2^{\beta_r} + \frac{\delta}{2\delta-1} 2^{\beta_s}\left(2^{\beta_r \frac{2\delta-1}{\delta}} - 1\right)\right) \frac{1}{\gamma_{rd}\gamma_{sd}P_rP_s} + O\left(\frac{1}{P^3}\right), \text{ if } \delta \neq \frac{1}{2} \\ \left(1 - 2^{2\beta} + 2\ln(2)\beta 2^{2\beta}\right) \frac{1}{\gamma_{rd}\gamma_{sd}P_rP_s} + O\left(\frac{1}{P^3}\right), \quad \text{if } \delta = \frac{1}{2} \end{cases} \tag{28}$$

$$\begin{aligned} \Pr\{\mathcal{O}_1\} &= \varsigma\int_0^1 f(\delta)d\delta \int_0^{\frac{2^\beta-1}{P_s}} f_{\alpha_{sd}}(t)dt \int_0^{\frac{2^{\frac{\beta}{1-\delta}}}{P_r(1+P_st)^{\frac{\delta}{1-\delta}}} - \frac{P_st+1}{P_r}} f_{\alpha_{rd}}(r)dr \\ &= \varsigma\int_0^1 f(\delta)d\delta \int_0^{\frac{2^\beta-1}{P_s}} \frac{1}{\gamma_{sd}}\left(1 - \exp\left(-\frac{2^{\frac{\beta}{1-\delta}}(1+P_st)^{\frac{\delta}{\delta-1}} - P_st - 1}{P_r\gamma_{rd}}\right)\right)\exp\left(\frac{-t}{\gamma_{sd}}\right)dt \\ &= \varsigma\int_0^1 f(\delta)d\delta \int_0^{2^\beta-1} \frac{1}{P_s\gamma_{sd}}\left[1 - \exp\left(-\frac{2^{\frac{\beta}{1-\delta}}(1+t)^{\frac{\delta}{\delta-1}} - t - 1}{P_r\gamma_{rd}}\right)\right]\exp\left(\frac{-t}{P_s\gamma_{sd}}\right)dt. \end{aligned} \tag{34}$$

$$\bar{\mathrm{P}} = \begin{cases} \left(1 - 2^{\beta_s} - 0.5\left(1 - 2^{\beta_s}\right)^2 + \frac{1-\delta}{2-3\delta}\left(2^{2\beta_s} - 2^{\beta_r}\right)\right) \frac{1}{\gamma_{rd}\gamma_{sd}P_rP_s} + O\left(\frac{1}{P^3}\right), \text{ if } \delta \neq \frac{2}{3} \\ \left(1 - 2^{1.5\beta} - 0.5\left(1 - 2^{1.5\beta}\right)^2 + 1.5\ln(2)\beta 2^{3\beta}\right) \frac{1}{\gamma_{rd}\gamma_{sd}P_rP_s} + O\left(\frac{1}{P^3}\right), \quad \text{if } \delta = \frac{2}{3} \end{cases} \tag{37}$$

After some manipulation it can be shown that the integration region is equivalent to

$$\psi = \Big\{(t,r,\delta) : t < \frac{2^\beta - 1}{P_s}, \quad r < \frac{2^{\frac{\beta}{1-\delta}}}{P_r(1+P_st)^{\frac{\delta}{1-\delta}}} - \frac{P_st+1}{P_r}, \ \delta < 1\Big\} \tag{33}$$

This yields (34), on top of this page.

Appendix D
Probability of $\bar{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)$

The probability of $\bar{\mathcal{O}}(P_s\alpha_{sd}, P_r\alpha_{rd}, \delta, \beta)$ can be written as follows

$$\begin{aligned} \bar{\mathrm{P}} &\triangleq \Pr\left\{(1 + P_s\alpha_{sd} + P_r\alpha_{rd})^{1-\delta}(1 + P_s\alpha_{sd})^{2\delta-1} < 2^\beta\right\} \\ &= \int\int_\psi f_{\alpha_{sd}}(t) f_{\alpha_{rd}}(r)dtdr \end{aligned} \tag{35}$$

where the integration region (ψ) is given by

$$\psi \triangleq \left\{(t,r) : (1 + P_st + P_rr)^{1-\delta}(1 + P_st)^{2\delta-1} < 2^\beta\right\}$$

After some manipulation it can be shown that the integration region is equivalent to

$$\psi = \left\{(t,r) : t < \frac{2^{\frac{\beta}{\delta}} - 1}{P_s}, r < \frac{2^{\frac{\beta}{1-\delta}}}{P_r(1+P_st)^{\frac{2\delta-1}{1-\delta}}} - \frac{P_st+1}{P_r}\right\}$$

Thereby, (35) can be written as

$$\bar{\mathrm{P}} = \int_0^{\frac{2^{\frac{\beta}{\delta}}-1}{P_s}} f_{\alpha_{sd}}(t) \int_0^{\frac{2^{\frac{\beta}{1-\delta}}}{P_r(1+P_st)^{\frac{2\delta-1}{1-\delta}}} - \frac{P_st+1}{P_r}} f_{\alpha_{rd}}(r)drdt \tag{36}$$

By using a series expansion, the probability of $\bar{\mathcal{O}}$ can be calculated as (37) (on top of this page) where $\beta_s = \frac{\beta}{\delta}$ and $\beta_r = \frac{\beta}{1-\delta}$.

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