Finite-SNR Analysis and Optimization of Decode-and-Forward Relaying over Slow Fading Channels

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Abstract

We provide analytical results on the finite-SNR outage performance of packet-based decode-and-forward relaying over a quasi-static fading channel, with different types of transmitter channel state information (CSI). At the relay we consider repetition coding (RC) and parallel coding (PC). At the destination we consider receivers based on selection combining (SC), code combining (CC), and maximum-ratio combining (MRC) (the latter only for the case of RC at the relay). Based on available CSI, we optimize the number of channel uses consumed by the source and by the relay for each packet. In doing so, we consider three different protocols that make use of different combinations of long-term CSI, 1-bit CSI and complete CSI, respectively, at the source node. Several interesting observations emerge. For example, we show that for high SNR, SC and CC provide the same outage probabilities when the source has perfect CSI.

Index Terms

Cooperative communications, relay channel, decode-and-forward, block length optimization, combining techniques, channel state information.

I. Introduction

Collaboration via a relay node for realizing transmit diversity in wireless networks has been recently proposed [1]–[14], [17]. The idea is that when the transmission of data from a source ($\mathcal{S}$) to a destination ($\mathcal{D}$) encounters unfavorable channel conditions, a relay ($\mathcal{R}$) may be employed to help by listening to the transmission from $\mathcal{S}$ and then forwarding it to $\mathcal{D}$. $\mathcal{D}$ may then combine whatever was heard directly from $\mathcal{S}$ and from $\mathcal{R}$. Finding the capacity of such a three-node network (consisting of $\mathcal{S}$, $\mathcal{R}$, and $\mathcal{D}$) is still an open problem, and therefore the "best" collaborative mode in general is unknown. One important and relatively simple way of collaborating is decode-and-forward (DF) relaying [2]. DF works by letting $\mathcal{R}$ decode the data packet, re-encode it, and transmit it to $\mathcal{D}$. In this paper we deal only with DF relaying. The main reason for this is that DF naturally permits the $\mathcal{S}-\mathcal{R}$ and $\mathcal{R}-\mathcal{D}$ links to operate with different spectral efficiencies, by choosing appropriate modulation and coding schemes. We also limit the discussion to half-duplex relays, i.e., relays that cannot transmit and listen on the same frequency simultaneously. Furthermore, we will deal only with quasi-static (slow fading) channels, since in this case it is possible to gain instantaneous channel state information (CSI) at $\mathcal{S}$ and $\mathcal{R}$ by using a feedback link from $\mathcal{D}$.

The two fundamental resources when conveying data from $\mathcal{S}$ to $\mathcal{D}$ are the number of channel uses (also referred to as the time-bandwidth product, the dimension, or the number of degrees of freedom of the channel) and the available transmit energy per packet. We will denote the number of available channel uses by T and the total energy per data frame by $E = PT$, where P is the transmit power. With DF, the transmission of a packet takes place in two phases. In the first phase, $\mathcal{S}$ transmits its data using T_s channel uses. During this phase, both $\mathcal{R}$ and $\mathcal{D}$ listen to the transmitted signal. In the second phase, provided that

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$\mathcal{R}$ successfully decoded the packet, $\mathcal{R}$ retransmits the packet using an appropriate transmission format. The second phase consumes $T_r = T - T_s$ channel uses. The channels used for the transmissions by $\mathcal{S}$ and $\mathcal{R}$ are orthogonal.

We shall assume that $\mathcal{S}$ and $\mathcal{R}$ operate under individual power constraints and that they transmit with a fixed power. This is natural in most applications. Additionally, there is some evidence that the possibility of trading power between $\mathcal{S}$ and $\mathcal{R}$ (under a joint power constraint) brings only a marginal gain when the relation between T_r and T_s is optimally chosen [12], [16]. This is so at least in the bandwidth-limited regime, i.e., in the regime of information theory where rate grows (only) logarithmically with power. In addition, one can argue that trading power between $\mathcal{S}$ and $\mathcal{R}$ is unrealistic in practice since these two nodes may have their own battery, which naturally gives an individual power constraint. Note also, that the optimization problem does not change if $\mathcal{S}$ and $\mathcal{R}$ have different individual power constraints: different individual power constraints can simply be incorporated into the model by appropriately adjusting the channel gains.

A. Contributions

This paper deals with the finite-SNR outage performance of DF-based relaying schemes. We establish a number of outage probability results in closed form. We also examine how the number of channel uses allotted to $\mathcal{S}$ and $\mathcal{R}$ (that is, T_s and T_r, respectively) should be optimally chosen in order to minimize the outage probability. For simplicity, we will refer to such channel use allocation as “bandwidth allocation”.

We will consider two different coding schemes at the relay:

- Repetition coding (RC): With RC, $\mathcal{R}$ forwards the packet using the same channel code as was used by $\mathcal{S}$. Thus the transmissions by $\mathcal{S}$ and $\mathcal{R}$ consume the same number of channel uses so we have $T_s = T_r = \frac{T}{2}$.[1]
- Parallel coding (PC): With PC, $\mathcal{R}$ uses a channel code different from that used by $\mathcal{S}$. For the analysis we will assume that $\mathcal{S}$ and $\mathcal{R}$ pick two independent Gaussian codebooks at random, and that these codes do not necessarily have the same rate. Hence, with PC we may have $T_s \neq T_r$; say $T_s = \delta T$ and $T_r = (1-\delta)T$ in general where $0 \leq \delta \leq 1$. That is δ and $1-\delta$ reflect the fractions of the time-bandwidth product used by $\mathcal{S}$ and by $\mathcal{R}$.

For both RC and PC, we consider two receiver structures at $\mathcal{D}$:

- Selection combining (SC): With SC, $\mathcal{D}$ considers the transmission successful *either* if it can decode the packet from $\mathcal{S}$ directly *or* if it can hear the transmission from $\mathcal{R}$.
- Optimum combining (OC): With OC, $\mathcal{D}$ optimally combines the information heard from $\mathcal{S}$ and that heard from $\mathcal{R}$. This reduces to maximum-ratio combination (MRC) when RC is used at $\mathcal{R}$, but it involves a code combining (CC) receiver when PC is used at $\mathcal{R}$.

When optimizing bandwidth (for the protocols where this is possible), we consider three different protocols that exploit different amounts of CSI at $\mathcal{S}, \mathcal{R}$ and $\mathcal{D}$ (see Table I). One of the main contributions of the paper is then to examine (for finite SNR) how much transmitter CSI can improve the performance of a DF relay link, when bandwidth allocation is optimally done.

B. Relation to Previous Literature

Our work contains two novel aspects. First, we provide analytical, finite-SNR performance results for a number of DF relaying protocols in closed form. This stands in contrast to most existing work on performance analysis of relay links [3]–[14] which either resorts to simulations, or to asymptotic measures such as diversity-multiplexing tradeoff. The second contribution is that we provide a framework for resource (channel use) optimization for a variety of combinations of CSI availability (cf. Table I). This extends previous work on resource allocation for relay channels, which has mostly focused on power

[1] There exist coding schemes based on repetition coding but for which $\mathcal{R}$ only repeats *a fraction* of the data heard from $\mathcal{S}$. See [16] for a detailed discussion of such a method.

TABLE I
THE DIFFERENT CSI-AVAILABILITY SITUATIONS CONSIDERED IN THE PAPER

	Case 1 (Sec. III)	Case 2 (Sec. IV)	Case 3 (Sec. V)
$\mathcal{S}$–$\mathcal{D}$	none*	one bit	one bit*
$\mathcal{S}$–$\mathcal{R}$	long-term	long-term	instantaneous (perfect)
$\mathcal{R}$–$\mathcal{D}$	long-term	long-term	none

*Knowledge of more complete CSI does not change the optimal bandwidth allocation in theses cases.

optimization for fixed bandwidth allocation [3]–[9]. In relation to existing papers that investigate channel use (bandwidth) optimization for DF relaying over quasi-static channels [10]–[12], we note the following. The work of [10] does not investigate the impact of feedback on the outage performance. Reference [11] optimizes delay-limited capacity while in this paper we consider outage probability (at fixed rate). Additionally, [10], [11] do not investigate different combining techniques at the destination. Reference [12] only considers repetition coding at the relay. Also, the performance results presented in [10]–[12] heavily rely on simulation. In addition to the aforementioned papers, there is a body of literature on bandwidth-power optimization for the *ergodic* relay channel (see for example [13], [14]), but this is a fundamentally different problem from that considered in the current paper. In [16] we proposed a DF scheme based on partial repetition. Therein we also performed a finite-SNR analysis of the proposed scheme and of some reference schemes. Relative to [16], the main contributions of the current paper are that we (i) study the effect of instantaneous channel quality feedback, and (ii) study the performance of different combining schemes at $\mathcal{S}$ (more precisely, SC, OC and CC).

C. Outline and Organization of the Paper

We next provide a brief outline of the remaining part of the paper:

- Section II presents the system model in more detail.
- Section III studies the performance of DF relaying with only long-term CSI at $\mathcal{S}$. It turns out that in this case, $\mathcal{S}$ does not need to know *anything* about the quality of the $\mathcal{S}-\mathcal{D}$ link; such knowledge does not affect performance. Hence we write "none" in the corresponding entry of Table I.
- Section IV investigates DF relaying with one-bit instantaneous CSI of the $\mathcal{S}-\mathcal{D}$ links and long-term CSI for the other links.
- Section V proceeds to consider the case when $\mathcal{S}$ has perfect CSI of the $\mathcal{S}-\mathcal{D}$ and $\mathcal{S}-\mathcal{R}$ links. It turns out that in this case, the same performance is achieved with one-bit CSI knowledge of the $\mathcal{S}-\mathcal{D}$ channel quality, and with full CSI knowledge of this link. (This is also reflected in Table I.)

For all schemes, we present analytical expressions for the outage performance. Section VI (see especially Table II) summarizes and numerically compares these results. In Section VII, some numerical examples are presented to verify the analytical results. Finally, Section VIII concludes the paper.

II. SYSTEM MODEL AND PRELIMINARIES

Figure 1 shows a schematic of the relay channel (consisting of a source $\mathcal{S}$, a relay $\mathcal{R}$ and a destination $\mathcal{D}$) that we study in this paper. The transmission consists of two phases. In the first phase, $\mathcal{S}$ transmits a signal x. The relay $\mathcal{R}$ receives $y_{sr} = h_{sr}x + z_{sr}$ and $\mathcal{D}$ receives $y_{sd} = h_{sd}x + z_{sd}$. During the second phase, the relay transmits the signal x_r and $\mathcal{D}$ receives $y_{rd} = h_{rd}x_r + z_{rd}$. The variables h_{sr}, h_{sd}, and h_{rd} denote the channel gains of the $\mathcal{S}$–$\mathcal{R}$, $\mathcal{S}$–$\mathcal{D}$ and $\mathcal{R}$–$\mathcal{D}$ links.

Throughout we will assume that all channels h_{ij} are Rayleigh fading, but constant during the transmission of one block. That is, the fading is quasi-static. We further assume that the magnitudes of the channel gains are independent, but not necessarily identically distributed. The variables z_{sr}, z_{sd}, and z_{rd}

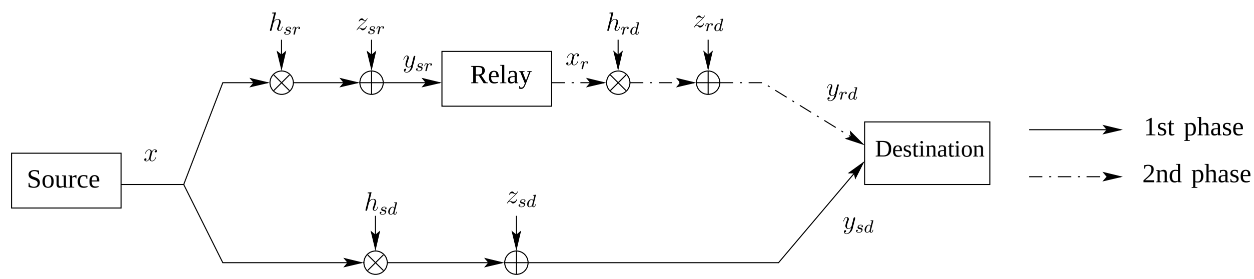

Fig. 1. Illustration of the three-node relay channel studied in this paper. The $\mathcal{S}-\mathcal{D}$ and $\mathcal{R}-\mathcal{D}$ links are orthogonal.

denote mutually independent, zero-mean white additive Gaussian noise with unit variance per complex dimension. We denote the received SNRs at the nodes by α_{sr}, α_{rd}, α_{sd}, and their means by γ_{sr}, γ_{rd}, γ_{sd} where $\gamma_{ij} \triangleq \mathbb{E}[\alpha_{ij}]$, $i \in \{s, r\}$, $j \in \{r, d\}$.

We will use a simple path loss model which assumes that $\gamma_{ij} = \frac{P_i \mathbb{E}[|h_{ij}|^2]}{N_0} = \frac{P}{N_0 d_{ij}^{\alpha}} = \frac{\rho}{d_{ij}^{\alpha}}$ where P is the transmit power, N_0 is the noise variance, d_{ij} is the distance between node i and j, α is the path loss exponent, and $\rho \triangleq \frac{P}{N_0}$ [15].[2] Throughout, we assume that all *receiving* nodes have access to perfect CSI. That is, $\mathcal{R}$ knows α_{sr} and $\mathcal{D}$ knows α_{sd}, α_{sr}, and α_{rd}. This is a weak assumption in slow fading.

As performance measure we will use the link outage probability, assuming capacity-achieving signaling with a Gaussian codebook. With this measure, a link with received SNR α_{ij} and spectral efficiency β bits per channel use [bpcu] is in outage when

$$\mathcal{O}(\alpha_{ij}, \beta) \Longleftrightarrow \log_2(1 + \alpha_{ij}) < \beta.$$

In particular, this means that if the relay is not used (transmission over the direct link only), we are in outage if $\mathcal{O}(\alpha_{sd}, \beta)$. This occurs with probability

$$\begin{aligned} \mathrm{P}_{\mathrm{out}} = \Pr\{\mathcal{O}(\alpha_{sd}, \beta)\} &= \Pr\left\{\alpha_{sd} < 2^{\beta} - 1\right\} = \int_0^{2^{\beta}-1} \frac{1}{\gamma_{sd}} \exp\left(-\frac{t}{\gamma_{sd}}\right) dt \\ &= \frac{2^{\beta} - 1}{\gamma_{sd}} + O\left(\frac{1}{\rho^2}\right). \end{aligned} \tag{1}$$

where $\rho = \frac{P}{N_0}$ is defined above.

While we assume Rayleigh fading throughout, most results that we present extend to a wide class of other fading distributions. The important property of the fading distribution that we use is that the distribution of the channel gain satisfies

$$P(|h|^2 < x) = ax + O(x^2), \qquad \text{for some nonzero } a \tag{2}$$

This is satisfied for Rayleigh fading in particular. Hence, then

$$P(\text{signal path bad}) \sim P(|h|^2 < N_0) \sim \frac{1}{N_0}$$

(which gives the classical slope-1 line when the logarithmic error probability is plotted against the SNR in dB). With mth order diversity, we obtain the classical slope-m curve given by

$$P(m \text{ signal paths bad}) \sim [P(|h|^2 < N_0)]^m \sim \left(\frac{1}{N_0}\right)^m$$

and again, this holds whenever (2) is satisfied.

We also stress that while the three-node relay channel considered here is a fairly simple model, it is widely used in classical and contemporary literature, and it provides fundamental insights.

[2]In the case that $\mathcal{S}$ and $\mathcal{R}$ use different transmit powers, we incorporate this difference into the corresponding channel gains.

III. Case 1: DF Relaying With Only Long-Term CSI at $\mathcal{S}$

The first case of interest is when $\mathcal{S}$ has access to *long-term* CSI, i.e., the path loss and the statistics of the fading distribution. Under the assumptions made in Section II this is equivalent to knowing the geometry (i.e., the distances d_{ij}). In this case it will turn out that for high SNR, the optimal bandwidth allocation depends on the long-term CSI for the $\mathcal{S}-\mathcal{R}$ and $\mathcal{R}-\mathcal{D}$ links, but it does not depend on the CSI for the $\mathcal{S}-\mathcal{D}$ link. (See Table I.)

Some of the results in this section are novel, and a few were presented in [16], [17] (the latter are reviewed briefly for easy reference, but without derivations).

A. Repetition Coding at $\mathcal{R}$ and Selection Combining at $\mathcal{D}$

With RC, $\mathcal{S}$ and $\mathcal{R}$ use the same code so $T_s = T_r = \frac{T}{2}$. With selection combining at $\mathcal{D}$, we consider that the link fails if the $\mathcal{S}$–$\mathcal{D}$ link fails and the $\mathcal{S}$–$\mathcal{R}$–$\mathcal{D}$ links fails simultaneously. The corresponding outage event is [12]

$$\underbrace{\mathcal{O}(\alpha_{sd},2\beta)}_{\triangleq\mathcal{O}_1}\bigcap\left[\underbrace{\mathcal{O}(\alpha_{sr},2\beta)}_{\triangleq\mathcal{O}_2}\bigcup\underbrace{\mathcal{O}(\alpha_{rd},2\beta)}_{\triangleq\mathcal{O}_3}\right]. \tag{3}$$

Since α_{sd}, α_{rd}, and α_{sr} are independent, the probability of (3) can be shown to be [17]

$$\begin{aligned}\mathrm{P_{out}} &= \Pr\left(\mathcal{O}_1\right)\Big[\Pr(\mathcal{O}_2)+\Pr(\mathcal{O}_3)-\Pr(\mathcal{O}_2)\Pr(\mathcal{O}_3)\Big] \\ &= \left(2^{2\beta}-1\right)^2\frac{\gamma_{rd}/\gamma_{sd}+\gamma_{sr}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right). \end{aligned}\tag{4}$$

B. Repetition Coding at $\mathcal{R}$ and Optimal Combining at $\mathcal{D}$

With RC at $\mathcal{R}$, the optimal receiver at $\mathcal{D}$ consists of MRC. The outage event is [16]:

$$\mathcal{O}(\alpha_{sd},2\beta)\bigcap\Big[\mathcal{O}(\alpha_{sr},2\beta)\bigcup\mathcal{O}(\alpha_{rd}+\alpha_{sd},2\beta)\Big]. \tag{5}$$

The outage event in (5) is reminiscent of that of the selective decode-and-forward (SDF) scheme presented in [2]. However with SDF, when the $\mathcal{S}-\mathcal{D}$ link is in outage, $\mathcal{S}$ repeats its message during the second phase as well. Therefore, to realize SDF, one bit of CSI feedback from $\mathcal{R}$ to $\mathcal{S}$ is required. By contrast, throughout this section we have assumed that there is no such instantaneous CSI feedback available at $\mathcal{S}$. The outage probability corresponding to (5) is [16]

$$\mathrm{P_{out}} = \left(2^{2\beta}-1\right)^2\frac{\gamma_{rd}/\gamma_{sd}+0.5\gamma_{sr}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right). \tag{6}$$

We see from (4) and (6) that RC-based DF relaying provides a diversity order of two, both with SC and with MRC. Comparing (4) with (6), it can be seen that if $d_{sr}=d_{rd}=d_{sd}$ then regardless of the value of β, MRC provides a $10\log_{10}(2/1.5)=0.62$ dB gain over SC at high SNR. This is in the complete agreement with the simulation results presented in Section VII (see Fig. 7).

C. Parallel Coding at $\mathcal{R}$ and Selection Combining at $\mathcal{D}$

With PC, $\mathcal{R}$ re-encodes the message using an independent random code which is different from the code used at $\mathcal{S}$. Suppose that $\mathcal{S}$ consumes $T_s=\delta T$ channel uses and $\mathcal{R}$ consumes the rest (i.e., $T_r=(1-\delta)T$ channel uses). With SC at $\mathcal{D}$, the outage event is

$$\mathcal{O}(\alpha_{sd},\beta_s)\bigcap\Big[\mathcal{O}(\alpha_{sr},\beta_s)\bigcup\mathcal{O}(\alpha_{rd},\beta_r)\Big], \tag{7}$$

where

$$\beta_s\triangleq\frac{\beta}{\delta}\quad\text{and}\quad\beta_r\triangleq\frac{\beta}{1-\delta}$$

The probability of the outage event (7) can be found by direct calculation:

$$\mathrm{P_{out}} \;=\; \frac{2^{\beta_s}-1}{\gamma_{sd}}\left[\frac{2^{\beta_s}-1}{\gamma_{sr}}+\frac{2^{\beta_r}-1}{\gamma_{rd}}\right]+O\left(\frac{1}{\rho^3}\right). \tag{8}$$

If $\mathcal{S}$ and $\mathcal{R}$ use equal transmit power, (8) suggests that we can optimize δ according to

$$\min_{\delta,\ 0<\delta<1}\left(2^{\frac{\beta}{\delta}}-1\right)\left[\left(2^{\frac{\beta}{\delta}}-1\right)d_{sr}^{\alpha}+\left(2^{\frac{\beta}{1-\delta}}-1\right)d_{rd}^{\alpha}\right]. \tag{9}$$

This optimization problem is convex and can be solved efficiently. The optimum δ depends only on d_{sr} and d_{rd}. Fig. 2(a) shows plots of the optimal δ as a function of the spectral efficiency for different d_{sr} when $d_{rd}=d_{sd}=1$ and $\alpha=4$. As expected, when $d_{sr}=0$ (i.e., the $\mathcal{S}-\mathcal{R}$ link is error free) the optimal δ is 0.5. The optimal δ increases as d_{sr} increases, for fixed d_{rd} and d_{sd}. The increase in δ helps $\mathcal{R}$ to recover the transmitted message more often and thereby it is more useful in the collaboration. Fig. 2(b) shows the gain obtained by optimization of δ versus setting $\delta=0.5$, as a function of d_{sr} for different β. It can be seen that the gain increases with increasing spectral efficiency and increasing d_{sr}. Moreover, the gain is negligible when $d_{sr}<0.5$. The closer $\mathcal{R}$ is located to $\mathcal{S}$, the smaller the gain is. This is so because for small d_{sr}, we have $\delta_{\mathrm{opt}}\approx 0.5$.

D. Parallel Coding at $\mathcal{R}$ and Optimal Combining at $\mathcal{D}$

Next consider the case when $\mathcal{D}$ uses the optimal receiver, consisting of code combining. The outage event is [16]

$$\mathcal{O}(\alpha_{sd},\beta/\delta)\bigcap\left[\mathcal{O}(\alpha_{sr},\beta/\delta)\bigcup\tilde{\mathcal{O}}(\alpha_{sd},\alpha_{rd},\delta)\right], \tag{10}$$

where

$$\tilde{\mathcal{O}}(\alpha_{sd},\alpha_{rd},\delta)\Leftrightarrow\left\{\delta\log_2\left(1+\alpha_{sd}\right)+(1-\delta)\log_2\left(1+\alpha_{rd}\right)<\beta\right\}. \tag{11}$$

and the probability of an outage is [16]

$$\mathrm{P_{out}}=\begin{cases}\left(2^{\beta_s}-1\right)^2\frac{1}{\gamma_{sd}\gamma_{sr}}+\left(1-2^{\beta_r}+\frac{\delta}{2\delta-1}2^{\beta_s}\left(2^{\beta_r\frac{2\delta-1}{\delta}}-1\right)\right)\frac{1}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta\neq\frac{1}{2}\\[2ex] \left(2^{2\beta}-1\right)^2\frac{1}{\gamma_{sd}\gamma_{sr}}+\left(1-2^{2\beta}+2\ln(2)\beta 2^{2\beta}\right)\frac{1}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta=\frac{1}{2}\end{cases} \tag{12}$$

Using (12), the optimal δ $(0<\delta<1)$ can be obtained by

$$\min_{\delta,\ 0\le\delta\le1} d_{sr}^{\alpha}\left(2^{\beta_s}-1\right)^2+d_{rd}^{\alpha}\left(1-2^{\beta_r}+\frac{\delta}{2\delta-1}2^{\beta_s}\left(2^{\beta_r\frac{2\delta-1}{\delta}}-1\right)\right). \tag{13}$$

The optimal δ depends only on d_{sr} and d_{rd}. Fig. 3(a) shows the optimal choice of δ as a function of the spectral efficiency β for different d_{sr} when the path loss exponent is $\alpha=4$ and $d_{sd}=d_{rd}=1$. For $d_{sr}=0$, the optimal δ is 0.5, since by symmetry the $\mathcal{R}-\mathcal{D}$ and $\mathcal{S}-\mathcal{D}$ links will be equally good on the average. However, as d_{sr} increases the optimal δ increases as well. For example, for the symmetric case (i.e., $d_{sr}=d_{rd}=d_{sd}=1$), $\delta_{\mathrm{opt}}\approx 0.7$ which means that the first phase should be allocated almost twice as much resources as the second phase.

Fig. 3(b) shows the relative power gain over repetition coding with MRC that can be obtained by optimally choosing δ. The figure also shows the power gain of parallel coding when $\delta=0.5$ and $d_{sd}=d_{sr}=d_{rd}=1$. This gain is very small at low spectral efficiency, but it increases with increasing β. By comparing (12) and (6), one can see that if δ is fixed to 0.5 the maximum achievable gain over repetition coding is upper bounded by $5\log_{10}(1.5)\approx 0.88$ dB. However, with optimized δ the gain grows without bound as β increases. This shows how important the optimization of δ is for the performance.

IV. CASE 2: DF RELAYING WITH ONE BIT OF INSTANTANEOUS CSI AT $\mathcal{S}$

It is possible to improve on the performance obtained in Section III, by letting $\mathcal{D}$ transmit one single bit of CSI that indicates (in advance) whether transmission over the direct link would succeed or not. The condition for a successful $\mathcal{S}-\mathcal{D}$ transmission is precisely that α_{sd} must be large enough for $\mathcal{O}(\alpha_{sd},\beta)$ not to occur. If this CSI-flag-bit indicates that the direct link will succeed, then $\mathcal{S}$ should simply use all available channel uses (T) for the direct link transmission (i.e,. not use $\mathcal{R}$ at all). Otherwise, the protocol resorts to the one analyzed in Section III. As in Section III we will analyze four different coding schemes and combining techniques.

A. Repetition Coding and Selection Combing at $\mathcal{D}$

The outage event with SC at $\mathcal{D}$ using RC (hence $\delta=0.5$) at $\mathcal{R}$ is

$$\mathcal{O}(\alpha_{sd},\beta)\bigcap\left[\mathcal{O}(\alpha_{sr},2\beta)\bigcup\mathcal{O}(\alpha_{rd},2\beta)\right]. \tag{14}$$

The outage probability is found by direct calculation to be

$$\mathrm{P_{out}} \;=\; \left(2^{\beta}-1\right)\left(2^{2\beta}-1\right)\frac{\gamma_{sr}/\gamma_{sd}+\gamma_{rd}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right). \tag{15}$$

B. DF with Repetition Coding at $\mathcal{R}$ and MRC at $\mathcal{D}$

The outage event with RC at $\mathcal{R}$ and MRC at $\mathcal{D}$ is

$$\underbrace{\mathcal{O}(\alpha_{sd},\beta)}_{\mathcal{O}_1}\bigcap\left[\underbrace{\mathcal{O}(\alpha_{sr},2\beta)}_{\mathcal{O}_2}\bigcup\underbrace{\mathcal{O}\left(\alpha_{rd}+\alpha_{sd},2\beta\right)}_{\mathcal{O}_3}\right]. \tag{16}$$

The outage probability is

$$\mathrm{P_{out}} \;=\; \Pr\left\{\mathcal{O}_1\bigcap\mathcal{O}_2\right\}+\Pr\left\{\mathcal{O}_3|\mathcal{O}_1\right\}\Pr\left\{\mathcal{O}_1\bigcap\mathcal{O}_2^c\right\}. \tag{17}$$

To compute (17) consider

$$\begin{aligned}\tilde{P} &\triangleq \Pr\left\{\mathcal{O}_3|\mathcal{O}_1\right\}\\ &= \frac{\Pr\left\{\mathcal{O}_3,\mathcal{O}_1\right\}}{\Pr\left\{\mathcal{O}_1\right\}}\\ &= \int_0^{2^{\beta}-1}\Pr\left\{\alpha_{sd}+\alpha_{rd}<2^{2\beta}-1,\alpha_{sd}=t\right\}\frac{f_{\alpha_{sd}}(t)}{\Pr\left\{\mathcal{O}_1\right\}}dt\\ &= \int_0^{2^{\beta}-1}\Pr\left\{\alpha_{sd}+\alpha_{rd}<2^{2\beta}-1,\alpha_{sd}=t\right\}g_{\alpha_{sd}}(t)dt \end{aligned} \tag{18}$$

In (18), $f_{\alpha_{sd}}(\cdot)$ denotes the pdf of α_{sd} and $g_{\alpha_{sd}}(t)$ is given by

$$g_{\alpha_{sd}}(t)\triangleq\begin{cases}\frac{1}{\varsigma\gamma_{sd}}\exp\left(-\frac{t}{\gamma_{sd}}\right) & 0<t<2^{\beta}-1\\ 0 & \text{otherwise}\end{cases}$$

where $\varsigma \triangleq \Pr\{\mathcal{O}_1\} = \Pr\{\alpha_{sd} < 2^{\beta} - 1\} = 1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right)$. Thus,

$$\begin{aligned}\tilde{P} &= \int_0^{2^{\beta}-1} \Pr\left\{\alpha_{rd} < 2^{2\beta} - 1 - t\right\} \frac{1}{\varsigma\gamma_{sd}} \exp\left(-\frac{t}{\gamma_{sd}}\right) dt \\ &= \frac{1}{\varsigma}\int_0^{2^{\beta}-1} \left[1 - \exp\left(\frac{t - \left(2^{2\beta}-1\right)}{\gamma_{rd}}\right)\right] \frac{1}{\gamma_{sd}} \exp\left(\frac{-t}{\gamma_{sd}}\right) dt \\ &= \frac{1}{\varsigma}\left[1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right)\right] - \underbrace{\int_0^{2^{\beta}-1} \frac{1}{\varsigma\gamma_{sd}} \exp\left(\frac{t - \left(2^{2\beta}-1\right)}{\gamma_{rd}} - \frac{t}{\gamma_{sd}}\right) dt}_{\triangleq A}\end{aligned}$$

where A can be further simplified by considering two cases, namely $\gamma_{sd} \neq \gamma_{rd}$ and $\gamma_{sd} = \gamma_{rd}$. We obtain

$$\tilde{P} = \begin{cases} \frac{1}{\varsigma}\left[1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right)\right] + \frac{\gamma_{rd}}{\varsigma(\gamma_{sd}-\gamma_{rd})} \exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}}\right) \left[1 - \exp\left(\frac{(\gamma_{sd}-\gamma_{rd})}{\gamma_{sd}\gamma_{rd}}(2^{\beta}-1)\right)\right] & \text{if } \gamma_{sd} \neq \gamma_{rd} \\ \frac{1}{\varsigma}\left[1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right) - \frac{2^{\beta}-1}{\gamma_{sd}} \exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}}\right)\right] & \text{if } \gamma_{sd} = \gamma_{rd} \end{cases} \tag{19}$$

Having found $\tilde{P}$, we can obtain the probability of the outage event in (16) as

$$\mathrm{P_{out}} = \begin{cases} \left(1 - \exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}}\right)\right)\left(1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right)\right) + \exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}}\right)\Bigg(1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right) + \\ \frac{\gamma_{rd}}{\gamma_{sd}-\gamma_{rd}} \exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}}\right)\left[1 - \exp\left(\frac{(\gamma_{sd}-\gamma_{rd})}{\gamma_{sd}\gamma_{rd}}(2^{\beta}-1)\right)\right]\Bigg) & \text{if } \gamma_{sd} \neq \gamma_{rd} \\ \left(1 - \exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}}\right)\right)\left(1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right)\right) \\ + \exp\left(\frac{1-2^{2\beta}}{\gamma_{sr}}\right)\left[1 - \exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right) - \frac{2^{\beta}-1}{\gamma_{sd}} \exp\left(\frac{1-2^{2\beta}}{\gamma_{rd}}\right)\right] & \text{if } \gamma_{sd} = \gamma_{rd} \end{cases} \tag{20}$$

Using a series expansion, we find that

$$\mathrm{P_{out}} = \left(2^{\beta}-1\right)\left(2^{2\beta}-1\right)\frac{\gamma_{sr}/\gamma_{sd} + \gamma_{rd}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}} - 0.5\left(2^{\beta}-1\right)^2 \frac{1}{\gamma_{sd}\gamma_{rd}} + O\left(\frac{1}{\rho^3}\right). \tag{21}$$

By comparing (21) with (15), we see that the maximum possible gain of MRC over selection combining is achieved at low spectral efficiency. This gain is 0.29 dB at high SNR if $d_{sr} = d_{sd} = d_{rd} = 1$. This gain then starts decreasing when increasing the spectral efficiency. For example, for spectral efficiency $\beta = 0.5$ and 2 bpcu, the gain at high SNR is 0.24 and 0.11 dB respectively. The performance of the scheme with one bit of CSI feedback with SC approaches that of its counterpart with MRC as the spectral efficiency increases. This is due to the fact that the gain of MRC over SC is essentially a *power* gain which is less important than a bandwidth gain at large spectral efficiencies.

C. Parallel Coding at $\mathcal{R}$ and Selection Combing at $\mathcal{D}$

With PC at $\mathcal{R}$ and SC at $\mathcal{D}$, and δ defined as before, the outage event is

$$\mathcal{O}(\alpha_{sd}, \beta) \bigcap \left[\mathcal{O}(\alpha_{sr}, \beta_s) \bigcup \mathcal{O}(\alpha_{rd}, \beta_r)\right], \tag{22}$$

The probability of (22) is

$$\mathrm{P_{out}} = \frac{2^{\beta}-1}{\gamma_{sd}}\left[\frac{2^{\beta_s}-1}{\gamma_{sr}} + \frac{2^{\beta_r}-1}{\gamma_{rd}}\right] + O\left(\frac{1}{\rho^3}\right). \tag{23}$$

If $\mathcal{S}$ and $\mathcal{R}$ use equal transmit power, (23) suggests that we can optimize δ according to

$$\min_{\delta,\ 0<\delta<1} \left(2^{\frac{\beta}{\delta}}-1\right) d_{sr}^{\alpha}+\left(2^{\frac{\beta}{1-\delta}}-1\right) d_{rd}^{\alpha}. \tag{24}$$

Note that optimal choice of δ does not depend on d_{sd}. This is natural, because it is implicit in SC that if the $\mathcal{S}-\mathcal{R}-\mathcal{D}$ link is used, then $\mathcal{D}$ discards the transmission heard directly from $\mathcal{S}$. Moreover, we see that $\delta_{\text{opt}}=0.5$ when $d_{sr}=d_{rd}$.

D. Parallel Coding at $\mathcal{R}$ and Code Combining at $\mathcal{D}$

With PC at $\mathcal{R}$ and using the optimal receiver (code combining) at $\mathcal{D}$, the outage event is

$$\mathcal{O}(\alpha_{sd},\beta)\bigcap\left[\mathcal{O}(\alpha_{sr},\beta/\delta)\bigcup\tilde{\mathcal{O}}(\alpha_{sd},\alpha_{rd},\delta)\right], \tag{25}$$

where $\tilde{\mathcal{O}}(\alpha_{sd},\alpha_{rd},\delta)$ is given by (11). The probability of (25) can be found to be

$$\mathrm{P_{out}}=\begin{cases}\left(2^{\beta_s}-1\right)\left(2^{\beta}-1\right)\frac{1}{\gamma_{sd}\gamma_{sr}}+ \\ \left(1-2^{\beta}+\frac{\delta-1}{2\delta-1}2^{\beta_r}\left(2^{\beta\frac{2\delta-1}{\delta-1}}-1\right)\right)\frac{1}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta\neq\frac{1}{2} \\ \left(2^{2\beta}-1\right)\left(2^{\beta}-1\right)\frac{1}{\gamma_{sd}\gamma_{sr}}+ \\ \left(1-2^{\beta}+\ln(2)\beta 2^{2\beta}\right)\frac{1}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta=\frac{1}{2}\end{cases} \tag{26}$$

Using (26), the optimal δ $(0<\delta<1)$ can be obtained via

$$\min_{\delta,\ 0<\delta<1} d_{sr}^{\alpha}\left(2^{\beta_s}-1\right)\left(2^{\beta}-1\right)+d_{rd}^{\alpha}\frac{\delta-1}{2\delta-1}2^{\beta_r}\left(2^{\beta\frac{2\delta-1}{\delta-1}}-1\right). \tag{27}$$

For the symmetric case (i.e., $d_{sr}=d_{sd}=d_{rd}=1$), we have $\delta_{\text{opt}}\approx 0.6$.

V. Case 3: DF Relaying with Perfect Instantaneous CSI at $\mathcal{S}$

In this section we analyze the outage performance of adaptive PC-based DF relaying protocols that exploit *instantaneous* transmitter CSI for the $\mathcal{S}-\mathcal{R}$ and $\mathcal{S}-\mathcal{D}$ links. (It will turn out, that only one bit of CSI for the $\mathcal{S}-\mathcal{D}$ link is indeed necessary in this case.) Now, the parameter δ should be chosen based on the instantaneous CSI, such that the outage probability is minimized.

A. Parallel Coding and Selection Combining at $\mathcal{D}$

The outage event with SC at $\mathcal{D}$ is

$$\underbrace{\mathcal{O}(\alpha_{sd},\beta)}_{\triangleq\mathcal{O}_1}\bigcap\underbrace{\left[\mathcal{O}(\alpha_{sr},\beta_s)\bigcup\mathcal{O}(\alpha_{rd},\beta_r)\right]}_{\triangleq\mathcal{O}_2}, \tag{28}$$

where $\beta_s=\frac{\beta}{\delta}$ and $\beta_r=\frac{\beta}{1-\delta}$ as before. If the direct link is in outage, $\mathcal{S}$ should choose δ as small as possible but large enough so that the message reaches $\mathcal{R}$. In other words, enough channel uses should be allocated to $\mathcal{S}$ such that

$$\mathcal{O}^c(\alpha_{sr},\beta_s)\Leftrightarrow\log_2\left(1+\alpha_{sr}\right)\geq\beta_s. \tag{29}$$

Since $\beta_s\geq\beta$ and $\delta\leq 1$, the optimal δ is obtained by

$$\delta_{\text{opt}}=\min\left\{1,\frac{\beta}{\log_2(1+\alpha_{sr})}\right\}. \tag{30}$$

Fig. 4 presents a flowchart for the optimal block length allocation. This optimal assignment requires two things: perfect knowledge of the $\mathcal{S}-\mathcal{R}$ channel gain (i.e., α_{sr}) and additionally *only* 1 bit of CSI feedback from $\mathcal{D}$ which indicates whether the transmission over the direct link will be successful or not. If so, $\mathcal{S}$ uses $\delta_{\text{opt}} = 1$. Otherwise, $\mathcal{S}$ chooses δ according to (30).

Since $\mathcal{O}_1$ and $\mathcal{O}_2$ are independent, we have

$$\text{P}_{\text{out}} = \Pr\{\mathcal{O}_1\} \Pr\{\mathcal{O}_2\}. \tag{31}$$

Using the result in Appendix A, the probability of the outage event in (28) with δ chosen according to (30) is given by

$$\begin{aligned}\text{P}_{\text{out}} &= \left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sd}}\right)\right)\left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sr}}\right)\right)+\left[1-\exp\left(-\frac{2^\beta-1}{\gamma_{sd}}\right)\right]\frac{\kappa(\gamma_{sr},\gamma_{rd})}{\gamma_{sr}} \\ &= \left[\left(2^\beta-1\right)^2+\kappa(\gamma_{sr},\gamma_{rd})\left(2^\beta-1\right)\right]\frac{1}{\gamma_{sd}\gamma_{sr}}+O\left(\frac{1}{\rho^3}\right),\end{aligned} \tag{32}$$

where

$$\kappa(\gamma_{sr},\gamma_{rd}) \triangleq (\ln 2)\int_\beta^\infty 2^t \exp\left(\frac{1-2^t}{\gamma_{sr}}\right)\left[1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right]dt. \tag{33}$$

The function $\kappa(\gamma_{sr},\gamma_{rd})$ can be evaluated numerically. An example plot of $\kappa(\gamma_{sr},\gamma_{rd})$ versus power for different spectral efficiencies (β) was provided in [17]. It can be shown that $\kappa(\gamma_{sr},\gamma_{rd})$ reaches a maximum for a particular β, and it then decreases slowly as the power increases. The outage probability can be upper bounded by

$$\text{P}_{\text{out}} \le \left[\left(2^\beta-1\right)^2+\tilde{\kappa}\left(2^\beta-1\right)\right]\frac{1}{\gamma_{sd}\gamma_{sr}}, \tag{34}$$

where $\tilde{\kappa}$ is the maximum of $\kappa(\gamma_{sr},\gamma_{rd})$. The following result sheds some light on the asymptotic behavior of $\kappa(\gamma_{sr},\gamma_{rd})$:

Proposition 1 (Behavior of $\kappa(\gamma_{sr},\gamma_{rd})$):

$$\begin{aligned}\lim_{\rho\to\infty}\kappa(\gamma_{sr},\gamma_{rd}) &= \left(\frac{d_{rd}}{d_{sr}}\right)^\alpha\left(2^\beta-1\right) \\ \lim_{\rho\to 0}\kappa(\gamma_{sr},\gamma_{rd}) &= 0\end{aligned} \tag{35}$$

Proof: See Appendix B. ■

Using Proposition 1, the outage probabilities at low ($\text{P}_{\text{out}}^{\text{low}}$) and high SNR ($\text{P}_{\text{out}}^{\text{high}}$) are

$$\begin{aligned}\text{P}_{\text{out}}^{\text{low}} &\approx \left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sd}}\right)\right)\left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sr}}\right)\right), \ \rho \ll 1, \\ \frac{\gamma_{sd}\gamma_{sr}\gamma_{rd}}{\gamma_{sr}+\gamma_{rd}}\cdot\text{P}_{\text{out}}^{\text{high}} &\to \left(2^\beta-1\right)^2 \quad \text{as} \quad \rho\to\infty.\end{aligned} \tag{36}$$

B. Parallel Coding at $\mathcal{R}$ and Code Combining at $\mathcal{D}$

We next consider PC at $\mathcal{R}$ and CC at $\mathcal{D}$. The outage event is given by

$$\mathcal{O}(\alpha_{sd},\beta)\bigcap\left[\mathcal{O}(\alpha_{sr},\beta_s)\bigcup\tilde{\mathcal{O}}(\alpha_{sd},\alpha_{rd},\delta)\right], \tag{37}$$

where $\tilde{\mathcal{O}}(\alpha_{sd},\alpha_{rd},\delta)$ is defined by (11) and as before, $\beta_s = \frac{\beta}{\delta}$. The following proposition gives the optimal choice of δ.

Proposition 2: The δ that minimizes the probability of the outage event in (37) is

$$\delta_{\text{opt}} = \min\left\{1, \frac{\beta}{\log_2(1+\alpha_{sr})}\right\}. \tag{38}$$

Proof: If the $\mathcal{S}-\mathcal{D}$ link is in outage, $\mathcal{R}$ must collaborate. In other words, enough channel uses should be allocated to $\mathcal{S}$ such that $\log_2(1+\alpha_{sr}) \geq \beta_s$. Since $\beta_s \geq \beta$ and $\delta \leq 1$, the feasible set of solutions is then given by

$$\frac{\beta}{\log_2(1+\alpha_{sr})} \leq \delta \leq 1.$$

Among all possible solutions we should pick the one which maximizes

$$\delta \log_2(1+\alpha_{sd}) + (1-\delta)\log_2(1+\alpha_{rd}).$$

It is easy to see that if $\alpha_{sd} > \alpha_{rd}$, we have $\delta_{opt} = 1$, which means that we should resort to non-collaborative transmission. But in this case we are in outage anyway, since we use $\mathcal{R}$ only when the $\mathcal{S}-\mathcal{D}$ link is in outage. However, if $\alpha_{rd} > \alpha_{sd}$, we should use a minimum amount of channel uses for the first phase. Therefore, the optimal δ is given by (38). ∎

The block length allocation scheme according to Fig. 4 is therefore valid for DF with PC as well. Since $\beta_s \geq \beta$, $\mathcal{R}$ cannot cooperate when $\mathcal{O}(\alpha_{sr}, \beta)$. Moreover when $\mathcal{R}$ cooperates, the $\mathcal{S}-\mathcal{D}$ link cannot support a spectral efficiency greater than β. Thus, the outage probability is given by

$$\begin{aligned} \mathrm{P_{out}} &= \Pr\left\{\mathcal{O}(\alpha_{sr},\beta)\bigcap\mathcal{O}(\alpha_{sd},\beta)\right\} + \underbrace{\Pr\left\{\tilde{\mathcal{O}}(\alpha_{sd},\alpha_{rd},\delta)|\mathcal{O}^c(\alpha_{sr},\beta)\bigcap\mathcal{O}(\alpha_{sd},\beta)\right\}}_{\triangleq\bar{\mathrm{P}}} \times \\ & \quad \Pr\left\{\mathcal{O}^c(\alpha_{sr},\beta)\bigcap\mathcal{O}(\alpha_{sd},\beta)\right\}. \end{aligned}$$

The quantity $\bar{\mathrm{P}}$ is evaluated in Appendix C. Thus, the outage probability can be calculated as

$$\begin{aligned} \mathrm{P_{out}} &= \left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sd}}\right)\right)\left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sr}}\right)\right) + \frac{\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})}{\gamma_{sr}\gamma_{rd}} \\ &= \left[\left(2^\beta-1\right)^2 + \nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})\right]\frac{1}{\gamma_{sd}\gamma_{sr}} + O\left(\frac{1}{\rho^3}\right), \end{aligned} \tag{39}$$

where

$$\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd}) \triangleq (\ln 2)^2 \int_\beta^\infty\int_0^\beta \exp\left(\frac{1-2^t}{\gamma_{sr}} + \frac{1-2^r}{\gamma_{sd}}\right)\left[1-\exp\left(\frac{1-2^{\frac{t-r}{t-\beta}\beta}}{\gamma_{rd}}\right)\right] 2^{t+r}drdt.$$

The function $\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})$ can be evaluated numerically. It can be shown that $\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})$ reaches a maximum for a particular β, and it then decreases slowly as the power increases. The outage probability can be upper bounded by

$$\mathrm{P_{out}} \leq \left[\left(2^\beta-1\right)^2 + \tilde{\nu}\left(2^\beta-1\right)\right]\frac{1}{\gamma_{sd}\gamma_{sr}}, \tag{40}$$

where $\tilde{\nu}$ is the maximum of $\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})$. For high and low SNR we have the following behavior:

Proposition 3 (Behavior of $\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})$):

$$\begin{aligned} \lim_{\rho\to\infty}\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd}) &= \left(\frac{d_{rd}}{d_{sr}}\right)^\alpha\left(2^\beta-1\right)^2 \\ \lim_{\rho\to 0}\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd}) &= 0 \end{aligned} \tag{41}$$

Proof: See Appendix D. ∎

Using Proposition 3, the outage probabilities at low ($\mathrm{P_{out}^{low}}$) and high SNR ($\mathrm{P_{out}^{high}}$) are

$$\begin{aligned} \mathrm{P_{out}^{low}} &\approx \left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sd}}\right)\right)\left(1-\exp\left(-\frac{2^\beta-1}{\gamma_{sr}}\right)\right), \ \rho \ll 1, \\ \frac{\gamma_{sd}\gamma_{sr}\gamma_{rd}}{\gamma_{sr}+\gamma_{rd}}\cdot\mathrm{P_{out}^{high}} &\to \left(2^\beta-1\right)^2 \quad \text{as} \quad \rho\to\infty. \end{aligned} \tag{42}$$

Comparing (42) and (36), we see that DF relaying with perfect CSI with selection combining (SC) and with code combining (CC) have the same outage probabilities both at low and at high SNR. Fig. 5 shows these outage probabilities for $d_{rd} = d_{sd} = 1$, and $\alpha = 4$. The plots are obtained by numerically evaluating (32) and (39). It can be seen that the outage probabilities at low SNR are the same, and at high SNR both schemes approach the asymptotic curve given by (36) and (42). It is worth mentioning, however, that CC is more complex than MRC and SC.

VI. Summary and Comparisons

Table II presents the outage probabilities of the schemes that we have studied in the paper. Clearly, all DF-based collaborative schemes provide a diversity order of two. Note that all outage probabilities can be written as

$$f(\beta, \delta) K(\gamma_{sd}, \gamma_{sr}, \gamma_{rd}) + O\left(\frac{1}{\rho^3}\right)$$

where $f(\beta, \delta)$ depends on the link spectral efficiency β and on δ (the relative block-length allocated to the first phase), and where $K(\gamma_{sd}, \gamma_{sr}, \gamma_{rd})$ depends on the SNR and on the geometry of the relay network. By neglecting the $O(1/\rho^3)$ terms at the end of all equations, we can compare the high-SNR performance of all schemes and investigate the effect of bandwidth allocation. For example, if we compare the outage probability of repetition coding (RC) with that of selection combining (SC) and MRC in Case 1, we see that both have the same $f(\beta, \delta = 0.5)$ while MRC gives a lower value of $K(\gamma_{sd}, \gamma_{sr}, \gamma_{rd})$ (cf. Table II). By doing a similar comparison for Case 2, we see that the gain of MRC over SC vanishes as β increases (the first term of the outage probability of MRC is identical to that for SC, and it dominates for large β; cf. Table II).

We next show some quantitative comparisons of the outage probabilities for the different schemes. Fig. 6(a) shows the relative gains of the collaborative schemes over non-adaptive repetition coding with MRC at $\mathcal{D}$ when $d_{sd} = d_{rd} = 1$, for $\beta = 0.25$ bpcu. It is seen that for small d_{sr} (i.e., when $\mathcal{R}$ is close to $\mathcal{S}$), the gain of PC with one bit of CSI feedback is very close to that of DF relaying with perfect CSI. Repetition coding in conjunction with one bit of CSI feedback performs very close to its counterpart with parallel coding when $\mathcal{R}$ is close to $\mathcal{D}$. Fig. 6(b) shows the corresponding relative gains for $\beta = 2$ bpcu. It can be seen that the performance of repetition coding even with one bit of CSI feedback is poor at high spectral efficiency. However, parallel coding with one bit of CSI feedback provides results which are comparable to those with perfect CSI when $\mathcal{R}$ is close to $\mathcal{S}$. Comparing Figs. 6(a) and 6(b), one can see that the gain of the proposed schemes with one bit of CSI feedback increases with spectral efficiency.

VII. Simulation Results

We used Monte-Carlo simulation to verify our analytical results. The channels $\mathcal{S} - \mathcal{R}$, $\mathcal{S} - \mathcal{D}$, and $\mathcal{R} - \mathcal{D}$ were assumed to be independent Rayleigh fading with means $d_{ij}^{-\alpha}$, $\alpha = 4$, where d_{ij} is the distance between node i and node j. Fig. 7 shows the outage probability versus power when $\beta = 2$ bpcu for the symmetric case. In addition to the analytical and the simulation results, high-SNR approximations[3] are plotted as well. We see that the high-SNR approximations are tight. For this particular spectral efficiency, relaying with one bit of CSI feedback outperforms conventional decode-and-forward with MRC at $\mathcal{D}$ by 2.9 dB. With perfect CSI feedback and SC, the gain is 4.5 dB at outage probability 10^{-3}. DF with PC in conjunction with perfect CSI can further improve the performance. This improvement varies between 1 and 0.5 dB in the shown SNR range, but will vanish at higher SNR. We also see that the scheme with one bit of CSI feedback outperforms non-collaborative transmission for the entire SNR range.

[3]The high-SNR approximations are obtained by neglecting the terms $O(1/\rho^3)$ in the analytical expressions; see Section VI.

VIII. Conclusions

We have provided analytical, finite-SNR results on the outage performance of DF relaying and additionally used these results to optimize the number of channel uses allocated for the source and relay transmissions, respectively. We considered three basic cases: the source had either long-term channel knowledge, 1-bit channel knowledge, or perfect (instantaneous) information about the channel gain.

We showed that for fixed SNR, the effect of optimizing the bandwidth allocation between the source and the relay provides a substantial power gain. However, this optimization does not affect the diversity order, which is two in all cases, since there is no optimization across independently fading channel realizations. A number of other interesting observations have also emerged. For example, we have seen that the performance of selection combining can be comparable to that of MRC and that of code combining. We have also demonstrated that for the case of perfect channel knowledge at the source, selection combining and code combining provide the same performance as SNR approaches infinity. However, at moderate SNR, code combining provides better performance than selection combining.

Appendix A
Calculation of $\mathrm{P}_{\text{out}}^{\text{2hop}}$

The probability that the $\mathcal{S}-\mathcal{R}-\mathcal{D}$ path is in outage is given by

$$\begin{aligned}\mathrm{P}_{\text{out}}^{\text{2hop}} &\triangleq \Pr\left\{\mathcal{O}(\alpha_{sr},\beta_s)\bigcup\mathcal{O}(\alpha_{rd},\beta_r)\right\}\\ &= \Pr\left\{\alpha_{sr}<2^{\beta}-1\right\}+\Pr\left\{\alpha_{rd}<2^{\beta_r}-1\middle|\alpha_{sr}>2^{\beta}-1\right\}\Pr\left\{\alpha_{sr}>2^{\beta}-1\right\}.\end{aligned}$$

We first calculate the following intermediate quantity:

$$\begin{aligned}\breve{\mathrm{P}} &\triangleq \Pr\left\{\alpha_{rd}<2^{\beta_r}-1\middle|\alpha_{sr}>2^{\beta}-1\right\}=\Pr\left\{\alpha_{rd}<2^{\frac{\log_2(1+\alpha_{sr})}{\log_2(1+\alpha_{sr})-\beta}\beta}-1\middle|\alpha_{sr}>2^{\beta}-1\right\}\\ &= \Pr\left\{\log_2(1+\alpha_{rd})<\frac{\beta\log_2(1+\alpha_{sr})}{\log_2(1+\alpha_{sr})-\beta}\middle|\log_2(1+\alpha_{sr})>\beta\right\}\\ &= \Pr\left\{y<\frac{\beta x}{x-\beta}\middle|x>\beta\right\}, \qquad (43)\end{aligned}$$

where $x\triangleq\log_2(1+\alpha_{sr})$ and $y\triangleq\log_2(1+\alpha_{rd})$. The pdfs of the random variables x and y are given by

$$f_x(t)=\frac{\ln 2}{\gamma_{sd}}\exp\left(\frac{1-2^t}{\gamma_{sd}}\right)2^t,\quad f_y(t)=\frac{\ln 2}{\gamma_{rd}}\exp\left(\frac{1-2^t}{\gamma_{rd}}\right)2^t,\quad t\geq 0. \qquad (44)$$

Thus,

$$\begin{aligned}\breve{\mathrm{P}} &= \int_{\beta}^{\infty}\Pr\left\{y<\frac{\beta x}{x-\beta}\right\}f_x(x)dydx=\int_{\beta}^{\infty}\left[1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right]f_x(t)dt\\ &= \frac{\ln 2}{\gamma_{sr}}\int_{\beta}^{\infty}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)\left[1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right]dt.\end{aligned}$$

We thus obtain

$$\mathrm{P}_{\text{out}}=1-\exp\left(-\frac{2^{\beta}-1}{\gamma_{sr}}\right)+\exp\left(-\frac{2^{\beta}-1}{\gamma_{sr}}\right)\frac{\kappa(\gamma_{sr},\gamma_{rd})}{\gamma_{sr}}, \qquad (45)$$

where $\kappa(\gamma_{sr},\gamma_{rd})$ is given by (33).

APPENDIX B
PROOF OF PROPOSITION 1

A. Limit at infinity

First we establish $\lim_{\rho\to\infty}\kappa(\gamma_{sr},\gamma_{rd})$. We do this by finding an upper and a lower bound.

Lower bound: Observe that

$$s(t) \triangleq 1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)$$

is a decreasing function over $(\beta,\infty]$. We thus have, for $t\geq 0$:

$$s(t)\geq \lim_{t\to\infty} 1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)=1-\exp\left(\frac{1-2^{\beta}}{\gamma_{rd}}\right). \quad (46)$$

Using (46), we obtain

$$\begin{aligned}\kappa(\gamma_{sr},\gamma_{rd}) &\geq \left[1-\exp\left(\frac{1-2^{\beta}}{\gamma_{rd}}\right)\right](\ln 2)\int_{\beta}^{\infty}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)dt\\ &= \gamma_{sr}\left[1-\exp\left(\frac{1-2^{\beta}}{\gamma_{rd}}\right)\right]\exp\left(\frac{1-2^{\beta}}{\gamma_{sr}}\right). \quad (47)\end{aligned}$$

Upper bound: Let ε, $\varepsilon>0$ and M, $M>\beta+\varepsilon$ be arbitrary and $\kappa(\gamma_{sr},\gamma_{rd})=I_1+I_2+I_3$, where

$$\begin{aligned}I_1 &\triangleq (\ln 2)\int_{\beta}^{\beta+\varepsilon}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)\left[1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right]dt,\\ I_2 &\triangleq (\ln 2)\int_{\beta+\varepsilon}^{M}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)\left[1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right]dt,\\ I_3 &\triangleq (\ln 2)\int_{M}^{\infty}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)\left[1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right]dt.\end{aligned}$$

Since $1-\exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\leq 1$, we have

$$I_1 \leq (\ln 2)\int_{\beta}^{\beta+\varepsilon}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)dt=\gamma_{sr}\left[\exp\left(\frac{1-2^{\beta}}{\gamma_{sr}}\right)-\exp\left(\frac{1-2^{\beta+\varepsilon}}{\gamma_{sr}}\right)\right]. \quad (48)$$

Now we bound I_2. Recall that $s(t)$ is a decreasing function over $[\beta+\varepsilon,M]$. Thereby, we have

$$\begin{aligned}I_2 &\leq \left[1-\exp\left(\frac{1-2^{\frac{\beta(\beta+\varepsilon)}{\varepsilon}}}{\gamma_{rd}}\right)\right](\ln 2)\int_{\beta+\varepsilon}^{M}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)dt\\ &= \gamma_{sr}\left[1-\exp\left(\frac{1-2^{\frac{\beta(\beta+\varepsilon)}{\varepsilon}}}{\gamma_{rd}}\right)\right]\left[\exp\left(\frac{1-2^{\beta+\varepsilon}}{\gamma_{sr}}\right)-\exp\left(\frac{1-2^{M}}{\gamma_{sr}}\right)\right]. \quad (49)\end{aligned}$$

Similarly, we obtain

$$\begin{aligned}I_3 &\leq \left[1-\exp\left(\frac{1-2^{\frac{\beta M}{M-\beta}}}{\gamma_{rd}}\right)\right](\ln 2)\int_{M}^{\infty}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)dt\\ &\leq \frac{\gamma_{sr}}{\gamma_{rd}}\left(2^{\frac{\beta M}{M-\beta}}-1\right)\int_{M}^{\infty}\frac{\ln 2}{\gamma_{sr}}2^t\exp\left(\frac{1-2^t}{\gamma_{sr}}\right)=\frac{\gamma_{sr}}{\gamma_{rd}}\left(2^{\frac{\beta M}{M-\beta}}-1\right)\exp\left(\frac{1-2^{M}}{\gamma_{sr}}\right)\\ &\leq \frac{\gamma_{sr}}{\gamma_{rd}}\left(2^{\frac{\beta M}{M-\beta}}-1\right) \quad (50)\end{aligned}$$

where we used $1 - e^x \leq -x$.

Putting (47)-(50) together, we have

$$\delta_1 \leq \kappa(\gamma_{sr}, \gamma_{rd}) \leq \delta_2 \tag{51}$$

where

$$\begin{aligned}
\delta_1 &= \gamma_{sr}\left[1 - \exp\left(\frac{1-2^\beta}{\gamma_{rd}}\right)\right]\exp\left(\frac{1-2^\beta}{\gamma_{sr}}\right),\\
\delta_2 &= \gamma_{sr}\left[\exp\left(\frac{1-2^\beta}{\gamma_{sr}}\right) - \exp\left(\frac{1-2^{\beta+\varepsilon}}{\gamma_{sr}}\right)\right] + \\
&\quad \gamma_{sr}\left[1-\exp\left(\frac{1-2^{\frac{\beta(\beta+\varepsilon)}{\varepsilon}}}{\gamma_{rd}}\right)\right]\left[\exp\left(\frac{1-2^{\beta+\varepsilon}}{\gamma_{sr}}\right) - \exp\left(\frac{1-2^M}{\gamma_{sr}}\right)\right] + \frac{\gamma_{sr}}{\gamma_{rd}}\left(2^{\frac{\beta M}{M-\beta}} - 1\right).
\end{aligned}$$

The inequality (51) holds for any $\varepsilon > 0$ and $M > \beta + \varepsilon$, and any γ_{sr} and γ_{rd}. Taking limits from the upper bound and the lower bound in (51), we obtain

$$\lim_{\rho\to\infty}\delta_1 = \frac{\gamma_{sr}}{\gamma_{rd}}\left(2^\beta - 1\right), \tag{52}$$

and

$$\lim_{\rho\to\infty}\delta_2 = 2^\beta\left(2^\varepsilon - 1\right) + \frac{\gamma_{sr}}{\gamma_{rd}}\left(2^{\frac{\beta M}{M-\beta}} - 1\right). \tag{53}$$

In (53), ε and M are arbitrary numbers. By choosing ε and M small and large enough, respectively, one can make the right hand side of (53) arbitrarily close to the right hand side of (52). That is, the upper bound and the lower bound in (51) meet each other as ρ approaches infinity. Thus $\lim_{\rho\to\infty}\kappa(\gamma_{sr}, \gamma_{rd})$ exists and its value is

$$\lim_{\rho\to\infty}\kappa(\gamma_{sr}, \gamma_{rd}) = \frac{\gamma_{sr}}{\gamma_{rd}}\left(2^\beta - 1\right). \tag{54}$$

B. Limit at zero

Finally we prove that $\lim_{\rho\to 0}\kappa(\gamma_{sr}, \gamma_{rd}) = 0$. This follows directly since

$$\begin{aligned}
0 \leq \kappa(\gamma_{sr}, \gamma_{rd}) &= (\ln 2)\int_\beta^\infty 2^t \exp\left(\frac{1-2^t}{\gamma_{sr}}\right)\underbrace{\left[1 - \exp\left(\frac{1-2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right]}_{\leq 1} dt\\
&\leq (\ln 2)\int_\beta^\infty 2^t \exp\left(\frac{1-2^t}{\gamma_{sr}}\right) dt = \gamma_{sr}\underbrace{\exp\left(\frac{1-2^\beta}{\gamma_{sr}}\right)}_{\leq 1}\\
&\leq \gamma_{sr} \to 0, \text{ as } \rho \to 0.
\end{aligned}$$

APPENDIX C
CALCULATION OF $\bar{\mathrm{P}}$

Consider

$$\bar{\mathrm{P}} \triangleq \Pr\left\{\frac{\beta}{x}z + \left(1 - \frac{\beta}{x}\right)y < \beta \middle| x > \beta, z < \beta\right\}$$

where $x = \log_2(1 + \alpha_{sr})$, $y = \log_2(1 + \alpha_{rd})$, and $z = \log_2(1 + \alpha_{sd})$. The pdfs of the random variables x and y are given by (44) and the pdf of z conditioned on the event $\{z < \beta\}$ is

$$g(z) = \begin{cases} \frac{\ln 2}{\varsigma\gamma_{sd}}\exp\left(\frac{1-2^z}{\gamma_{sd}}\right)2^z & 0 < z < \beta \\ 0 & \text{otherwise} \end{cases}$$

where $\varsigma \triangleq \Pr\{z < \beta\} = 1 - \exp\left(\frac{1-2^\beta}{\gamma_{sd}}\right)$. After some manipulations, one obtains

$$\bar{\mathrm{P}} = \int_\beta^\infty \int_0^\beta \left[1 - \exp\left(\frac{1 - 2^{\frac{t-r}{t-\beta}\beta}}{\gamma_{rd}}\right)\right] g(r) f_x(t) dr dt. \quad (55)$$

Appendix D
Proof of Proposition 3

A. Limit at infinity

The $\nu(\gamma_{sr}, \gamma_{rd}, \gamma_{sd})$ is given by

$$\nu(\gamma_{sr}, \gamma_{rd}, \gamma_{sd}) = \gamma_{sd}\gamma_{sr} \int_\beta^\infty \int_0^\beta \left[1 - \exp\left(\frac{1 - 2^{\frac{t-r}{t-\beta}\beta}}{\gamma_{rd}}\right)\right] f_z(r) f_x(t) dr dt,$$

where $f_x(t)$ is given in (44) and $f_z(r) = \frac{\ln 2}{\gamma_{sr}} \exp\left(\frac{1-2^r}{\gamma_{sr}}\right) 2^r, \quad r \geq 0$. To this end, we establish an upper and a lower bound on $\nu(\gamma_{sr}, \gamma_{rd}, \gamma_{sd})$.

Lower bound: Note that we have

$$1 - \exp\left(\frac{1 - 2^{\frac{t-r}{t-\beta}\beta}}{\gamma_{rd}}\right) \geq 1 - \exp\left(\frac{1 - 2^\beta}{\gamma_{rd}}\right)$$

when $0 \leq r \leq \beta$ and $t \geq \beta$. Thus we obtain

$$\begin{aligned} \nu(\gamma_{sr}, \gamma_{rd}, \gamma_{sd}) &\geq \gamma_{sd}\gamma_{sr} \left[1 - \exp\left(\frac{1 - 2^\beta}{\gamma_{rd}}\right)\right] \int_\beta^\infty \int_0^\beta f_z(r) f_x(t) dr dt \\ &= \gamma_{sd}\gamma_{sr} \left[1 - \exp\left(\frac{1 - 2^\beta}{\gamma_{rd}}\right)\right] \left[1 - \exp\left(\frac{1 - 2^\beta}{\gamma_{sd}}\right)\right] \exp\left(\frac{1 - 2^\beta}{\gamma_{sr}}\right) \\ &= \left(2^\beta - 1\right)^2 \frac{\gamma_{sr}}{\gamma_{rd}} + O\left(\frac{1}{\rho}\right). \end{aligned} \quad (56)$$

Upper bound: We have

$$1 - \exp\left(\frac{1 - 2^{\frac{t-r}{t-\beta}\beta}}{\gamma_{rd}}\right) \leq 1 - \exp\left(\frac{1 - 2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right),$$

when $0 \leq r \leq \beta$ and $t \geq \beta$. Then

$$\begin{aligned} \nu(\gamma_{sr}, \gamma_{rd}, \gamma_{sd}) &\leq \gamma_{sd}\gamma_{sr} \int_\beta^\infty \int_0^\beta \left[1 - \exp\left(\frac{1 - 2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right] f_z(r) f_x(t) dr dt \\ &= \gamma_{sd}\gamma_{sr} \left[1 - \exp\left(\frac{1 - 2^\beta}{\gamma_{sd}}\right)\right] \int_\beta^\infty \left[1 - \exp\left(\frac{1 - 2^{\frac{\beta t}{t-\beta}}}{\gamma_{rd}}\right)\right] f_x(t) dt \\ &= \gamma_{sd} \left[1 - \exp\left(\frac{1 - 2^\beta}{\gamma_{sd}}\right)\right] \kappa(\gamma_{sr}, \gamma_{rd}). \end{aligned} \quad (57)$$

Using (56), (57) and (54), we obtain

$$\lim_{\rho \to \infty} \nu(\gamma_{sr}, \gamma_{rd}, \gamma_{sd}) = \frac{\gamma_{sr}}{\gamma_{rd}} \left(2^\beta - 1\right)^2. \quad (58)$$

B. *Limit at zero*

Now, we prove that $\lim_{\rho\to 0}\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})=0$:

$$\begin{aligned}
0\le\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd}) &= \gamma_{sd}\gamma_{sr}\int_{\beta}^{\infty}\int_{0}^{\beta}\underbrace{\left[1-\exp\left(\frac{1-2^{\frac{t-r}{t-\beta}\beta}}{\gamma_{rd}}\right)\right]}_{\le 1} f_z(r)f_x(t)drdt \\
&\le \gamma_{sd}\gamma_{sr}\int_{\beta}^{\infty}\int_{0}^{\beta} f_z(r)f_x(t)drdt \\
&= \gamma_{sd}\gamma_{sr}\underbrace{\left[1-\exp\left(\frac{1-2^{\beta}}{\gamma_{rd}}\right)\right]}_{\le 1}\underbrace{\left[1-\exp\left(\frac{1-2^{\beta}}{\gamma_{sd}}\right)\right]}_{\le 1}\underbrace{\exp\left(\frac{1-2^{\beta}}{\gamma_{sr}}\right)}_{\le 1} \\
&\le \gamma_{sd}\gamma_{sr}\to 0,\ \text{as}\ \rho\to 0.
\end{aligned}$$

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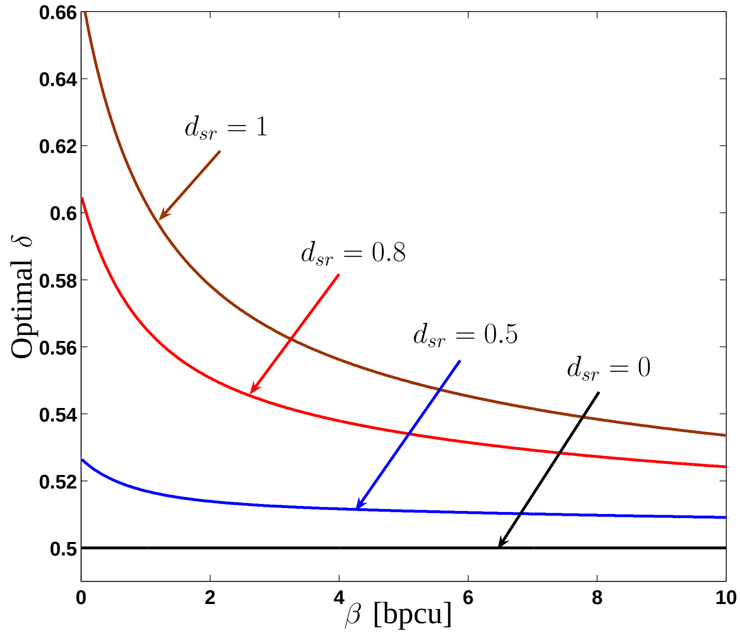
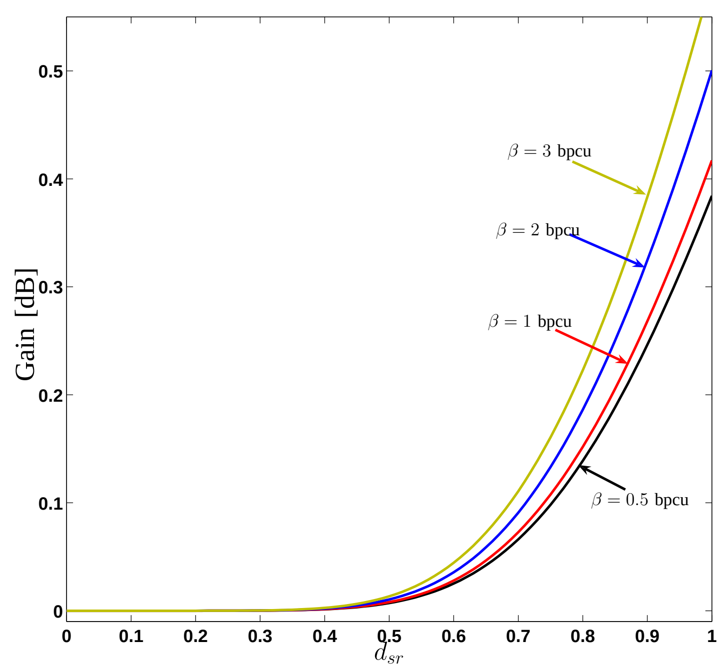
TABLE II

SUMMARY OF RESULTS: OUTAGE BEHAVIOR OF THE DF RELAYING SCHEMES. IN THE TABLE, γ_{ij} IS THE AVERAGE SNR FROM NODE i TO NODE j AND ρ IS THE TRANSMIT POWER. (FOR THE DEFINITIONS OF CASES 1, 2 AND 3, SEE TABLE I.)

Transmission scheme		$\mathrm{P_{out}}$: Outage Probability
Direct transmission ($\mathcal{S}-\mathcal{D}$)		$\frac{2^{\beta}-1}{\gamma_{sd}}+O\left(\frac{1}{\rho^2}\right)$
Case 1 (Section III)	Repetition coding and SC	$\left(2^{2\beta}-1\right)^2 \frac{\gamma_{rd}/\gamma_{sd}+\gamma_{sr}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right)$
	Repetition coding and MRC	$\left(2^{2\beta}-1\right)^2 \frac{\gamma_{rd}/\gamma_{sd}+0.5\gamma_{sr}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right)$
	Parallel coding and SC	$\frac{2^{\beta_s}-1}{\gamma_{sd}}\left[\frac{2^{\beta_s}-1}{\gamma_{sr}}+\frac{2^{\beta_r}-1}{\gamma_{rd}}\right]+O\left(\frac{1}{\rho^3}\right)$
	Parallel coding and CC	$\begin{cases} \frac{\left(2^{\beta_s}-1\right)^2}{\gamma_{sd}\gamma_{sr}}+\frac{\left(1-2^{\beta_r}+\frac{\delta}{2\delta-1}2^{\beta_s}\left(2^{\beta_r\frac{2\delta-1}{\delta}}-1\right)\right)}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta\neq\frac{1}{2} \\ \frac{\left(2^{2\beta}-1\right)^2}{\gamma_{sd}\gamma_{sr}}+\frac{\left(1-2^{2\beta}+2\ln(2)\beta 2^{2\beta}\right)}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta=\frac{1}{2} \end{cases}$
Case 2 (Section IV)	Repetition coding and SC	$\left(2^{\beta}-1\right)\left(2^{2\beta}-1\right)\frac{\gamma_{sr}/\gamma_{sd}+\gamma_{rd}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right)$
	Repetition coding and MRC	$\left(2^{\beta}-1\right)\left(2^{2\beta}-1\right)\frac{\gamma_{sr}/\gamma_{sd}+\gamma_{rd}/\gamma_{sd}}{\gamma_{sr}\gamma_{rd}}-0.5\left(2^{\beta}-1\right)^2\frac{1}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right)$
	Parallel coding and SC	$\frac{2^{\beta}-1}{\gamma_{sd}}\left[\frac{2^{\beta_s}-1}{\gamma_{sr}}+\frac{2^{\beta_r}-1}{\gamma_{rd}}\right]+O\left(\frac{1}{\rho^3}\right)$
	Parallel coding and CC	$\begin{cases} \left(2^{\beta_s}-1\right)\left(2^{\beta}-1\right)\frac{1}{\gamma_{sd}\gamma_{sr}}+ & \\ \left(1-2^{\beta}+\frac{\delta-1}{2\delta-1}2^{\beta_r}\left(2^{\beta\frac{2\delta-1}{\delta-1}}-1\right)\right)\frac{1}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta\neq\frac{1}{2} \\ \left(2^{2\beta}-1\right)\left(2^{\beta}-1\right)\frac{1}{\gamma_{sd}\gamma_{sr}} & \\ \left(1-2^{\beta}+\ln(2)\beta 2^{2\beta}\right)\frac{1}{\gamma_{sd}\gamma_{rd}}+O\left(\frac{1}{\rho^3}\right), & \text{if } \delta=\frac{1}{2} \end{cases}$
Case 3 (Section V)	Parallel coding and SC	$\left[\left(2^{\beta}-1\right)^2+\kappa(\gamma_{sr},\gamma_{rd})\left(2^{\beta}-1\right)\right]\frac{1}{\gamma_{sd}\gamma_{sr}}+O\left(\frac{1}{\rho^3}\right)$
	Parallel coding and CC	$\left[\left(2^{\beta}-1\right)^2+\nu(\gamma_{sr},\gamma_{rd},\gamma_{sd})\right]\frac{1}{\gamma_{sd}\gamma_{sr}}+O\left(\frac{1}{\rho^3}\right)$

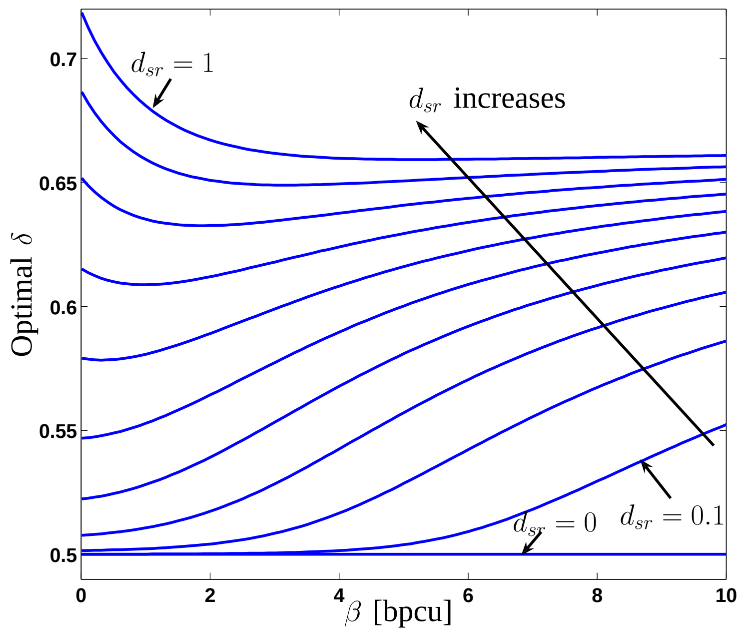


(a) The optimum δ as a function of the spectral efficiency for different d_{sr}, when $d_{sd} = d_{rd} = 1$, $\alpha = 4$ and the destination performs selection combining.

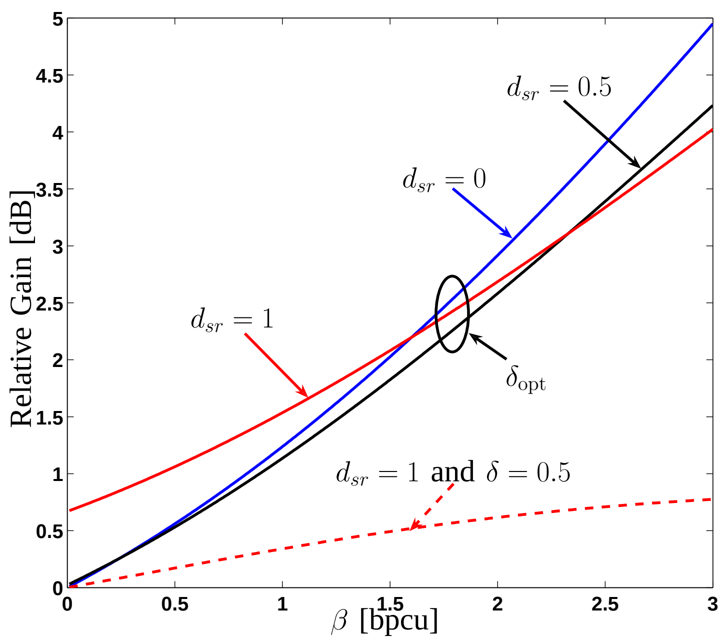


(b) The gain of parallel coding with SC and optimum δ over SC with $\delta = 0.5$, as a function of d_{sr} for different β, when $d_{sd} = d_{rd} = 1$ and $\alpha = 4$.

Fig. 2. Comparisons for Case 1. PC at $\mathcal{R}$ and SC at $\mathcal{D}$.



(a) The optimum δ as a function of the spectral efficiency for different d_{sr}, when $d_{sd} = d_{rd} = 1$, $\alpha = 4$.



(b) The relative gain of parallel coding with code combining and optimal δ over repetition coding with MRC, as a function of the spectral efficiency for different d_{sr}, when $d_{sd} = d_{rd} = 1$ and $\alpha = 4$.

Fig. 3. Further comparisons for Case 1. PC at $\mathcal{R}$ and CC at $\mathcal{D}$.

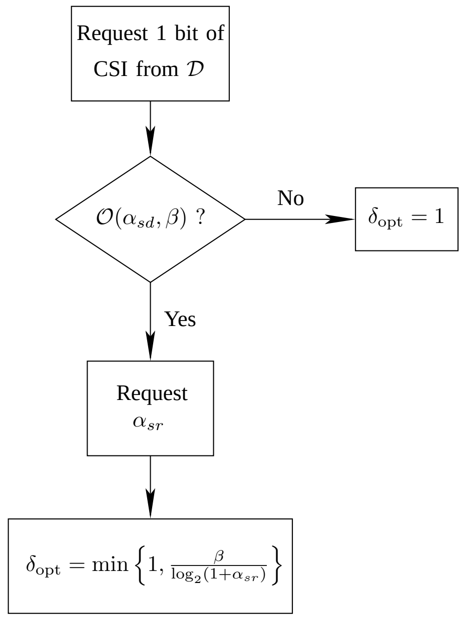


Fig. 4. Flowchart for optimal block length allocation. (For Cases 2 and 3.)

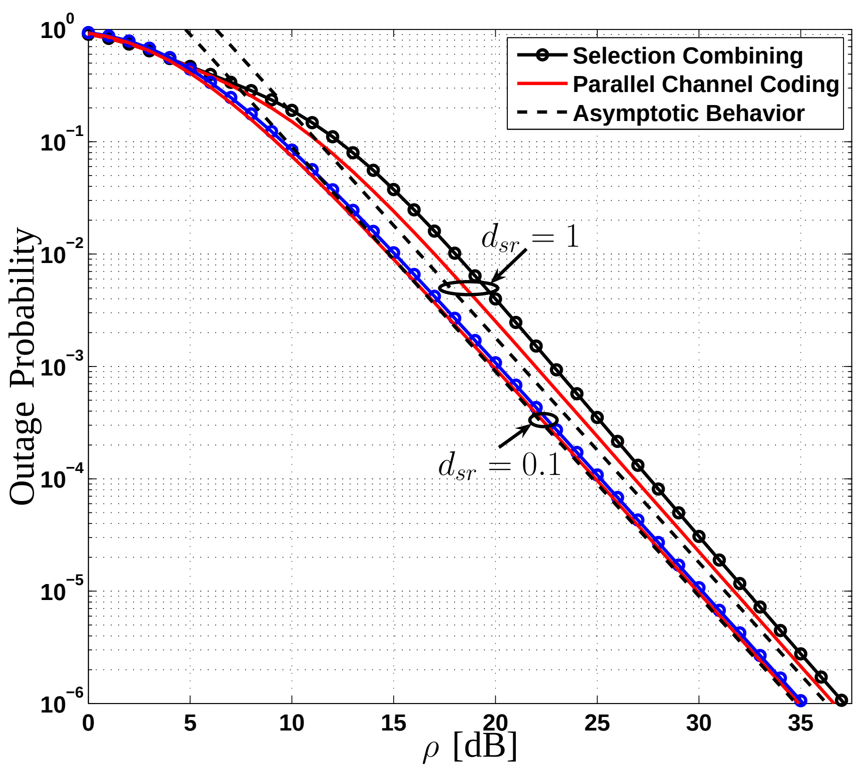
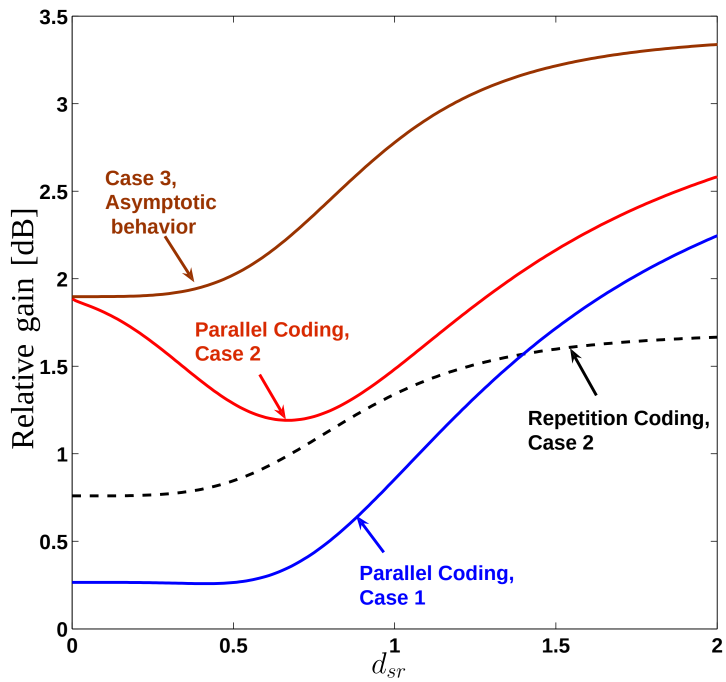
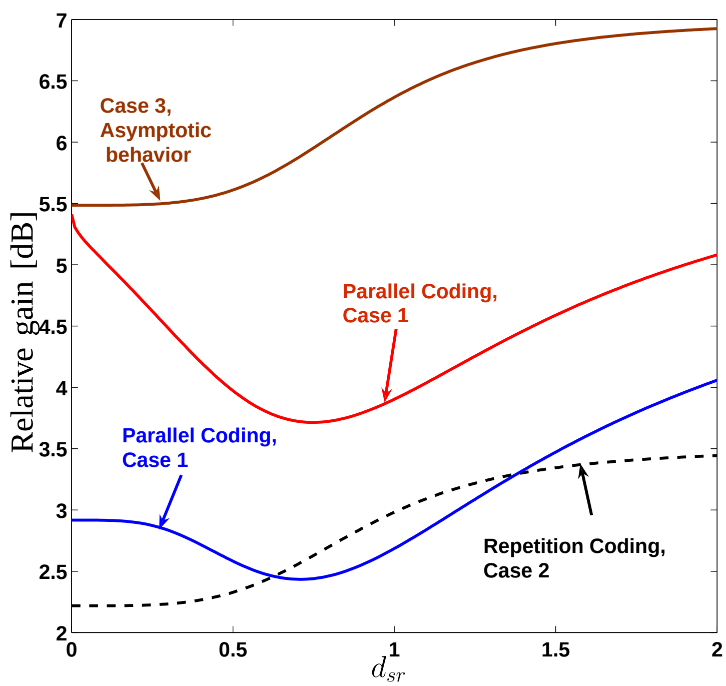


Fig. 5. Performance comparison for Case 3. PC-based DF schemes with SC and CC. Here $d_{rd} = d_{sd} = 1$, and $\alpha = 4$. The asymptotic curve is obtained using (36) and (42).



(a) $\beta = 0.25$ bpcu



(b) $\beta = 2$ bpcu

Fig. 6. Comparison of DF relaying schemes when $d_{sr} = d_{rd} = 1$. The gain is with respect to repetition coding with MRC.

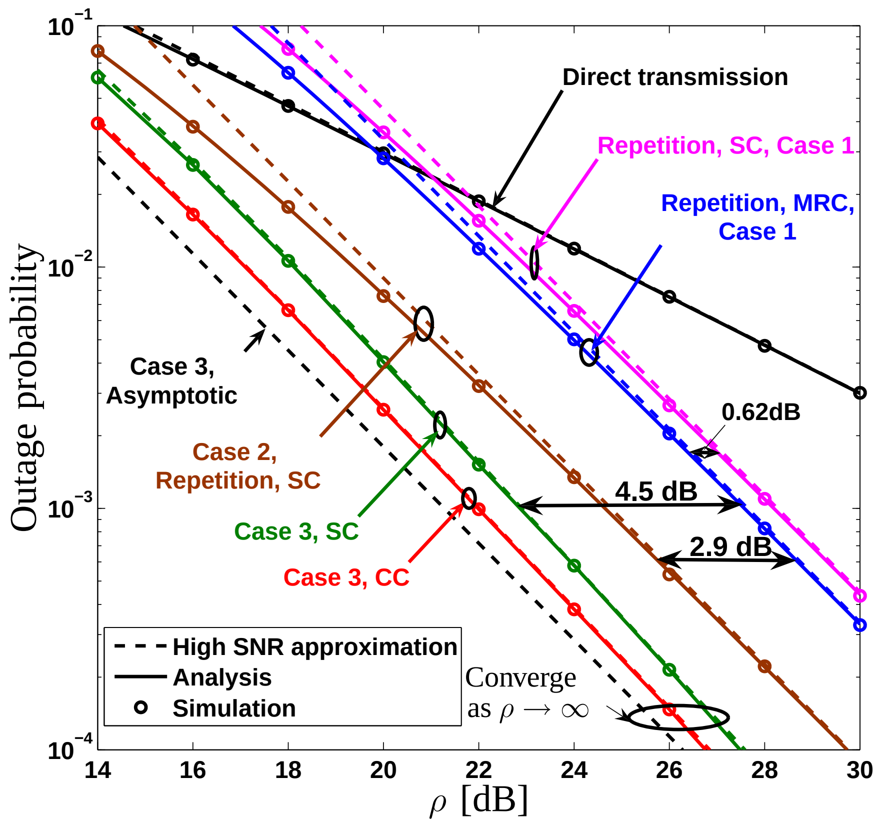


Fig. 7. Outage probability of various DF relaying schemes for spectral efficiency $\beta = 2$ bpcu. The asymptotic curve for Case 3 is obtained using (36) and (42).