TSTE14
Analog Filters
160609

1a) Passive - can only dissipate power
$\Rightarrow$ Passive filters always stable

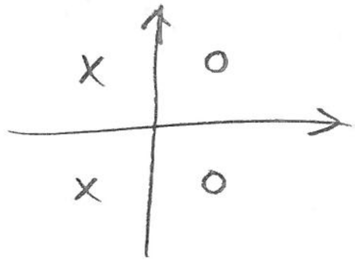
b) Cascaded two-ports

c) Overdesigned (higher filter order) due to bandedge symmetries and the same attenuation in both passbands / stopbands.
BS / BP

d) 11th-order Butterworth - 11 components
7th-order Cauer - 10 components

e) $Z = P$ mirrored, $z = a + jb$, $p = -a + jb$

Ex second-order



2) Filter order (Nomogram $\frac{\omega_s}{\omega_c} = 4{,}7$) $N = 4$

Table C040506) $14 \le \theta \le 23$ Select $\theta = 18$ (for ex.)

Normalized poles $-0{,}3749 \pm j1{,}3374$

(rounded) $-0{,}9954 \pm j0{,}5928$

zeros $\pm j3{,}8206$

2 at ∞

Denormalize, mult. by $\omega_c \Rightarrow$

poles $(-4{,}711 \pm j16{,}8062) \times 10^6$

$(-12{,}5086 \pm j7{,}4493) \times 10^6$

zeros $48{,}0111 \times 10^6$

2 at ∞

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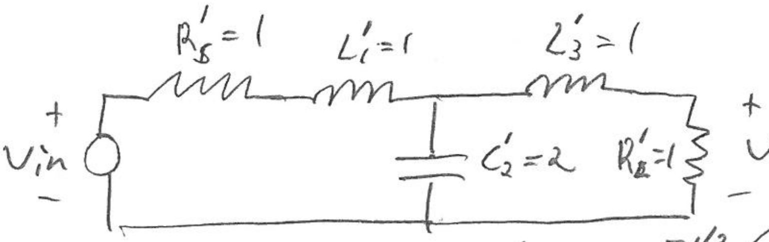
3) LP prototype $\Omega_c = \frac{\omega_I^2}{\omega_c} = /\text{use } \omega_I^2 = \omega_c/ = 1$, $\Omega_s = \frac{\omega_I^2}{\omega_s} = \frac{\omega_c}{\omega_s} = 10$

Filter order (Nomogram) $N = 3$



and $R = R_L = R_S = 10\,k\Omega$

Denormalize with $\Omega_c \cdot \varepsilon^{-1/3}$, $A_{max} = 10 \cdot \log_{10}(1+\varepsilon^2) = 0{,}2$

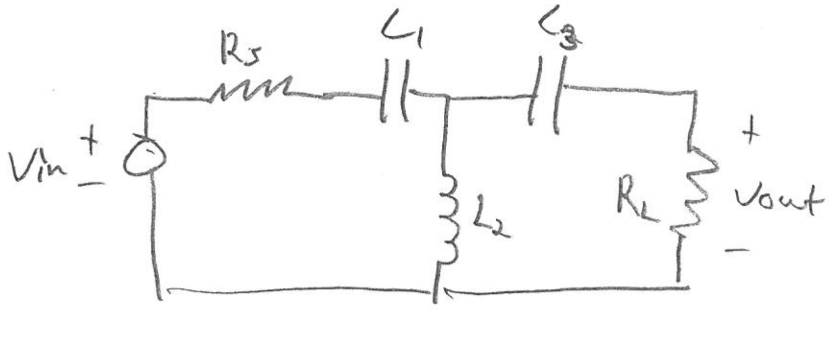
$\varepsilon = \sqrt{10^{0{,}1A_{max}} - 1} = 0{,}2171 \Rightarrow \varepsilon^{-1/3} = 1{,}664$

$\Rightarrow L_1 = L_3 = 6{,}010$ kH, $C_2 = 0{,}1202$ mF

LP $\rightarrow$ HP

$C_1 = C_3 = \frac{1}{\omega_I^2 L_1} = 2{,}648$ pF

$L_2 = \frac{1}{\omega_I^2 C_2} = 0{,}1324$ mH



4a) $H_3(s) = \frac{P_1}{s+P_1} + \frac{s}{s+P_2} = \frac{P_1(s+P_2) + s(s+P_1)}{(s+P_1)(s+P_2)} = \frac{s^2+2P_1 s+P_1P_2}{(s+P_1)(s+P_2)}$

poles: $-P_1$ & $-P_2$

zeros: $s = -\frac{P_1}{2} \pm \sqrt{(\frac{P_1}{2})^2 - P_1P_2}$

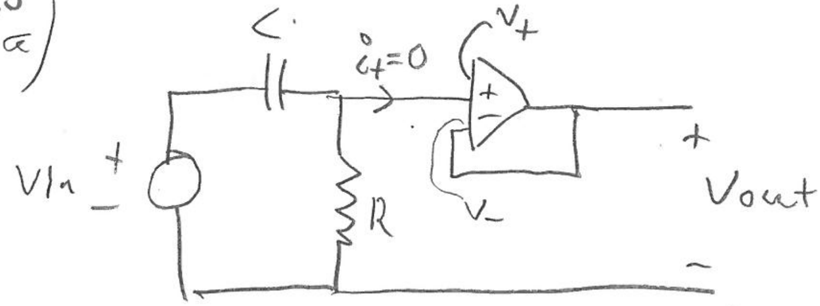
b) $H_4(s) = \frac{P_1}{s+P_1} \cdot \frac{s}{s+P_2}$, poles: $-P_1$ & $-P_2$

zero: 0

c) $|H_3(j0)| = |1+0| = 1$, $|H_3(j\infty)| = |0+1| = 1$

$|H_4(j0)| = |1 \cdot 0| = 0$, $|H_4(j\infty)| = |0 \cdot 1| = 0$

5 a)



$$V_{out}(s) = A(s)\left(V_+(s) - V_-(s)\right) \quad (1)$$

$$V_+(s) \underset{\text{voltz. div.}}{=} V_{in}(s) \cdot \frac{R}{R + \frac{1}{sC}} = \underbrace{\frac{sRC}{1+sRC}}_{H_1(s)} \cdot V_{in}(s) \quad (2)$$

$$V_-(s) = V_{out}(s) \quad (3)$$

$$(1)-(3) \Rightarrow V_{out}(s) = A(s)\left(H_1(s)V_{in}(s) - V_{out}(s)\right)$$

$$\Longrightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{H_1(s) \cdot A(s)}{1 + A(s)} = H_1(s)H_2(s), \quad H_2(s) = \frac{A(s)}{1+A(s)}$$

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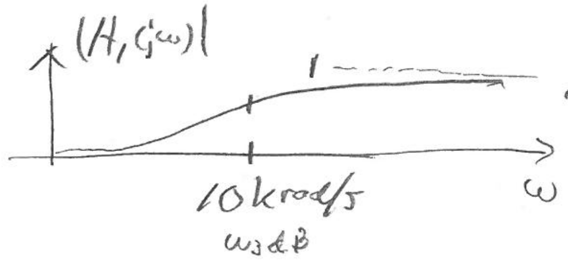
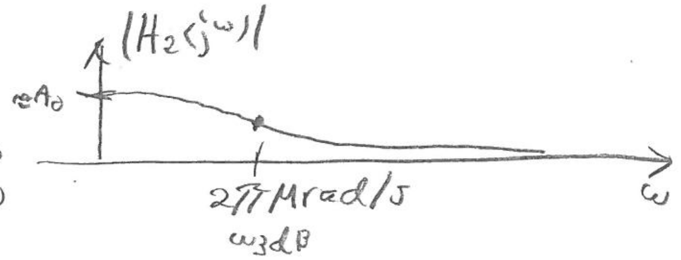
b) $H_1(j\omega) = \dfrac{j\omega RC}{1 + j\omega RC}$, ω_{3dB}: $\omega RC = 1 \Rightarrow \omega_{3dB} = \dfrac{1}{RC} = 10 \text{krad/s}$

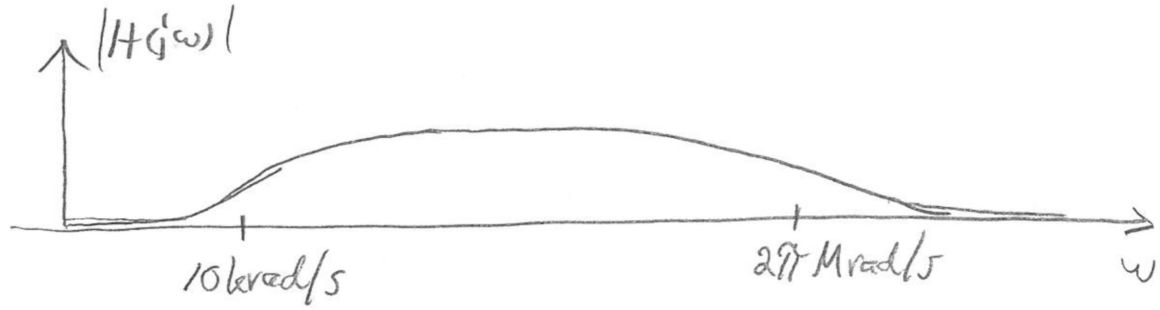
max: $|H(j\infty)| = 1$

$$H_2(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)} = \frac{\frac{A_0}{1 + j\omega/\omega_0}}{1 + \frac{A_0}{1 + j\omega/\omega_0}} = \frac{A_0}{A_0 + 1 + j\omega/\omega_0} \approx$$

$$\omega_{3dB}: \frac{\omega}{\omega_0} = 1 + A_0 \Rightarrow \omega_{3dB} = \omega_0(1 + A_0) \underset{A_0 \gg 1}{\approx} \omega_0 A_0 = 2\pi \text{M rad/s}$$







6) a) $H_{BP}(s) = H_{LP}(\tilde{s})$ for $\tilde{s} = s + \frac{\omega_I^2}{s}$

$s = j\omega \Rightarrow \tilde{s} = j\omega + \frac{\omega_I^2}{j\omega} = j\left(\omega - \frac{\omega_I^2}{\omega}\right) = j\Omega$ for $\Omega = \omega - \frac{\omega_I^2}{\omega}$

$\therefore H_{BP}(j\omega) = H_{LP}(j\Omega)$ for $\Omega = \omega - \frac{\omega_I^2}{\omega}$

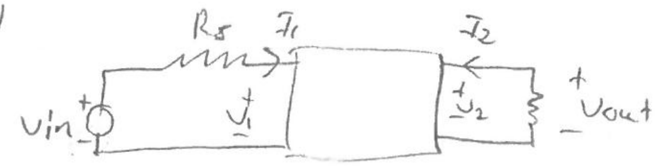
b) $\Omega = \omega - \frac{\omega_I^2}{\omega}$ and $\Omega_c = \omega_{c2} - \omega_{c1}$ and $\omega_I^2 = \omega_{c1}\omega_{c2}$

$$\left.\begin{array}{l}
\omega = \omega_{c1}: \quad \Omega = \omega_{c1} - \frac{\omega_{c1}\omega_{c2}}{\omega_{c1}} = \omega_{c1} - \omega_{c2} = -\Omega_c \\
\omega = \omega_{c2}: \quad \Omega = \omega_{c2} - \frac{\omega_{c1}\omega_{c2}}{\omega_{c2}} = \omega_{c2} - \omega_{c1} = \Omega_c \\
\text{\& } \Omega = \omega - \frac{\omega_I^2}{\omega} \text{ increasing function on } [\omega_{c1}, \omega_{c2}] \\
\therefore \omega \in [\omega_{c1}, \omega_{c2}] \Leftrightarrow \Omega \in [-\Omega_c, \Omega_c]
\end{array}\right\} (1)$$

$$\left.\begin{array}{l}
\omega = -\omega_{c1}: \quad \Omega = -\omega_{c1} + \frac{\omega_{c1}\omega_{c2}}{\omega_{c1}} = \omega_{c2} - \omega_{c1} = \Omega_c \\
\omega = -\omega_{c2}: \quad \Omega = -\omega_{c2} + \frac{\omega_{c1}\omega_{c2}}{\omega_{c2}} = \omega_{c1} - \omega_{c2} = -\Omega_c \\
\text{\& } \Omega = \omega - \frac{\omega_I^2}{\omega} \text{ increasing function on } [-\omega_{c2}, -\omega_{c1}] \\
\therefore \omega \in [-\omega_{c2}, -\omega_{c1}] \Leftrightarrow \Omega \in [-\Omega_c, \Omega_c]
\end{array}\right\} (2)$$

Alt.: (2) follows from (1) and the fact that

$\omega \to -\omega$ corresponds to $\Omega \to -\Omega$

7) 

((s) ignored for simplicity)

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \quad (1)$$

$$V_2 = V_{out} = -R_L I_2 \quad (2)$$

$$V_{in} = V_1 + R_S I_1 \quad (3)$$

(1) and (2) $\Rightarrow$ $\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_{out} \\ V_{out}/R_L \end{pmatrix} \quad (4)$

(3) and (4) $\Rightarrow$ $V_{in} = \left(A + \frac{B}{R_L} + C R_S + D \cdot \frac{R_S}{R_L} \right) V_{out}$

$$\Rightarrow H = \frac{V_{out}}{V_{in}} = \frac{1}{A + \frac{B}{R_L} + C R_S + D \frac{R_S}{R_L}} = \frac{R_L}{AR_L + B + C R_L R_S + D R_S}$$