Solutions to TSTE14 ~~[illegible]~~ Analog Filters 160321

1a) Linear $a_1 x_1(t) + a_2 x_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$ if $x(t) \rightarrow y(t)$

Time-invariant $x(t - t_0) \rightarrow y(t - t_0)$

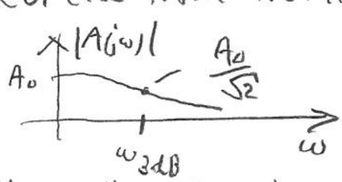
b) LP: 2 zeros at $s = \infty$, HP: 2 zeros at $s = 0$,

BP: one zero at $s = 0$, one zero at $s = \infty$

c) Overdesigned (higher filter order) due to band edge symmetries and the same attenuation in both passbands/stopbands
BS BP

d) Low sensitivity $\Rightarrow$ small variations in $H(j\omega)$ when component values deviates from nominal values

e) $\omega_t = A_0 \cdot \omega_{3dB}$



$$A(j\omega) = \frac{A_0}{1 + \frac{j\omega}{\omega_{3dB}}}$$

Approx. the max bandwidth of the filter in which the OPamp is used

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2) Filter order (nomogram $\frac{\omega_s}{\omega_c} = 2$) $N = 5$

Table C0515θ, $30 \le \theta \le 36$ Select $\theta = 33$ (for ex)

Normalized poles: $-0{,}6046827$

$-0.1335634 \pm j \times 1.0725098$

$-0{,}4279530 \pm j \times 0.7336827$

Normalized zeros $\pm j \times 1.9153952$

$\pm j \times 2.9552877$

one at ∞

Denormalize, mult by ω_c $\Rightarrow$

poles: -3.7993334×10^6

$(-0{,}8392036 \pm j \times 6.7387778) \times 10^6$

$(-2{,}6889080 \pm j \times 4.6098644) \times 10^6$

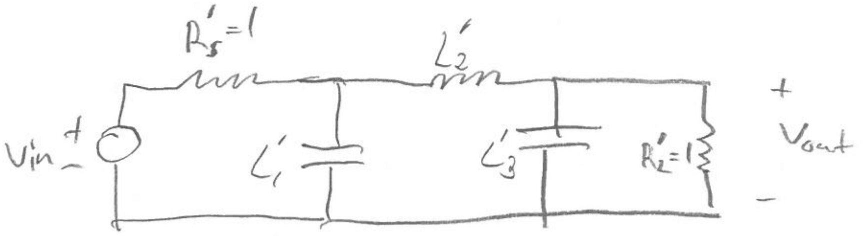
zeros $\pm j \times 1.2034783 \times 10^7$

$\pm j \times 1.8568620 \times 10^7$

one at ∞

3) LP prototype $\Omega_c = \frac{\omega_I^2}{\omega_c}$ | use $\omega_I^2 = \omega_c$ | $= 1$, $\Omega_s = \frac{\omega_I^2}{\omega_s} = \frac{\omega_c}{\omega_s} = 5$

$\Rightarrow$ Filter order $N = 3$



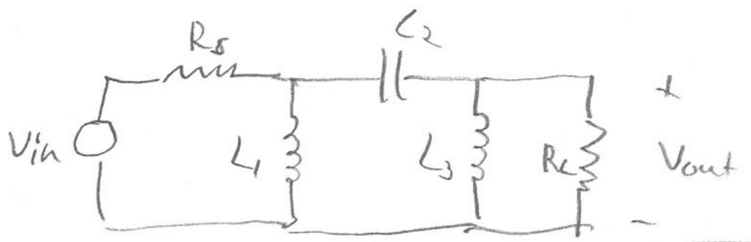
$C_1' = C_3' = 1{,}5963$

$L_2' = 1.0967$

Denormalize with $\Omega_c = 1$, $R = R_S = R_L = 2\,k\Omega \Rightarrow$

$C_1 = C_3 = \frac{C_1'}{\Omega_c \cdot R} = 0{,}79815\ mF$, $L_2 = L_2' \cdot \frac{R}{\Omega_c} = 2{,}1934\ kH$

LP → HP



$L_1 = L_3 = \frac{1}{\omega_I^2 \cdot C_1} = \frac{1}{\omega_c \cdot C_1} = 39.881\ \mu H$

$C_2 = \frac{1}{\omega_c \cdot L_2} = 14{,}512\ pF$

4) a) BP: $a = c = 0$, $b \neq 0$, BS: $a \neq 0$, $c \neq 0$, $b = 0$

b) $H(j\omega) = \frac{e}{(j\omega)^2 + d \cdot j\omega + e}$, $|H(j\omega)| = \frac{|e|}{\sqrt{(e-\omega^2)^2 + d^2\omega^2}}$

$$\frac{\partial |H(j\omega)|}{\partial \omega} = |e| \cdot \frac{2(e-\omega^2)\cdot -2\omega + 2d^2\omega}{\left((e-\omega^2)^2 + d^2\omega^2\right)^{1,5}} = 0 \text{ when } \omega = 0$$ not the sol. we want here

and when $-4(e-\omega^2) + 2d^2 = 0 \Leftrightarrow \omega^2 = e - \frac{d^2}{2} \Leftrightarrow \omega = \sqrt{e - \frac{d^2}{2}}$

$\left(@\ \omega = -\sqrt{e - \frac{d^2}{2}},\ |H(j\omega)| = |H(-j\omega)| \right)$

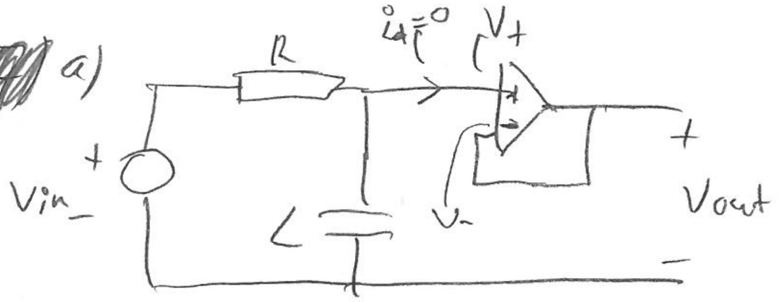
$\therefore \omega_{peak} = \sqrt{e - \frac{d^2}{2}}$

with $e = 10$, $d = 1$: $\omega_{peak} = \sqrt{10 - \frac{1}{2}} \approx 3.082207$

$\Rightarrow |H(j\omega_{peak})| \approx 3.2025$

(Note: with $e = r_p^2$, $d = -2\sigma_p = \frac{r_p}{Q}$: $\omega_{peak} = r_p\sqrt{1 - \frac{1}{2Q^2}}$)

5 a)



$$V_{out}(s) = A(s)\left(V_+(s) - V_-(s)\right) \qquad (1)$$

$$V_+(s) = V_{in}(s) \cdot \frac{1/sC}{R + 1/sC} = V_{in}(s) \cdot \underbrace{\frac{1}{1+sRC}}_{H_1(s)} \qquad (2)$$

volt. div. since $i_+ = 0$, i.e., $i_R = i_C$

$$V_-(s) = V_{out}(s) \qquad (3)$$

$$(1)-(3) \Rightarrow V_{out}(s) = A(s)\left(V_{in}(s) \cdot H_1(s) - V_{out}(s)\right)$$

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{H_1(s) \cdot A(s)}{1 + A(s)} = H_1(s)\, H_2(s)$$

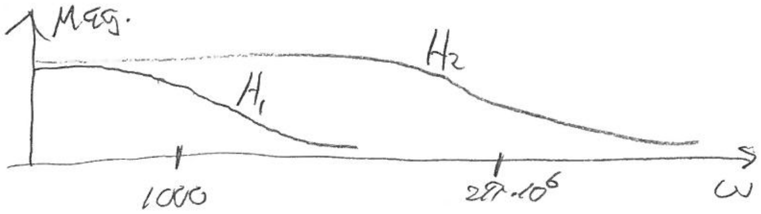
$H_2(s)$

b) $$H_1(j\omega) = \frac{1}{1 + j\omega RC}, \quad \omega_{3dB}: \ \omega RC = 1 \Rightarrow \omega_{3dB} = \frac{1}{RC} = 1000 \text{ rad/s}$$

$$H_2(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)} = \frac{\frac{A_0}{1 + j\omega/\omega_0}}{1 + \frac{A_0}{1 + j\omega/\omega_0}} = \frac{A_0}{1 + A_0 + j\omega/\omega_0}$$

$$\omega_{3dB}: \ \frac{\omega}{\omega_0} = 1 + A_0 \Rightarrow \omega_{3dB} = \omega_0(1 + A_0) \approx \omega_0 A_0 = 2\pi \cdot 10^6 \text{ rad/s}$$

$H(j\omega)$: ω_{3dB} approx the same as for $H_1(j\omega)$ since the bandwidth of $H_2(j\omega)$ much larger



6) a) $H_{BP}(s) = H_{LP}(\mathcal{S})$ for $\mathcal{S} = s + \frac{\omega_I^2}{s}$

$s = j\omega \Rightarrow \mathcal{S} = j\omega + \frac{\omega_I^2}{j\omega} = j\left(\omega - \frac{\omega_I^2}{\omega}\right) = j\Omega$ for $\Omega = \omega - \frac{\omega_I^2}{\omega}$

$\therefore H_{BP}(j\omega) = H_{LP}(j\Omega)$ for $\Omega = \omega + \frac{\omega_I^2}{\omega}$

b) $\Omega = \omega - \frac{\omega_I^2}{\omega}$ and $\Omega_c = \omega_{c2} - \omega_{c1}$ and $\omega_I^2 = \omega_{c1}\omega_{c2}$

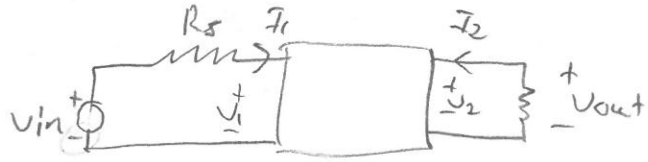
$$\left.\begin{array}{l}
\omega = \omega_{c1}: \quad \Omega = \omega_{c1} - \frac{\omega_{c1}\omega_{c2}}{\omega_{c1}} = \omega_{c1} - \omega_{c2} = -\Omega_c \\
\omega = \omega_{c2}: \quad \Omega = \omega_{c2} - \frac{\omega_{c1}\omega_{c2}}{\omega_{c2}} = \omega_{c2} - \omega_{c1} = \Omega_c \\
\text{\& } \Omega = \omega - \frac{\omega_I^2}{\omega} \text{ increasing function on } [\omega_{c1}, \omega_{c2}] \\
\therefore \omega \in [\omega_{c1}, \omega_{c2}] \Leftrightarrow \Omega \in [-\Omega_c, \Omega_c]
\end{array}\right\} (1)$$

$$\left.\begin{array}{l}
\omega = -\omega_{c1}: \quad \Omega = -\omega_{c1} + \frac{\omega_{c1}\omega_{c2}}{\omega_{c1}} = \omega_{c2} - \omega_{c1} = \Omega_c \\
\omega = -\omega_{c2}: \quad \Omega = -\omega_{c2} + \frac{\omega_{c1}\omega_{c2}}{\omega_{c2}} = \omega_{c1} - \omega_{c2} = -\Omega_c \\
\text{\& } \Omega = \omega - \frac{\omega_I^2}{\omega} \text{ increasing function on } [-\omega_{c2}, -\omega_{c1}] \\
\therefore \omega \in [-\omega_{c2}, -\omega_{c1}] \Leftrightarrow \Omega \in [-\Omega_c, \Omega_c]
\end{array}\right\} (2)$$

Alt.: (2) follows from (1) and the fact that

$\omega \to -\omega$ corresponds to $\Omega \to -\Omega$

7)



((s) ignored for simplicity)

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \qquad (1)$$

$$V_2 = V_{out} = -R_L I_2 \qquad (2)$$

$$V_{in} = V_1 + R_S I_1 \qquad (3)$$

$$(1) \text{ and } (2) \Rightarrow \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_{out} \\ V_{out}/R_L \end{pmatrix} \qquad (4)$$

$$(3) \text{ and } (4) \Rightarrow V_{in} = \left(A + \frac{B}{R_L} + C R_S + D \cdot \frac{R_S}{R_L} \right) V_{out}$$

$$\Rightarrow H = \frac{V_{out}}{V_{in}} = \frac{1}{A + \frac{B}{R_L} + C R_S + D \frac{R_S}{R_L}} = \frac{R_L}{A R_L + B + C R_L R_S + D R_S}$$