TSEI 10 Filters [redacted] - Solutions 170608

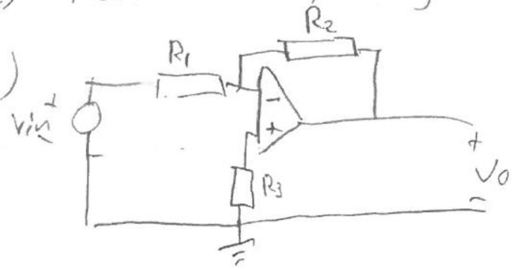
1/ a) Passive - can only dissipate power, always stable filters
Active - can generate power, can be unstable

b) Spec. may have to be sharpened $\Rightarrow$ higher filter order

c) Maximum power transfer principle $\Rightarrow$ lower sensitivity

d) Periodic frequency response

e)



$$V_{out} = V_{in} \cdot \underbrace{\left(\overbrace{-}^{\text{inverting}} \frac{R_2}{R_1} \right)}_{\text{Transfer function}}$$

2) HP filter spec. $\Rightarrow$ LP:

$$\Omega_c = \frac{\omega_I^2}{\omega_c} \Big/ \text{Select } \omega_I^2 = \omega_c \Big/ = 1, \quad \Omega_s = \frac{\omega_I^2}{\omega_s} = \frac{\omega_c}{\omega_s} = 3$$

$\Rightarrow$ Filter order $N = 4$

Normalized poles: $-0{,}175353 \pm j\,1.0162529$

$-0{,}4233398 \pm j\,0{,}420946$

zeros: 4 at $s = \infty$

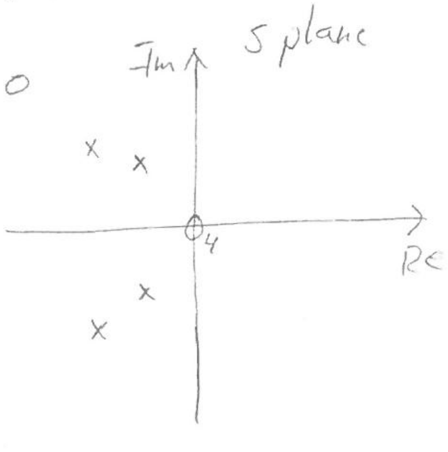
Denormalized = Normalized due to $\Omega_c = 1$

$$LP \to HP \quad s = \frac{\omega_I^2}{s'} = \frac{\omega_c}{s'} \quad \Rightarrow$$

poles: $(-0{,}124317 \pm j\,0{,}720473) \times 10^6$

$(-0{,}893566 \pm j\,0{,}890502) \times 10^6$

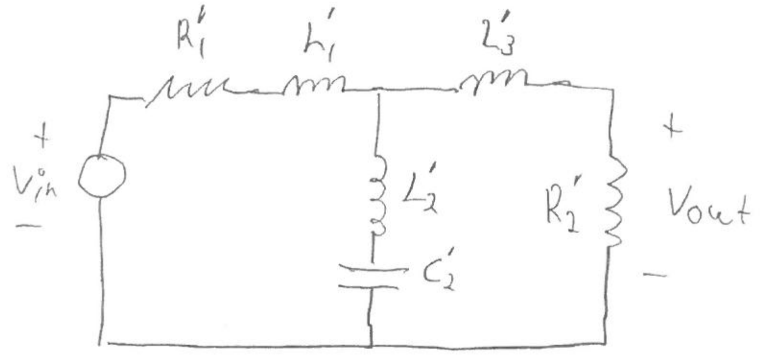
zeros: 4 at $s = 0$



3) $\frac{\omega_s}{\omega_c} = 5 \Rightarrow N = 3$

Table C0315θ, $12 \leq \theta \leq 15$

Select (for ex.) $\theta = 13 \Rightarrow$



$R_1' = R_2' = 1\ \Omega$

$L_1' = L_3' = 1{,}002943\ H$

$L_2' = 0{,}034496\ H$

$C_2' = 1{,}107291\ F$

Denormalize $L \to L \cdot \frac{R_1}{\omega_c}$, $C \to C \cdot \frac{1}{\omega_c R_1}$, $R_1 = R_2 = 50\,\Omega$

$\Rightarrow L_1 = L_3 = 1{,}33019 \times 10^{-4}\ H$

$L_2 = 4{,}57517 \times 10^{-6}\ H$

$C_2 = 5{,}87436 \times 10^{-8}\ F$

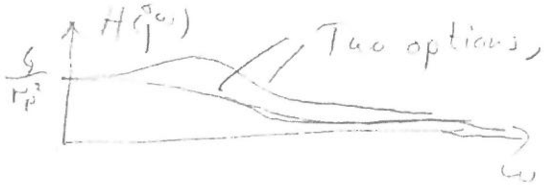
4a) $H(s) = H_1(s) H_2(s) = -\frac{\overset{\text{2 zeros at } s=0}{s^2}}{\underbrace{(s^2+24s+169)}_{\text{poles at } -12 \pm j5}\underbrace{(s^2+10s+169)}_{\text{poles at } -5 \pm j12}}$

b) $H(s) = H_1(s) + H_2(s) = -\frac{14s^2}{(s^2+24s+169)(s^2+10s+169)}$

∴ same poles and zeros as in a).

c) Same filter in both cases, except for the gain of 14 in b)

$H(j0) = 0$, $H(j\infty) = 0$ ∴ Bandpass filter

5a) 

Two options, always lowpass
$H(j\omega) \to 0$ when $\omega \to \infty$

b) $H(j0) = \frac{G}{r_p^2}$, $G = r_p^2 \Rightarrow H(j0) = 1$ and thus $|H(j\omega)| = 1$

c) $$|H(j\omega)| = \frac{|G|}{\sqrt{(r_p^2 - \omega^2)^2 + a^2\omega^2}}, \quad a = \frac{r_p}{Q}$$

$$\frac{\partial |H(j\omega)|}{\partial \omega} = |G| \cdot \frac{-4\omega(r_p^2 - \omega^2) + 2a^2\omega}{\left[(r_p^2 - \omega^2)^2 + a^2\omega^2\right]^{1/3}} = 0 \Rightarrow$$

$$2(r_p^2 - \omega^2) = a^2 \Leftrightarrow \omega^2 = r_p^2 - \frac{a^2}{2} = r_p^2 - \frac{r_p^2}{2Q^2} = r_p^2\left(1 - \frac{1}{2Q^2}\right)$$

$$\Leftrightarrow \omega = \omega_{peak} = \underset{(-)}{+} r_p \sqrt{1 - \frac{1}{2Q^2}}$$ (and $\omega = 0$, corresp. to minimum when $0 < \omega_{peak} < \infty$)

d) $$1 - \frac{1}{2Q^2} \geq 0 \Leftrightarrow Q \geq \frac{1}{\sqrt{2}}$$

6a) $I_{op} = 0 \Rightarrow V_+ = 0$ and $V_{out} = A(V_+ - V_-) = -A V_-$

and : $V_{in}(s) - I(s)\left(Z_1(s) + Z_2(s)\right) - V_{out}(s) = 0 \quad (1)$

from

$$I(s) = \frac{V_{in}(s) - V_-(s)}{Z_1(s)} = \frac{V_{in}(s)}{Z_1(s)} + \frac{V_{out}(s)}{Z_1(s)\,A(s)} \quad (2)$$

$$(1)\ \&\ (2) \Rightarrow V_{in}(s) - \left(\frac{V_{in}(s)}{Z_1(s)} + \frac{V_{out}(s)}{Z_1(s)A(s)}\right)\left(Z_1(s) + Z_2(s)\right) - V_{out}(s) = 0$$

$$\Leftrightarrow V_{in}(s)\underbrace{\left(1 - \frac{\left(Z_1(s) + Z_2(s)\right)}{Z_1(s)}\right)}_{= -\frac{Z_2(s)}{Z_1(s)}} = V_{out}(s)\left(1 + \frac{Z_1(s) + Z_2(s)}{Z_1(s)\,A(s)}\right)$$

$(\to 1 \text{ when } A \to \infty)$

$$\Leftrightarrow H(s) = \frac{V_2(s)}{V_1(s)} = -\frac{Z_2(s)}{Z_1(s)} \cdot \frac{1}{1 + \frac{Z_1(s) + Z_2(s)}{Z_1(s)\,A(s)}}$$

b) $G = \frac{Z_2}{Z_1} \Rightarrow H(j\omega) = -G \cdot \frac{1}{1 + \frac{1+G}{A(j\omega)}}$

$$= -G \cdot \frac{1}{1 + \frac{1+G}{A_0} \cdot \left(1 + \frac{j\omega}{\omega_0}\right)}$$

$$A_0 >> G \Rightarrow H(j\omega) \approx \frac{-G}{1 + j \cdot \frac{(1+G)\,\omega}{A_0\,\omega_0}}$$

$|H(j0)| = |G|$, $|H(j\omega)| = \frac{|G|}{\sqrt{2}}$ when $\frac{1+G}{A_0\omega_0}\omega = 1$

$$\Leftrightarrow \omega = \frac{A_0\,\omega_0}{1+G} = \omega_{3dB}$$

7) [redacted] BP: $s + \frac{\omega_I^2}{s} - p = 0 \iff s^2 - p \cdot s + \omega_I^2 = 0$

poles at $\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - \omega_I^2} = \left| p = -10, \omega_I^2 = 9 \right| = -5 \pm 4 = -9, -1$

zeros at $s = 0$ & $s = \infty$ $\left(s^2 + \frac{\omega_I^2}{s} - \infty = 0\right)$

BS: $\frac{\omega_I^2}{s + \frac{\omega_I^2}{s}} - p = 0 \iff \frac{\omega_I^2 s}{s^2 + \omega_I^2} - p = 0 \iff$

$\iff p s^2 - \omega_I^2 s + \omega_I^2 p = 0 \iff s^2 - \frac{\omega_I^2}{p} + \omega_I^2 = 0$

$\Rightarrow$ Poles at $\frac{\omega_I^2}{2p} \pm \sqrt{\left(\frac{\omega_I^2}{2p}\right)^2 - \omega_I^2}$ [redacted]

zeros at $s = \pm j \omega_I^2$ [redacted]

$\iff \frac{\omega_I^2 s}{s^2 + \omega_I^2} = \infty$

$\iff s^2 + \omega_I^2 = 0$

b) BP: $\frac{1}{sC} \rightarrow \frac{1}{\left(s + \frac{\omega_I^2}{s}\right) C} = \frac{1}{sC + \frac{\omega_I^2 C}{s}} =$

$= \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}}$ with $\frac{1}{Z_1} = sC \iff Z_1 = \frac{1}{sC}$

$\frac{1}{Z_2} = \frac{\omega_I^2 C}{s} \iff Z_2 = \frac{s}{\omega_I^2 C} = sL$

with $L = \frac{1}{\omega_I^2 C}$

Impedance of parallel connection of Z_1 & Z_2