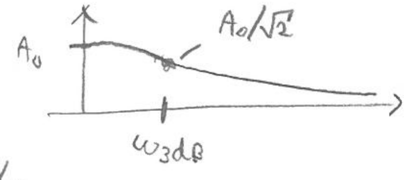
Solutions to TSEI10 Filters 170313

1a) Low sensitivity $\Rightarrow$ small variations in $H(j\omega)$ when component values deviate from nominal values

b) GB product $\approx \frac{A_0 \cdot \omega_{3dB}}{2\pi}$ [Hz]

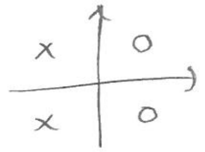


$$A(j\omega) = \frac{A_0}{1 + j\frac{\omega}{\omega_{3dB}}}$$

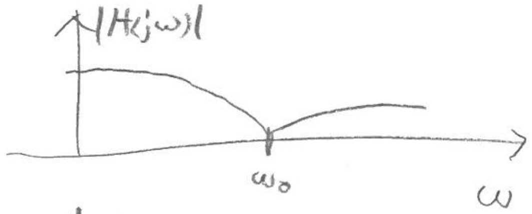
Approx. the maximum of the bandwidth [Hz] of the filter in which the OPamp is used.

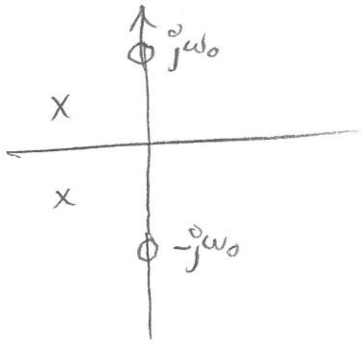
c) Chebyshev-I, 5th-order Cauer, π-net : 2 inductors
7th-order Ch-I, π-net : 3 inductors

d) Reflected poles/zeros

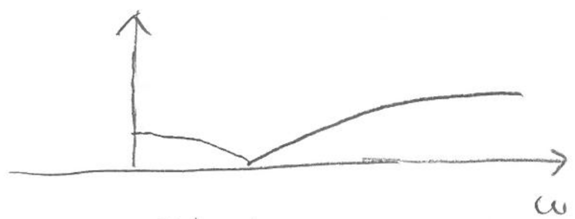


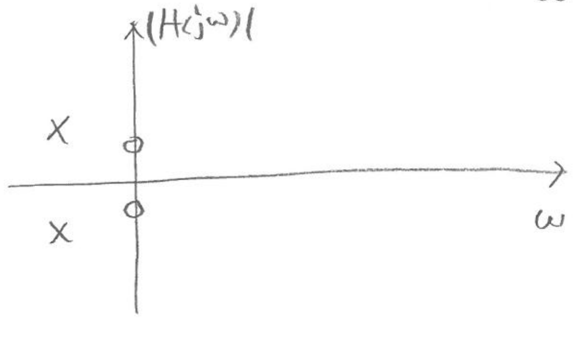
e) LP notch





HP notch





2) LP Cauer, $\frac{\omega_s}{\omega_c} = 2 \Rightarrow$ Filter order $N=5$

Normalized poles & zeros, Table C0515θ, $\frac{\omega_s}{\omega_c} \le 2$, $A_{min} \geq 30$

$\Rightarrow 30 \le \theta \le 36$

Select (e.g.) $\theta = 33 \Rightarrow$

Normalized poles: $-0{,}604683$ p_1 [rad/s]

$-0{,}133563 \pm j1.072510$ $p_{2,3}$

$-0{,}427953 \pm j0{,}733683$ $p_{4,5}$

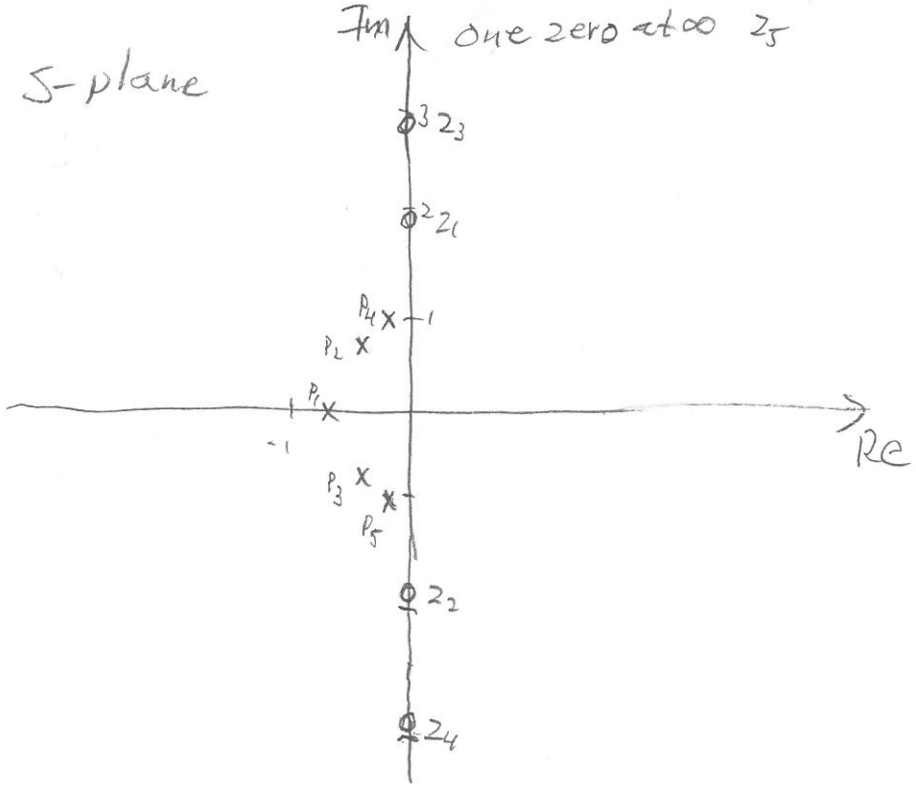
zeros: $\pm j1.915395$ $z_{1,2}$

$\pm j2.955288$ $z_{3,4}$

∞ z_5

Denormalized poles and zeros: Mult. by $\omega_c = 1$ Mrad/s

$\Rightarrow$ Same values but Mrad/s instead of rad/s



3) $\frac{Ch\text{-}I}{BP} \rightarrow LP$ spec. $\omega_{c1}\omega_{c2} = \omega_{s1}\omega_{s2} = 4\pi^2 \cdot 120 \times 10^6$ ok

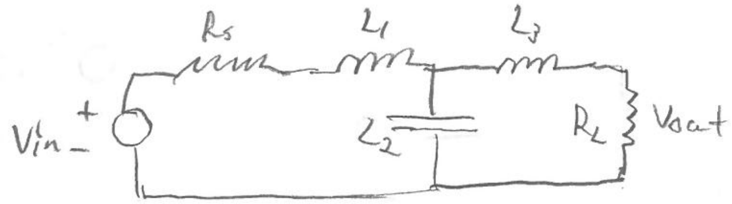
$\Omega_c = \omega_{c2} - \omega_{c1} = 2\pi \cdot 2$ krad/s

$\Omega_s = \omega_{s2} - \omega_{s1} = 2\pi \cdot 7$ krad/s

$\frac{\Omega_s}{\Omega_c} = 3{,}5 \Rightarrow$ Filter order $= 3$ $\begin{pmatrix} A_{max} = 0{,}5 \\ A_{min} = 30 \end{pmatrix}$

(Nomogram, Ch-I)

LP realization



Normalized values

$R_S = R_L = 1$

$L_1' = L_3' = 1{,}5963$ (Table $A_{max} = 0{,}5$, $r = 1$)

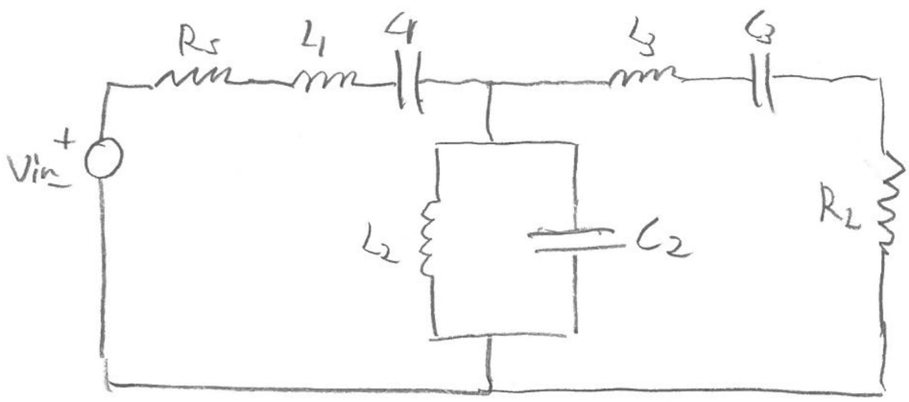
$C_2' = 1{,}0967$

Denormalize: $R_S = R_L = R = 100\ \Omega$

$L_1 = L_3 = L_1' \cdot \frac{R}{\Omega_c} = 12{,}70$ mH

$C_2 = C_2' \cdot \frac{1}{R \cdot \Omega_c} = 0{,}8727\ \mu F$

LP realiz $\rightarrow$ BP realiz.



$C_1 = C_3 = \frac{1}{\omega_I^2 \cdot L_1} = 16{,}62$ nF, $L_2 = \frac{1}{\omega_I^2 C_2} = 0{,}2419$ mH

$= \omega_{c1}\omega_{c2} = \omega_{s1}\omega_{s2} = 4\pi^2 \cdot 120 \times 10^6$

$L_1 = L_3$ and C_2 same as in LP

4a) $H(s) = H_1(s)H_2(s) = \underbrace{\dfrac{-s^2}{\underbrace{(s^2+20s+116)}_{\text{poles at } -10\pm j4}\underbrace{(s^2+8s+116)}_{\text{poles at } -4\pm j10}}}$ — 2 zeros at $s=0$

b) $H(s) = H_1(s)+H_2(s) = 12\cdot\dfrac{-s}{(s^2+20s+116)(s^2+8s+116)}$

same as $H(s)$ in a)

∴ same zeros and poles as in a)
only different gain (12)

c) $H(j0)=0$, $H(j\infty)=0$ ∴ Bandpass filter / both cases same filter except for the gain

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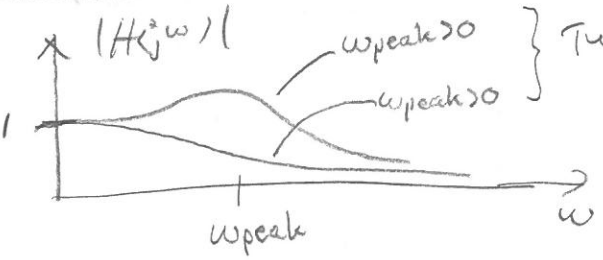
5a)



Two options, always lowpass
$|H(j\omega)| \to 0$ when $\omega\to\infty$

b) $|H(j\omega)| = \dfrac{r_p^2}{\sqrt{(r_p^2-\omega^2)^2 + a^2\omega^2}}$, $a = \dfrac{r_p}{Q}$

$$\frac{\partial |H(j\omega)|}{\partial\omega} = r_p^2 \cdot \frac{-4\omega(r_p^2-\omega^2)+2a^2\omega}{\left[(r_p^2-\omega^2)^2+a^2\omega^2\right]^{1.5}} = 0 \iff \omega = 0$$ not the solution we want

and $2(r_p^2-\omega^2) = a^2 = \dfrac{r_p^2}{Q^2}$ this is the solution we want

$$\iff \omega^2 = r_p^2 - \frac{r_p^2}{2Q^2} = r_p^2\left(1-\frac{1}{2Q^2}\right) \iff \omega = \omega_{peak} = (\pm)\, r_p\cdot\sqrt{1-\frac{1}{2Q^2}}$$

c) $Q \gg 1 \Rightarrow \omega_{peak} \approx \pm r_p \iff \omega_{peak}^2 \approx r_p^2 \Rightarrow$

$$|H(j\omega_{peak})| \approx \frac{r_p^2}{\sqrt{0 + \left(\frac{r_p}{Q}\right)^2\cdot r_p^2}} = \frac{r_p^2}{\frac{r_p^2}{Q}} = Q$$

6a) $I_{op} = 0 \Rightarrow V_+ = 0$ and $V_{out} = A(V_+ - V_-) = -AV_-$

and: $V_{in}(s) - I(s)\big(Z_1(s) + Z_2(s)\big) - V_{out}(s) = 0$ (1) from

$$I(s) = \frac{V_{in}(s) - V_-(s)}{Z_1(s)} = \frac{V_{in}(s)}{Z_1(s)} + \frac{V_{out}(s)}{Z_1(s)\,A(s)} \quad (2)$$

$$(1) \,\&\, (2) \Rightarrow V_{in}(s) - \left(\frac{V_{in}(s)}{Z_1(s)} + \frac{V_{out}(s)}{Z_1(s)A(s)}\right)\big(Z_1(s) + Z_2(s)\big) - V_{out}(s) = 0$$

$$\Longleftrightarrow V_{in}(s)\underbrace{\left(1 - \frac{\big(Z_1(s) + Z_2(s)\big)}{Z_1(s)}\right)}_{= -\frac{Z_2(s)}{Z_1(s)}} = V_{out}(s)\left(1 + \frac{Z_1(s) + Z_2(s)}{Z_1(s)\,A(s)}\right)$$

($\to 1$ when $A \to \infty$)

$$\Longleftrightarrow H(s) = \frac{V_2(s)}{V_1(s)} = -\frac{Z_2(s)}{Z_1(s)} \cdot \frac{1}{1 + \frac{Z_1(s) + Z_2(s)}{Z_1(s)\,A(s)}}$$

b) $$G = \frac{Z_2}{Z_1} \Rightarrow H(j\omega) = -G \cdot \frac{1}{1 + \frac{1+G}{A(j\omega)}}$$

$$= -G \cdot \frac{1}{1 + \frac{1+G}{A_0} \cdot \left(1 + \frac{j\omega}{\omega_0}\right)}$$

$$A_0 \gg G \Rightarrow H(j\omega) \approx \frac{-G}{1 + j \cdot \frac{(1+G)\,\omega}{A_0 \omega_0}}$$

$|H(j0)| = |G|$, $|H(j\omega)| = \frac{|G|}{\sqrt{2}}$ when $\frac{1+G}{A_0\omega_0}\omega = 1$

$$\Longleftrightarrow \omega = \frac{A_0 \omega_0}{1+G} = \omega_{3dB}$$

7a)

$$\begin{pmatrix} V_1(s) \\ I_1(s) \end{pmatrix} = \begin{pmatrix} A(s) & B(s) \\ C(s) & D(s) \end{pmatrix} \begin{pmatrix} V_2(s) \\ -I_2(s) \end{pmatrix} = \Big/ \; I_2(s) = -\frac{V_2(s)}{Z_2(s)} \; \Big/ =$$

$$= \begin{pmatrix} A(s) & B(s) \\ C(s) & D(s) \end{pmatrix} \begin{pmatrix} 1 \\ 1/Z_2(s) \end{pmatrix} V_2(s)$$

$$\Rightarrow Z_1(s) = \frac{V_1(s)}{I_1(s)} = \frac{A(s) + \frac{B(s)}{Z_2(s)}}{C(s) + \frac{D(s)}{Z_2(s)}} =: \frac{A(s)Z_2(s) + B(s)}{C(s)Z_2(s) + D(s)}$$

b) $Z_2(s) = \frac{K^{\text{const.}}}{s} \longleftrightarrow$ capacitor

Select $A = D = 0 \Rightarrow Z_1(s) = \frac{B}{C \cdot Z_2(s)} = \frac{B}{C \cdot K} \cdot s$

Select for example $K = 10^7$, $C = 1$, $B = 10^3$

$\Rightarrow \frac{B}{C \cdot K} = 10^{-4} \Rightarrow Z_1(s) = 10^{-4} \cdot s$

$= sL$ with $L = 10^{-4}$