TSE110
Analog Filters
160609

1a) Passive – can only dissipate power
⇒ Passive filters always stable

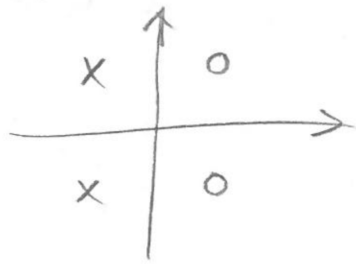
b) cascaded two-ports

c) Overdesigned (higher filter order) due to bandedge symmetries and the same attenuation in both passbands/stopbands.
BS BP

d) 11th-order Butterworth – 11 components
7th-order Cauer – 10 components

e) $z = p$ mirrored, $z = a + jb$, $p = -a + jb$

Ex second-order



2) Filter order (Nomogram $\frac{\omega_s}{\omega_c} = 4{,}7$) $N = 4$

Table C040506) $14 \leq \Theta \leq 23$ Select $\Theta = 18$ (for ex.)

Normalized poles $-0{,}3749 \pm j\,1{,}3374$

(rounded) $-0{,}9954 \pm j\,0{,}5928$

zeros $\pm j\,3{,}8206$

2 at ∞

Denormalize, mult. by $\omega_c \Rightarrow$

poles $(-4{,}711 \pm j\,16.8062) \times 10^6$

$(-12{,}5086 \pm j\,7{,}4493) \times 10^6$

zeros 48.0111×10^6

2 at ∞

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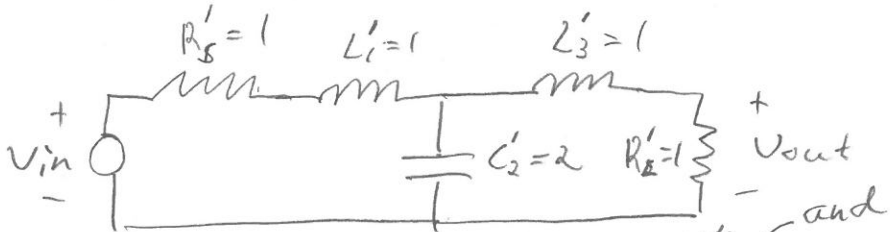
3) LP prototype $\Omega_c = \frac{\omega_I^2}{\omega_c} = \left|\text{use } \omega_I^2 = \omega_c\right| = 1$, $\Omega_s = \frac{\omega_I^2}{\omega_s} = \frac{\omega_c}{\omega_s} = 10$

Filter order (Nomogram) $N = 3$



and $R = R_L = R_S = 10\,k\Omega$

Denormalize with $\Omega_c \cdot \varepsilon^{-1/3}$, $A_{max} = 10 \cdot \log_{10}(1 + \varepsilon^2) = 0{,}2$

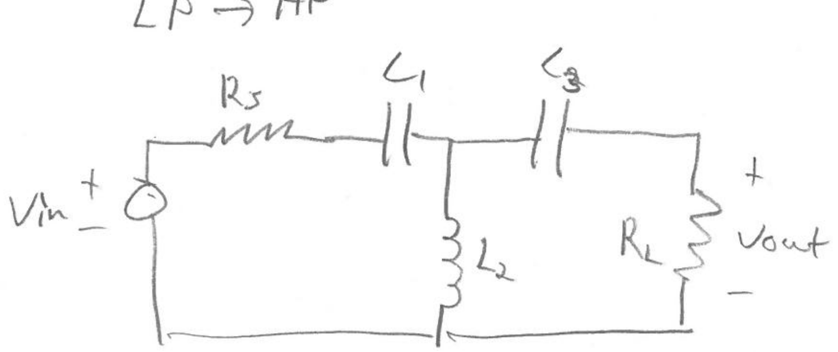
$\varepsilon = \sqrt{10^{0{,}1 A_{max}} - 1} = 0{,}2171 \Rightarrow \varepsilon^{-1/3} = 1{,}664$

$\Rightarrow L_1 = L_3 = 6.010\ kH$, $C_2 = 0{,}1202\ mF$

LP → HP

$C_1 = C_3 = \frac{1}{\omega_I^2 L_1} = 2{,}648\,pF$

$L_2 = \frac{1}{\omega_I^2 C_2} = 0{,}1324\,mH$



4a) $H_3(s) = \frac{p_1}{s+p_1} + \frac{s}{s+p_2} = \frac{p_1(s+p_2) + s(s+p_1)}{(s+p_1)(s+p_2)} = \frac{s^2+2p_1 s+p_1p_2}{(s+p_1)(s+p_2)}$

poles: $-p_1$ & $-p_2$

zeros: $s = -p_1 \pm \sqrt{p_1^2 - p_1p_2}$

b) $H_4(s) = \frac{p_1}{s+p_1} \cdot \frac{s}{s+p_2}$, poles: $-p_1$ & $-p_2$

zeros: 0 & ∞

c) $|H_3(j0)| = |1+0| = 1$, $|H_3(j\infty)| = |0+1| = 1$

$|H_4(j0)| = |1\cdot 0| = 0$, $|H_4(j\infty)| = |0\cdot 1| = 0$

5a) BP: $s + \frac{\omega_I^2}{s} - p = 0 \Leftrightarrow s^2 - p\cdot s + \omega_I^2 = 0$

poles at $\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - \omega_I^2} = \Big/ p = -10, \omega_I^2 = 9 \Big/ = -5 \pm 4 = -9, -1$

zeros at $s=0$ & $s=\infty$ $(s^2 + \frac{\omega_I^2}{s} - \infty = 0)$

BS: $\frac{\omega_I^2}{s + \frac{\omega_I^2}{s}} - p = 0 \Leftrightarrow \frac{\omega_I^2 s}{s^2+\omega_I^2} - p = 0 \Leftrightarrow$

$\Leftrightarrow ps^2 - \omega_I^2 s + \omega_I^2 p = 0 \Leftrightarrow s^2 - \frac{\omega_I^2}{p} + \omega_I^2 = 0$

$\Rightarrow$ Poles at $\frac{\omega_I^2}{2p} \pm \sqrt{\left(\frac{\omega_I^2}{2p}\right)^2 - \omega_I^2} = 0{,}45 \pm j\,2{,}966$

zeros at $s = \pm j\omega_I^2 = \pm j3$

$\Leftrightarrow \frac{\omega_I^2 s}{s^2+\omega_I^2} = \infty$

$\Leftrightarrow s^2 + \omega_I^2 = 0$

b) BP: $\frac{1}{sC} \rightarrow \frac{1}{\left(s + \frac{\omega_I^2}{s}\right)C} = \frac{1}{sC + \frac{\omega_I^2 C}{s}} =$

$= \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}}$ with $\frac{1}{Z_1} = sC \Leftrightarrow Z_1 = \frac{1}{sC}$

$\frac{1}{Z_2} = \frac{\omega_I^2 C}{s} \Leftrightarrow Z_2 = \frac{s}{\omega_I^2 C} = sL$

Impedance of parallel connection of Z_1 & Z_2

with $L = \frac{1}{\omega_I^2 C}$

6) $$\begin{pmatrix}V_1\\I_1\end{pmatrix}=\begin{pmatrix}A&B\\C&D\end{pmatrix}\begin{pmatrix}V_2\\-I_2\end{pmatrix}=\left(-I_2=\frac{V_2}{Z_2}\right)=\begin{pmatrix}A&B\\C&D\end{pmatrix}\begin{pmatrix}1\\1/Z_2\end{pmatrix}\cdot V_2$$

$$\Rightarrow Z_{in}=\frac{V_1}{I_1}=\frac{A+\frac{B}{Z_2}}{C+\frac{D}{Z_2}}$$

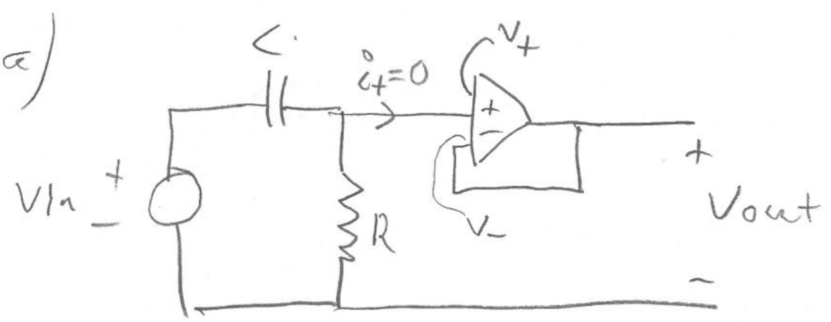
$Z_2(s)=\frac{1}{s\cdot C_1}$, Select $A=D=0 \Rightarrow$

Capacitor

$$\Rightarrow Z_{in}(s)=\frac{B\cdot C_1\cdot s}{C}\overset{\text{according to problem}}{=}10^{-4}s$$

Select for ex. $C_1=1\,nF$, $B=10^5$, $C=1$

7 a)



$$V_{out}(s) = A(s)\left(V_+(s) - V_-(s)\right) \quad (1)$$

$$V_+(s) = V_{in}(s)\cdot \frac{R}{R+\frac{1}{sC}} = \underbrace{\frac{sRC}{1+sRC}}_{H_1(s)} \cdot V_{in}(s) \quad (2)$$

volt. div.

$$V_-(s) = V_{out}(s) \quad (3)$$

$$(1)-(3) \Rightarrow V_{out}(s) = A(s)\left(H_1(s)V_{in}(s) - V_{out}(s)\right)$$

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = H_1(s)\cdot\underbrace{\frac{A(s)}{1+A(s)}}_{H_2(s)} = H_1(s)H_2(s)$$

b) $H_1(j\omega) = \dfrac{j\omega RC}{1+j\omega RC}$, $\omega_{3dB}: \omega RC = 1 \Rightarrow \omega_{3dB} = \dfrac{1}{RC} = 10\,krad/s$

max: $|H(j\infty)| = 1$

$$H_2(j\omega) = \frac{A(j\omega)}{1+A(j\omega)} = \frac{\frac{A_0}{1+j\omega/\omega_0}}{1+\frac{A_0}{1+j\omega/\omega_0}} = \frac{A_0}{A_0+1+j\omega/\omega_0}$$

$A_0 >> 1$

$$\omega_{3dB}: \frac{\omega}{\omega_0} = 1+A_0 \Rightarrow \omega_{3dB} = \omega_0(1+A_0) \approx \omega_0 A_0 = 2\pi\,M\,rad/s$$