Solutions to TSEI10 Analog Filters 160321

1a) Linear $a_1x_1(t) + a_2x_2(t) \rightarrow a_1y_1(t) + a_2y_2(t)$ if $x(t) \rightarrow y(t)$

Time-invariant $x(t-t_0) \rightarrow y(t-t_0)$

b) LP: 2 zeros at $s=\infty$, BP: one zero at $s=0$

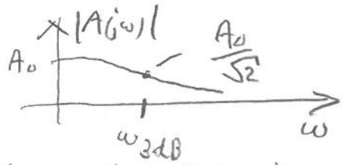
one zero at $s=\infty$

HP: 2 zeros at $s=0$,

c) overdesigned (higher filter order) due to band edge symmetries and the same attenuation in both passbands/stopbands
BS / BP

d) Low sensitivity $\Rightarrow$ small variations in $H(j\omega)$ when component values deviates from nominal values

e) $\omega_t = A_0 \cdot \omega_{3dB}$



$$A(j\omega) = \frac{A_0}{1 + \frac{j\omega}{\omega_{3dB}}}$$

Approx. the max bandwidth of the filter in which the OPamp is used

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2) Filter order (nomogram $\frac{\omega_s}{\omega_c} = 2$) $N = 5$

Table C0515θ, $30 \leq \theta \leq 36$ Select $\theta = 33$ (for ex)

Normalized poles: $-0{,}6046827$

$-0.1335634 \pm j \times 1.0725098$

$-0{,}4279530 \pm j \times 0.7336827$

Normalized zeros $\pm j \times 1.9153952$

$\pm j \times 2.9552877$

one at ∞

Denormalize, mult by ω_c $\Rightarrow$

poles: -3.7993334×10^6

$(-0{,}8392036 \pm j \times 6.7387778) \times 10^6$

$(-2{,}6889080 \pm j \times 4.6098644) \times 10^6$

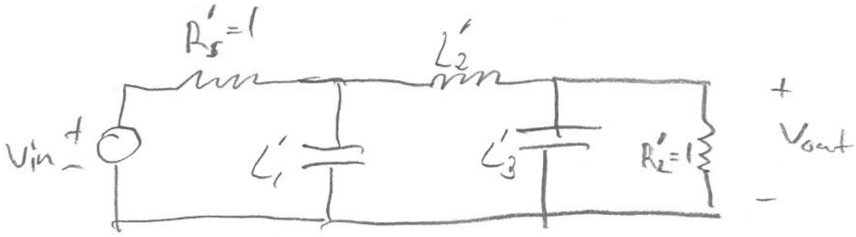
zeros $\pm j \times 1.2034783 \times 10^7$

$\pm j \times 1.8568620 \times 10^7$

one at ∞

3) LP prototype $\Omega_c = \frac{\omega_I^2}{\omega_c}$ $\{$ use $\omega_I^2 = \omega_c$ $\}$ $= 1$, $\Omega_s = \frac{\omega_I^2}{\omega_s} = \frac{\omega_c}{\omega_s} = 5$

$\Rightarrow$ Filter order $N = 3$



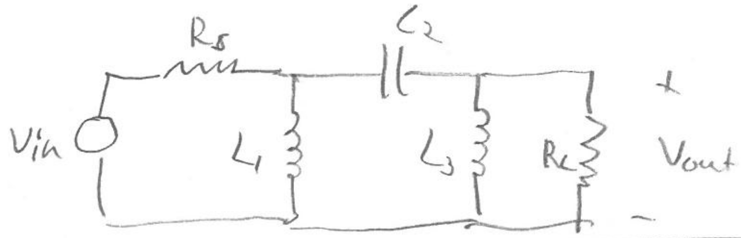
$C_1' = C_3' = 1{,}5963$

$L_2' = 1.0967$

Denormalize with $\Omega_c = 1$, $R = R_S = R_L = 2\,k\Omega$ $\Rightarrow$

$C_1 = C_3 = \frac{C_1'}{\Omega_c \cdot R} = 0{,}79815\ mF$, $L_2 = L_2' \cdot \frac{R}{\Omega_c} = 2{,}1934\ kH$

LP → HP



$L_1 = L_3 = \frac{1}{\omega_I^2 \cdot C_1} = \frac{1}{\omega_c \cdot C_1} = 39.881\,\mu H$

$C_2 = \frac{1}{\omega_c \cdot L_2} = 14{,}512\ pF$

4) a) BP: $a = c = 0$, $b \neq 0$, BS: $a \neq 0$, $c \neq 0$, $b = 0$

b) $H(j\omega) = \frac{e}{(j\omega)^2 + d \cdot j\omega + e}$, $|H(j\omega)| = \frac{|e|}{\sqrt{(e-\omega^2)^2 + d^2\omega^2}}$

$\frac{\partial |H(j\omega)|}{\partial \omega} = |e| \cdot \frac{2(e-\omega^2)\cdot -2\omega + 2d^2\omega}{\left((e-\omega^2)^2 + d^2\omega^2\right)^{1,5}} = 0$ when $\omega = 0$ not the sol. we want here

and when $-4(e-\omega^2) + 2d^2 = 0 \Leftrightarrow \omega^2 = e - \frac{d^2}{2} \Leftrightarrow \omega = \sqrt{e - \frac{d^2}{2}}$

(@ $\omega = -\sqrt{e - \frac{d^2}{2}}$, $|H(j\omega)| = |H(-j\omega)|$)

$\therefore \omega_{peak} = \sqrt{e - \frac{d^2}{2}}$

with $e = 10$, $d = 1$: $\omega_{peak} = \sqrt{10 - \frac{1}{2}} \approx 3.082207$

$\Rightarrow |H(j\omega_{peak})| \approx 3.2025$

(Note: with $e = r_p^2$, $d = -2\sigma_p = \frac{r_p}{Q}$: $\omega_{peak} = r_p\sqrt{1 - \frac{1}{2Q^2}}$)

5) a) BP: $s + \frac{\omega_I^2}{s} - P = 0 \Leftrightarrow s^2 - Ps + \omega_I^2 = 0 \Rightarrow$

poles at $\frac{P}{2} \pm \sqrt{\frac{P^2}{4} - \omega_I^2} = \left/ P = -4,\ \omega_I^2 = 4\pi^2 \right/ = -2 \pm j\sqrt{4\pi^2 - 4}$

$\approx -2 \pm j\,5.9564$

zeros at $s = 0$ & $s = \infty$ $\left(s + \frac{\omega_I^2}{s} - \infty = 0 \right)$

BS: $\frac{\omega_I^2}{s + \frac{\omega_I^2}{s}} - P = 0 \Leftrightarrow \frac{\omega_I^2 \cdot s}{s^2 + \omega_I^2} - P = 0$

$\Leftrightarrow P \cdot s^2 - \omega_I^2 s + \omega_I^2 \cdot P = 0 \Leftrightarrow s^2 - \frac{\omega_I^2}{P} s + \omega_I^2 = 0$

$\Rightarrow$ poles at $\frac{\omega_I^2}{2P} \pm \sqrt{\left(\frac{\omega_I^2}{2P}\right)^2 - \omega_I^2} \approx 4.9348 \pm j\,3.8892$

zeros at $s = \pm j\omega_I$ $\left(s^2 + \omega_I^2 = 0 \right) = \pm j\,2\pi \approx \pm j\,6.2832$

b) $\underset{LP}{\frac{1}{sC}} \rightarrow \underset{BS}{\frac{1}{\frac{\omega_I^2 \cdot C}{s + \frac{\omega_I^2}{s}}}} = \frac{s + \frac{\omega_I^2}{s}}{\omega_I^2 \cdot C} = \frac{s}{\omega_I^2 C} + \frac{1}{sC}$

$= sL + \frac{1}{sC}$, $L = \frac{1}{\omega_I^2 \cdot C}$

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6) a) $\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$ and $V_2 = -I_2 \cdot Z_2 \Leftrightarrow -I_2 = V_2 / Z_2$

$\Rightarrow Z_{in}(s) = \frac{V_1(s)}{I_1(s)} = \frac{A(s) V_2(s) + B(s) V_2(s)/Z_2(s)}{C(s) V_2(s) + D(s) V_2(s)/Z_2(s)} = \frac{A(s) + B(s)/Z_2(s)}{C(s) + D(s)/Z_2(s)}$

$\left(\overset{Alt.}{=} \frac{A(s) Z_2(s) + B(s)}{C(s) Z_2(s) + D(s)} \right)$

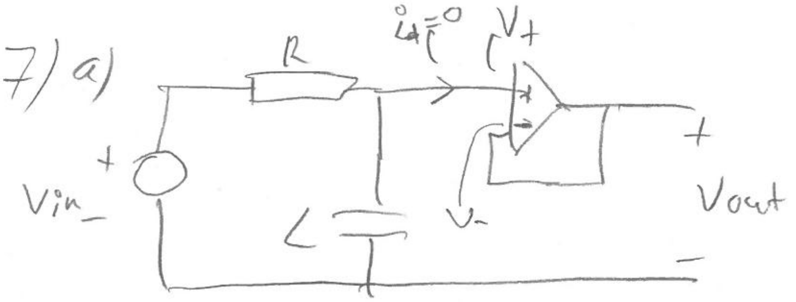
b) $Z_2(s) = \frac{10^8}{s}$, A, B, C, D const. $\Rightarrow \frac{V_1(s)}{I_1(s)} = \frac{A + B \cdot s \cdot 10^{-8}}{C + D \cdot s \cdot 10^{-8}}$

= Desired: $10^{-4} s$ Select $A = D = 0 \Rightarrow$

$\frac{V_1(s)}{I_1(s)} = \frac{B}{C} \cdot s \cdot 10^{-8}$ select $\frac{B}{C} = 10^4 \Rightarrow \frac{V_1(s)}{I_1(s)} = 10^{-4} s$

For ex. $B = 10^4$
$C = 1$

7) a)



$$V_{out}(s) = A(s)\left(V_+(s) - V_-(s)\right) \qquad (1)$$

$$V_+(s) = V_{in}(s)\cdot\frac{1/sC}{R+1/sC} = V_{in}(s)\cdot\underbrace{\frac{1}{1+sRC}}_{H_1(s)} \qquad (2)$$

↑ volt. div. since $i_+=0$, i.e., $i_R = i_C$

$$V_-(s) = V_{out}(s) \qquad (3)$$

$$(1)-(3) \Rightarrow V_{out}(s) = A(s)\left(V_{in}(s)\cdot H_1(s) - V_{out}(s)\right)$$

$$\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{H_1(s)\cdot A(s)}{1+A(s)} = H_1(s)\,H_2(s)$$

$H_2(s)$

b) $H_1(j\omega) = \dfrac{1}{1+j\omega RC}$, ω_{3dB}: $\omega RC = 1 \Rightarrow \omega_{3dB} = \dfrac{1}{RC} = 1000$ rad/s

$$H_2(j\omega) = \frac{A(j\omega)}{1+A(j\omega)} = \frac{\dfrac{A_0}{1+j\omega/\omega_0}}{1+\dfrac{A_0}{1+j\omega/\omega_0}} = \frac{A_0}{1+A_0+j\omega/\omega_0}$$

ω_{3dB}: $\dfrac{\omega}{\omega_0} = 1+A_0 \Rightarrow \omega_{3dB} = \omega_0(1+A_0) \approx \omega_0 A_0 = 2\pi\cdot 10^6$ rad/s

$H(j\omega)$: ω_{3dB} approx the same as for $H_1(j\omega)$ since the bandwidth of $H_2(j\omega)$ much larger