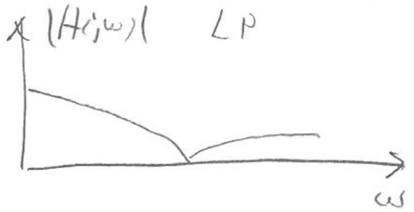
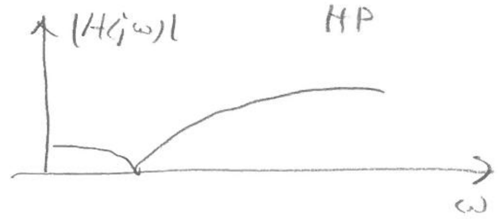
Exam in Analog Filters TSEI 10 2015-03-19 — SOLUTIONS

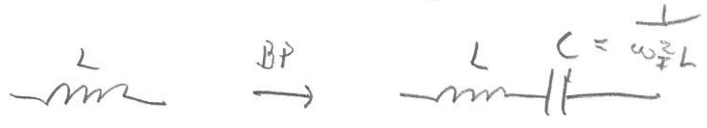
1a) Linear phase $\phi(\omega) = -k \cdot \omega$ All freq. components delayed by the same amount

b)





c)



d) $\omega_t = A_0 \cdot \omega_{3dB}$ $\quad A(s) = \dfrac{A_0}{1 + \dfrac{s}{\omega_{3dB}}}$

e) Very sensitive to component variations

2) Order $N = 5$

Nomalized poles $-0,3623196$

$-0,111963 \pm j\,1.0115574$

$-0,2931227 \pm j\,0.625177$

zeros 5 at $s = \infty$

Denormalized poles (mult. by ω_c)

$(-0,4553043) \times 10^5$

$(-0,1406968 \pm j\,1,271160) \times 10^5$

$(-0,3683489 \pm j\,0,7856204) \times 10^5$

zeros 5 at $s = \infty$

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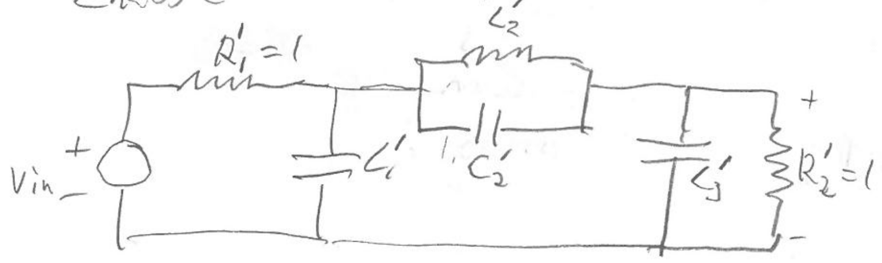
3) Order $N=3$, Use $A_{max} = 0,09833$ & $r^2 = 1$ table

($\Omega_c = \frac{\omega_I^2}{\omega_c}$ / set $\omega_I^2 = \omega_c$ / $= 1$ $\quad \Omega_s = \frac{\omega_I^2}{\omega_s} = \frac{\omega_c}{\omega_s} = 5 \Rightarrow N=3$

$N = 3$, $A_{max} = 0,09833$, $A_{min} \geq 44$, $\frac{\Omega_s}{\Omega_c} \leq 5 \Rightarrow 12 \leq \Theta \leq 14$

Choose $\Theta = 13 \Rightarrow$



$C_1' = C_3' = 1.002943$

$C_2' = 0,034496$

$L_2' = 1.107291$

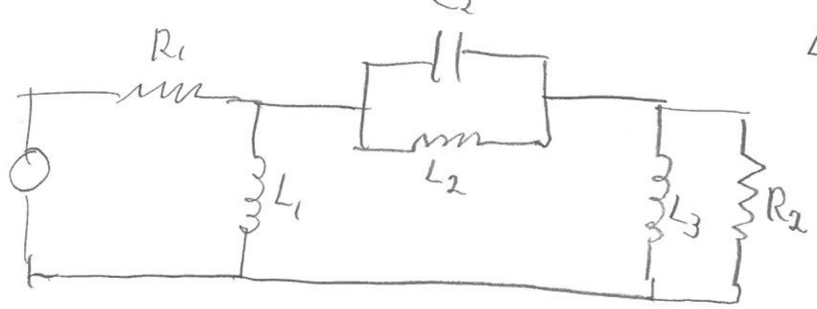
Denormalize ($\omega_c = 1$): $C = \frac{C'}{R_1}$, $L = L' \cdot R_1$

LP $\Rightarrow$ HP $\quad R_1 = R_2 = 50\,\Omega$

$L_1 = L_3 = \frac{R_1}{\omega_c \cdot C_1'} = 0,158688\,mH$

$L_2 = 4,61372\,mH$

$C_2 = \frac{1}{R_1 \cdot \omega_c \cdot L_2'} = 57.4934\,nF$



4a) BP: $s + \frac{\omega_I^2}{s} - s_{LP} = 0 \iff s^2 - s_{LP} \cdot s + \omega_I^2 = 0$

$\Rightarrow$ poles at $\frac{s_{LP}}{2} \pm \sqrt{\frac{s_{LP}^2}{4} - \omega_I^2} = -1 \pm \sqrt{1-4\pi^2}$

$= -1 \pm j\sqrt{4\pi^2-1} \approx -1 \pm j \cdot 6{,}203097$

zeros at 0 and ∞ $\left(s + \frac{\omega_I^2}{s} - \infty = 0\right)$

BS: $\frac{\omega_I^2}{s + \frac{\omega_I^2}{s}} - s_{LP} = 0 \iff \frac{\omega_I^2 \cdot s}{s^2 + \omega_I^2} - s_{LP} = 0$

$\iff s_{LP} \cdot s^2 - \omega_I^2 \cdot s + \omega_I^2 \cdot s_{LP} = 0$

$\iff s^2 - \frac{\omega_I^2}{s_{LP}} s + \omega_I^2 = 0$

$\Rightarrow$ Poles at $\frac{\omega_I^2}{2 \cdot s_{LP}} \pm \sqrt{\left(\frac{\omega_I^2}{2 \cdot s_{LP}}\right)^2 - \omega_I^2} = -\pi^2 \pm \sqrt{\pi^4 - 4\pi^2}$

$= -\pi^2 \pm \pi \cdot \sqrt{\pi^2 - 4} \approx -2{,}258384$ and $-17{,}4808$

zeros at $\pm j\omega_I = \pm j\, 2\pi \approx \pm j\, 6.283185$ $(s^2 + \omega_I^2 = 0)$

b) $H_{BS}(s) = \frac{G}{\frac{\omega_I^2 \cdot s}{s^2 + \omega_I^2} - s_{LP}} \Rightarrow H_{BS}(0) = \frac{G}{-s_{LP}} = 1$ if $G = -s_{LP} = 2$

5a) LP: $A = B = 0$, $C \neq 0$; HP $B = C = 0$, $A \neq 0$

BP: $A = C = 0$, $B \neq 0$

b) $2(j\omega_0)^2 + C = 0 \iff -2\omega_0^2 + C = 0 \Rightarrow C = 2 \cdot (2\pi \cdot 10)^2 = 800\pi^2$

$B = 0$

c) poles: $-\frac{D}{2} \pm \sqrt{\left(\frac{D}{2}\right)^2 - E}$ complex if $E > \left(\frac{D}{2}\right)^2$

6a) $V_+ = V_- = V_{out} = V_{in} \cdot \dfrac{R}{R + \frac{1}{sC}} \Rightarrow$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{RC \cdot s}{1 + RCs} = \frac{s}{s + \frac{1}{RC}}$$

HP filter

b) $V_{out}(s) = A(s) \cdot [V_+(s) - \underbrace{V_-(s)}_{= V_{out}(s)}] = A(s)V_+(s) - A(s)V_{out}(s)$

$$\Rightarrow V_{out}(s) = \frac{A(s)}{1 + A(s)} \cdot \underbrace{V_+(s)}_{= V_{in}(s) \cdot \frac{s}{s + \frac{1}{RC}}} \Rightarrow$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \underbrace{\frac{s}{s + \frac{1}{RC}}}_{HP} \cdot \underbrace{\frac{A(s)}{1 + A(s)}}_{LP} \Rightarrow \text{BP filter}$$

$$\frac{A(s)}{1 + A(s)} = \frac{\frac{A_0}{1 + \frac{s}{\omega_{3dB}}}}{1 + \frac{A_0}{1 + \frac{s}{\omega_{3dB}}}} = \frac{A_0}{1 + A_0 + \frac{s}{\omega_{3dB}}}$$

$$\left(= \frac{A_0}{1 + A_0} \approx 1 \text{ for } \omega = 0 \right.$$

$\rightarrow 0$ when $\omega \rightarrow \infty$

7/ $\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} = \left/ V_2 = -I_2 Z_2 \right/ = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ \frac{V_2}{Z_2} \end{pmatrix}$

$$\Rightarrow Z_{in}(s) = \frac{V_1(s)}{I_1(s)} = \frac{\left(A(s) + \frac{B(s)}{Z_2(s)} \right) \cancel{V_2(s)}}{\left(C(s) + \frac{D(s)}{Z_2} \right) \cancel{V_2(s)}} = \frac{2 - s}{2 + s}$$

Allpass filter, $|Z_{in}(j\omega)| = 1$ for all ω