LINKÖPING UNIVERSITY

# Exam in TSEI10 Filters

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| **Exam code:** | TEN1 |
| **Date:** | 2017-03-13 **Time:** 14–18 |
| **Place:** | KÅRA |
| **Examiner:** | Håkan Johansson |
| **Department:** | ISY |
| **Allowed aids:** | Pocket calculator<br>Wanhammar: Tables and formulas for analog and digital filters<br>Söderkvist: Formler och Tabeller<br>Ingelstam, Rönngren, Sjöberg: Tefyma<br>Ekbom: Tabeller och Formler NT<br>Nordling: Physics Handbook for Science and Engineering<br>Strid: Formler och Lexikon<br>Mathematical tables |
| **Number of tasks:** | 7 |
| **Grading:** | Maximum 70 points, 30 points required to pass the exam.<br>**Note** that a **motivation/solution** is required to get the maximal number of points for a problem.<br>**Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively. |
| **Solutions:** | Will be published no later than three days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE14/ |
| **Result:** | Available 2017-03-27 |

**1** **a.** Explain why it is important to use low-sensitivity filter structures. (2 p)

**b.** Explain what the gain-bandwidth product of an OP amplifier is, and why it is desired to have a large gain-bandwidth product. (2 p)

**c.** Which of the following two filters requires most inductors in a $\pi$-type lowpass ladder structure: a 7th-order Chebyshev-I or a 5th-order Cauer? (2 p)

**d.** What is the relation between the poles and zeros in an allpass filter? (2 p)

**e.** Sketch typical magnitude responses, and pole/zero configurations, of second-order lowpass and highpass notch sections. (2 p)

**2** Synthesize a Cauer filter that meets the following specification: $\omega_c = 1$ Mrad/s, $\omega_s = 2$ Mrad/s, $A_{\max} = 0.1$ dB ($\rho = 15\%$), and $A_{\min} = 50$ dB. Determine the poles and zeros, and indicate their locations in the $s$-plane. The filter order should not be higher than necessary. (10 p)

**3** Realize a Chebyshev-I filter that meets the following specification: $\omega_{c1} = 2\pi \times 10$ krad/s, $\omega_{c2} = 2\pi \times 12$ krad/s, $\omega_{s1} = 2\pi \times 8$ krad/s, $\omega_{s2} = 2\pi \times 15$ krad/s, $A_{\max} = 0.5$ dB, and $A_{\min} = 30$ dB. Use a T-type ladder structure. Determine the capacitor and inductor values assuming source and load resistors with the same value according to $R_S = R_L = 100\ \Omega$. (10 p)

**4** Assume two (normalized) filters with transfer functions as:

$$H_1(s) = \frac{s}{s^2 + 20s + 116}, \quad H_2(s) = \frac{-s}{s^2 + 8s + 116}$$

**a.** Compute the poles and zeros of the overall filter $H(s)$ when $H_1(s)$ and $H_2(s)$ are cascade connected, i.e., $H(s) = H_1(s)H_2(s)$. (4 p)

**b.** Compute the poles and zeros of the overall filter $H(s)$ when $H_1(s)$ and $H_2(s)$ are parallel connected, i.e., $H(s) = H_1(s) + H_2(s)$. (4 p)

**c.** For each of the two cases above, determine $H(j\omega)$ for $\omega = 0$ and $\omega = \infty$ and state the type of filter (lowpass, highpass, bandpass, or bandstop). (2 p)

**5** The transfer function of a second-order filter section is given as

$$H(s) = \frac{r_p^2}{s^2 + \frac{r_p}{Q}s + r_p^2}$$

**a.** Is this a lowpass, highpass, bandpass, or bandstop filter section? Motivate the answer by sketching the principle behavior of the frequency response. (2 p)

**b.** Express in terms of $r_p$ and $Q$, the angular frequency $\omega > 0$ (say $\omega_{\text{peak}}$) for which the magnitude response $|H(j\omega)|$ has its maximum. It is assumed here that $Q > 1/\sqrt{2}$. (6 p)

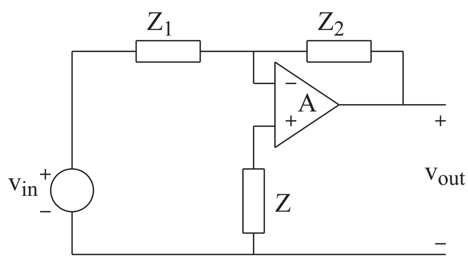
**c.** Show that $|H(j\omega_{\text{peak}})| \approx Q$ when $Q >> 1$. (2 p)

**6** **a.** Derive the transfer function $H(s) \triangleq V_{\text{out}}(s)/V_{\text{in}}(s)$ for the active circuit below, expressed in terms of $Z_1(s)$, $Z_2(s)$ and $A(s)$, where $A(s)$ is the transfer characteristics of the OP amplifier. The OP amplifier is thus non-ideal, but its input impedance can be assumed to be infinite in the derivations. (5 p)

**b.** Assume that $Z_2(s)/Z_1(s) = G$, with $Z_1(s)$ and $Z_2(s)$ being real-valued constants, and thus $G$ being a real-valued constant. This means that the circuit is an inverting amplifier. However, due to the non-ideal OP amplifier, $H(s)$ has a lowpass filter frequency response. Determine the 3dB angular frequency for $H(s)$, expressed in terms of $G$, $A_0$, and $\omega_0$ (see the OP-amp model below). Use reasonable approximations if necessary. For a lowpass filter $H(s)$, the 3dB angular frequency, say $\omega_{3dB}$, is defined by $|H(j\omega_{3dB})| = |H(j0)|/\sqrt{2}$. Assume that

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

and $A_0 >> G$. (5 p)



**7** **a.** For the terminated two-port below, determine the input impedance $Z_1(s) \triangleq V_1(s)/I_1(s)$, expressed in terms of $Z_2(s)$ and the two-port's chain matrix entries $A(s)$, $B(s)$, $C(s)$, and $D(s)$. (5 p)

**b.** Assume that it is desired to realize an inductor according to $Z_1(s) = 10^{-4}s$. Select $Z_2(s)$ and real values of $A$, $B$, $C$, and $D$ so that the desired $Z_1(s)$ is realized. The impedance $Z_2(s)$ should correspond to a capacitor. The solution is not unique, suggest one. (5 p)