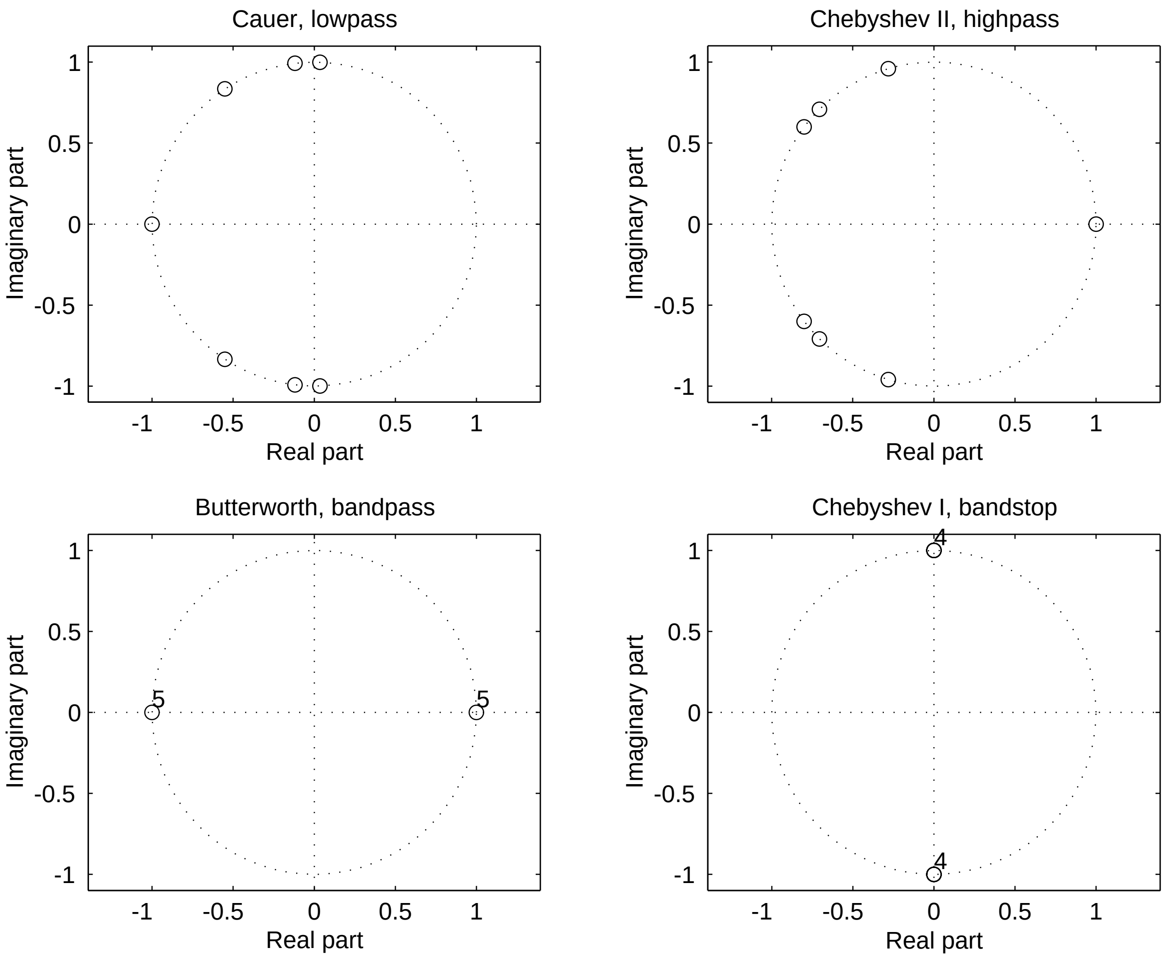
TSTE06, Digitala Filter 2013-05-29, Solutions

1 a)



b) Linear phase Type I FIR filter, can not be used as highpass filter, |H(1)| = |H(–1)|

c) FIR: + linear-phase, nonrecursive structures

– high order, long delays

IIR: + low order

– nonlinear-phase, recursive structures

2) Requirements for analog LP-filter:

$$\omega_{ac} = \frac{2}{T}\tan\left(\frac{\omega_c T}{2}\right) = \frac{2}{T}\tan\left(\frac{0{,}1786\pi}{2}\right) \approx \frac{2}{T}0{,}288095$$

$$\omega_{as} = \frac{2}{T}\tan\left(\frac{\omega_s T}{2}\right) = \frac{2}{T}\tan\left(\frac{0{,}7143\pi}{2}\right) \approx \frac{2}{T}2{,}076521$$

Filter order: nomogram => $N = 4$

Normalized poles:

$S'_{2,3} = -0{,}1753531 \pm j1{,}016253$

$S'_{3,4} = -0{,}4233398 \pm j0{,}4209457$

Zeros: 4 zeros i at S = inf

Denormalize with ω_{ac} =>

Poles:

$s_{2,3} = -0{,}0505183 \pm j0{,}2927774$

$s_{3,4} = -0{,}1219621 \pm j0{,}1212724$

Zeros: 4 zeros at s = inf

Transform into digital filter

$$z = \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}}$$

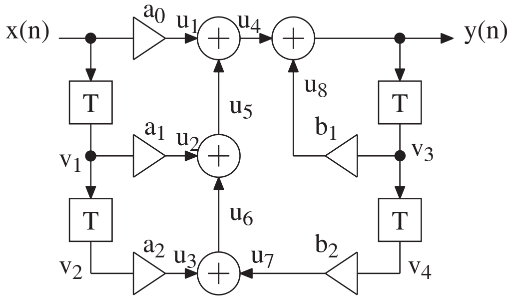
Poles:

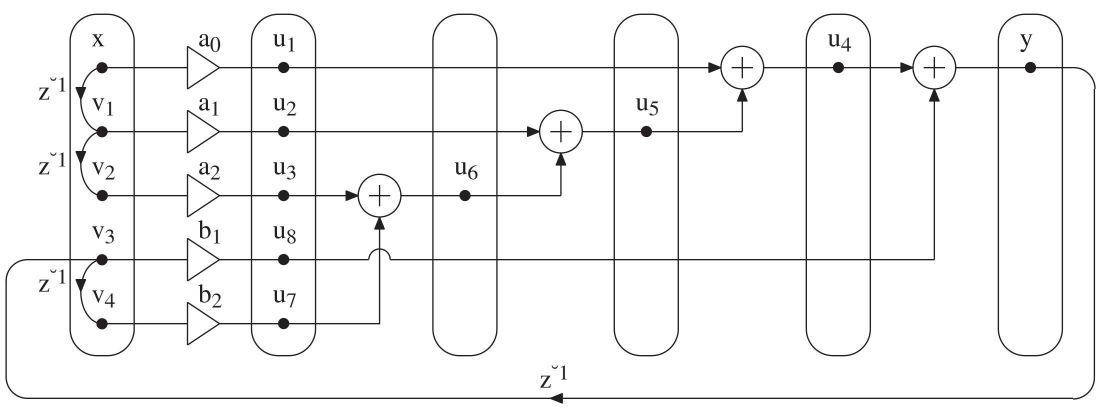
$z_{2,3} = 0{,}7666052 \pm j0{,}4923494$

$z_{3,4} = 0{,}7620053 \pm j0{,}1904544$

Zeros: 4 zeros at z = –1

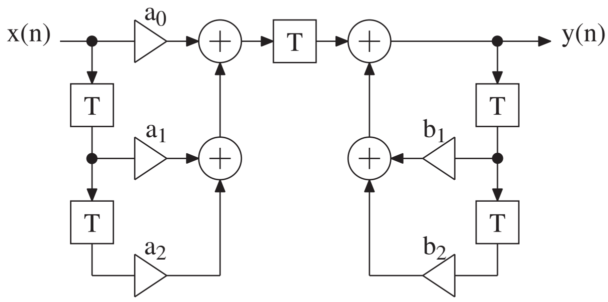
3 a)





b) $T_{CP} = T_{mult}+4T_{add} = 0.22$ μs, $T_{min} = 0.5[T_{mult}+4T_{add}] = 0.11$ μs

c)



4) Requirements for the analog reference filter:

$$\omega_{arc} = \frac{2}{T}\tan\left(\frac{\omega_c T}{2}\right) = \frac{2}{T}\tan\left(\frac{0,85\pi}{2}\right) \approx \frac{2}{T}4,165300$$

$$\omega_{ars} = \frac{2}{T}\tan\left(\frac{\omega_s T}{2}\right) = \frac{2}{T}\tan\left(\frac{0,52\pi}{2}\right) \approx \frac{2}{T}1,064882$$

LP specification

$$\Omega_c = \frac{\omega_I^2}{\omega_{arc}}, \quad \Omega_s = \frac{\omega_I^2}{\omega_{ars}}$$

Select $\Omega_c = 1 \Rightarrow \omega_I^2 = \omega_{arc}, \quad \Omega_s = \frac{\omega_{arc}}{\omega_{ars}}$

Requirements on ω_{ars} and A_{min} => $15 \leq \Theta \leq 15$

Normalized element values

$C'_1 = C'_3 = 0.9944$, $C'_2 = 0.0463$, $L'_2 = 1.0941$

Denormalize with $R = 1$, $\omega_0 = \Omega_c = 1$ =>

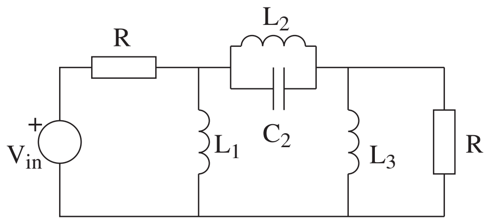
$C_1 = C_3 = 0.9944$, $C_2 = 0.0463$, $L_2 = 1.0941$

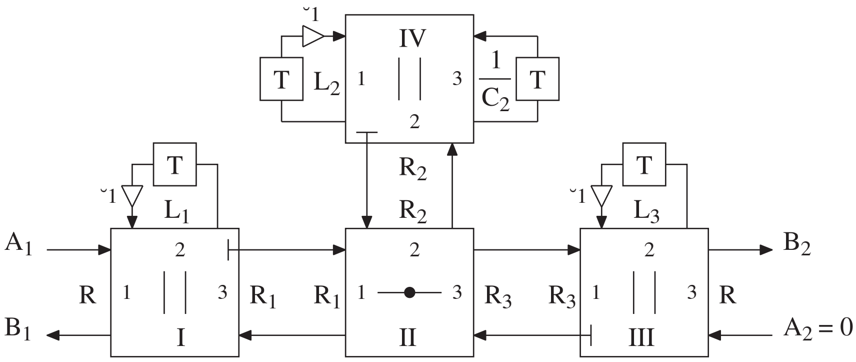
LP to HP transformation

$$C \rightarrow \frac{1}{\omega_I^2 L}, \quad L \rightarrow \frac{1}{\omega_I^2 C}$$

$L_1 = L_3 = 0.2414308$, $L_2 = 5.185286$, $C_2 = 0.2194304$

Reference highpass filter and corresponding wave flow graph





Adaptor coefficients

I: $R_1 = (1/L_1 + 1/R)^{-1} = 0{,}194478$

$$\alpha_1 = \frac{2/R}{1/R + 1/L_1 + 1/R_1} = 0{,}194478$$

$$\alpha_2 = 1 - \alpha_1 = 0{,}805522$$

$$\alpha_3 = 1$$

III: $R_3 = (1/L_3 + 1/R)^{-1} = 0{,}194478$

$$\alpha_1 = 1$$

$$\alpha_2 = \frac{2/L_3}{1/R + 1/L_3 + 1/R_3} = 0{,}805522$$

$$\alpha_3 = 1 - \alpha_2 = 0{,}194478$$

IV: $R_2 = (1/L_2 + C_2)^{-1} = 2{,}42551$

$$\alpha_1 = \frac{2/L_2}{1/R_2 + 1/L_2 + C_2} = 0{,}467769$$

$$\alpha_2 = 1$$

$$\alpha_3 = 1 - \alpha_1 = 0{,}532231$$

II:

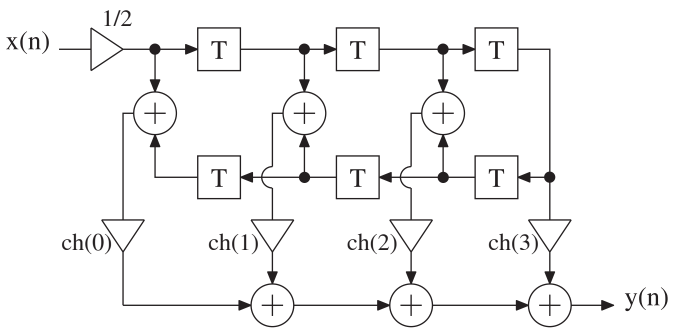
$$\alpha_1 = \frac{2R_1}{R_1 + R_2 + R_3} = 0{,}138199$$

$$\alpha_2 = \frac{2R_2}{R_1 + R_2 + R_3} = 1{,}723603$$

$$\alpha_3 = 1 - \alpha_1 - \alpha_2 = 0{,}138199$$

5 a) The filter is scaled according to the figure below where

$$c = \frac{1}{\sum_{n=0}^{6} \frac{1}{2}|h(n)|} = \frac{2}{0,51} \approx 3,92$$



b) SNR at the output before scaling:

$$\mathrm{SNR}_1 = 10\log_{10}\left(\frac{\sigma_x^2 \sum_{n=0}^{6} h^2(n)}{4\sigma_e^2}\right)$$

SNR at the output after scaling:

$$\mathrm{SNR}_2 = 10\log_{10}\left(\frac{\sigma_x^2 \sum_{n=0}^{6} \left(\frac{c}{2}\right)^2 h^2(n)}{4\sigma_e^2}\right)$$

Difference:

$$\mathrm{SNR}_2 - \mathrm{SNR}_1 = 10\log_{10}\left(\left(\frac{c}{2}\right)^2\right) = 20\log_{10}(3,92/2) \approx 5,84 \text{ dB}$$

6 a) Passband ripples: $\delta_c^{(F)} + \delta_c^{(G)} \le \delta_c^{(H)} = 0,01$ (second-order effects ignored)

Stopband ripples: $\delta_s^{(F)}, \delta_s^{(G)} \le \delta_s^{(H)} = 0,0005$ (second-order effects ignored)

Passband edges:
$$\omega_c^{(F)} T = \omega_c^{(H)} T = 0,925\pi$$
$$\omega_c^{(G)} T = M(\pi - \omega_c^{(H)} T) = 0.4\pi$$

Stopband edges: $\omega_s^{(F)}T = \dfrac{4\pi + \omega_s^{(G)}T}{M} = \dfrac{4\pi + 0.6\pi}{6} \approx 0.767\pi$

$$\omega_s^{(G)}T = M(\pi - \omega_s^{(H)}T) = 0.6\pi$$

b) H: N = 229.38, choose N = 230 =>116 mult.
G: N = 40.91, choose N = 42 => 22 mult., F: N = 38.76, choose N = 40 => 21 mult., in total 22 + 21 = 43 mult

c) $\omega_s^{(G)}T < \pi$ => $M < 10$. Thus the maximum value of M is eight as it must also be an even number when G(z) is a lowpass filter.

7 a) Since $A_0(z)$ and $A_1(z)$ are allpass filters, their frequency responses can be written as $A_0(e^{j\omega T}) = e^{j\Phi_0(\omega T)}$ and $A_1(e^{j\omega T}) = e^{j\Phi_1(\omega T)}$ where $\Phi_0(\omega T)$ and $\Phi_1(\omega T)$ denote the phase responses. This gives us

$$\begin{aligned} H(e^{j\omega T}) &= 0,5[A_0(e^{j\omega T}) + A_1(e^{j\omega T})] \\ &= 0,5[e^{j\Phi_0(\omega T)} + e^{j\Phi_1(\omega T)}] \\ &= e^{j0,5[\Phi_0(\omega T) + \Phi_1(\omega T)]}\cos\left(\frac{\Phi_0(\omega T) - \Phi_1(\omega T)}{2}\right) \end{aligned}$$

$$\begin{aligned} H_c(e^{j\omega T}) &= 0,5[A_0(e^{j\omega T}) - A_1(e^{j\omega T})] \\ &= 0,5[e^{j\Phi_0(\omega T)} - e^{j\Phi_1(\omega T)}] \\ &= je^{j0,5[\Phi_0(\omega T) + \Phi_1(\omega T)]}\sin\left(\frac{\Phi_0(\omega T) - \Phi_1(\omega T)}{2}\right) \end{aligned}$$

Hence,

$$\left|H(e^{j\omega T})\right|^2 + \left|H_c(e^{j\omega T})\right|^2 = \cos^2\left(\frac{\Phi_0(\omega T) - \Phi_1(\omega T)}{2}\right) + \sin^2\left(\frac{\Phi_0(\omega T) - \Phi_1(\omega T)}{2}\right) = 1$$

b) Due to the power complementary property we have

$$(1 - \delta_c)^2 + \delta_s^2 = 1 \Leftrightarrow \delta_c \approx 0,5\delta_s^2$$

This results in a required filter order of six, but since the order of lowpass and highpass lattice WDFs must be odd, the minimum order is seven.

c) $H(-z^2)$ corresponds to a bandpass filter; $H_c(-z^2)$ corresponds to a bandstop filter.