LINKÖPING UNIVERSITY

Exam in TSTE06 Digital Filters

Exam code:	TEN1
Date:	2017-06-03 **Time:** 14–18
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/
Result:	Available by 2017-06-17

1 **a.** Why do some ports have to be reflection-free when realizing wave digital filters using directly interconnected adaptors? (2 p)

b. Sketch typical magnitude responses for lowpass filters designed in the minimax sense and least-squares sense, respectively. (2 p)

c. Draw a block diagram of a digital system, using upsampler(s), downsampler(s) and filter(s), that increases the sampling rate by a factor of 1.2. Also sketch the ideal frequency response of the filter(s). (2 p)

d. An overall linear-phase FIR filter is formed by cascading two Type-III linear-phase FIR filters. What type is the overall filter? (2 p)

e. Indicate in the z-plane typical zero locations for the following digital filters: 9th-order highpass Cauer filter, 7th-order lowpass Chebyshev-II filter, 8th-order bandstop Chebyshev-I filter, 6th-order bandpass Butterworth filter.

(2 p)

2 Synthesize a digital Chebyshev-I filter that meets the specification in Figure 1. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. Do not use a higher filter order than the specification requires. (10 p)

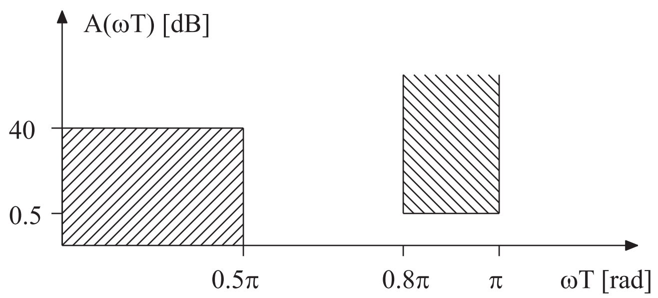

Figure 1:

3 A filter structure is given according to Figure 2.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Compute the minimal sample period $T_{\min}$ and critical path T_{CP}. Assume that $T_{\text{mult}} = 1\ \mu\text{s}$ and $T_{\text{add}} = 0.25\ \mu\text{s}$. (2 p)

c. Use pipelining so that the critical path is reduced by one multiplier. The pipelined structure should not have more than five delay elements in total.

(2 p)

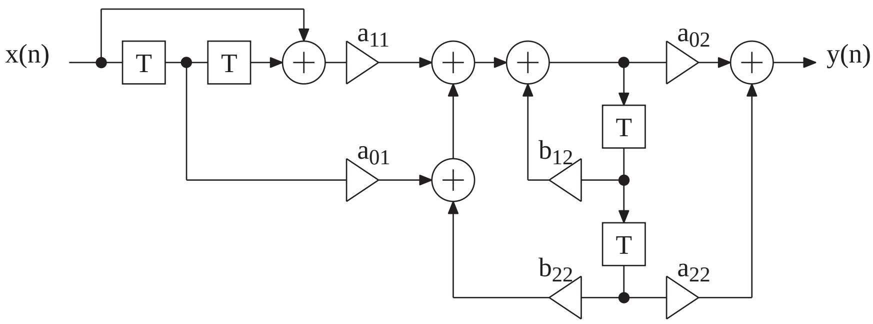

Figure 2:

4 A digital filter is used to filter an analog signal after sampling it with the sampling frequency $f_{sample} = 40$ MHz. The specification of the filter is: $f_c = 6$ MHz, $f_s = 15$ MHz, $A_{\max} = 0.1$ dB ($\rho = 15\%$), $A_{\min} = 40$ dB. Realize the digital filter with as low filter order as possible using a Cauer π-type ladder wave digital filter. Use Richards variable $\Psi = (z-1)/(z+1)$ and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients.

(10 p)

5 A first-order unity-gain (magnitude = 1) allpass filter is realized according to Figure 3.

a. Compute the variance of the roundoff noise at the output of the filter. Assume that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modeled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-14}$. Utilize that when white noise propagates through an LTI-system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. (5 p)

L_2-norm of an LTI system with the impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$||F(e^{j\omega T})||_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

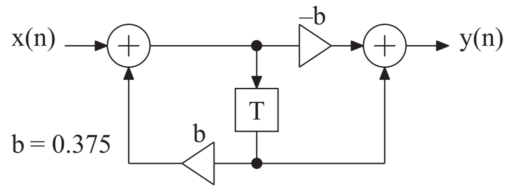

Figure 3:

6 A symmetric bandstop filter $H(z)$, thus with a stopband symmetric around $\pi/2$, is to be realized in the form of $H(z) = G(z^M)F(z)$ (frequency-response masking technique), where $G(z)$ is a lowpass filter and $F(z)$ is a bandstop filter, and M is an integer. It is also assumed that $M > 2$.

a. Under the above assumptions, and given the passband and stopband edges of the overall filter $H(z)$, which values of M are feasible? (5 p)

b. Given the passband and stopband edges of $H(z)$, and a feasible M, express the passband and stopband edges of $G(z)$ and $F(z)$ in terms of the edges of $H(z)$ and M. (5 p)

7 The normal-output transfer function $H(z)$ of a lattice WDF, and its complementary-output transfer function $H_c(z)$, are given by $H(z) = 0.5[A_0(z) + A_1(z)]$ and $H_c(z) = 0.5[A_0(z) - A_1(z)]$, where $A_0(z)$ and $A_1(z)$ are unity-gain (magnitude = 1) allpass transfer functions.

a. Show that $H(z)$ and $H_c(z)$ form a power-complementary filter pair, i.e., $|H(e^{j\omega T})|^2 + |H_c(e^{j\omega T})|^2 = 1$. (4 p)

b. Assume that $H(z)$ and $H_c(z)$ simultaneously [that is, with the same pair of $A_0(z)$ and $A_1(z)$] are to meet the specifications given below. Determine the filter order required for $H(z)$ (the filter order of $H_c(z)$ is automatically the same) so that both of $H(z)$ and $H_c(z)$ meet their respective specification. Assume Butterworth approximation. (3 p)

$$1 - 0.05 \leq |H(e^{j\omega T})| \leq 1, \quad \omega T \in [0, 0.1\pi]$$
$$|H(e^{j\omega T})| \leq 0.005, \quad \omega T \in [0.5\pi, \pi]$$

$$1 - 0.05 \leq |H_c(e^{j\omega T})| \leq 1, \quad \omega T \in [0.5\pi, \pi]$$
$$|H_c(e^{j\omega T})| \leq 0.005, \quad \omega T \in [0, 0.1\pi]$$

c. Determine the group delay responses of $H(z)$ and $H_c(z)$, respectively, expressed in terms of the group delay responses of $A_0(z)$ and $A_1(z)$. (3 p)