Tekniska Högskolan i Linköping
Institutionen för Systemteknik, Elektroniksystem

Exam in TSTE06/TEN1, Digital Filters

Time:	2015-06-02, 14-18
Place:	TB
Examiner:	Håkan Johansson
Aid:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler & Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller & Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler & Lexikon Mathematical tables
Number of problems:	7
Instructions:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem! **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Results:	Available 2015-06-16

1 a) An FIR filter has the transfer function $H(z) = z^{-2} + 2z^{-3} + z^{-4}$. Is it a lowpass or highpass filter? (2 p)

b) Explain which nodes need to be scaled, and why, in digital filters when two's-complement arithmetic is used. What is the difference between safe scaling and L_p-norm scaling? (2 p)

c) State and explain two methods of avoiding delay-free loops when realizing ladder wave digital filters. (2 p)

d) A linear-phase FIR filter transfer function is given as $H(z) = z^{-10}(z + z^{-1})^{10}$. What is the group delay of the filter? (2 p)

e) Indicate in the z-plane typical zero locations for the following digital filters: (2 p)

5th-order lowpass Butterworth filter, 7th-order highpass Chebyshev-I filter
10th-order bandpass Cauer filter, 12th-order bandstop Chebyshev-II filter

2) A digital filter is used to filter an analog signal after sampling it with the sampling frequency f_{sample} = 13.6 KHz. The specification of the filter is: f_c = 3.4KHz, f_s = 5.75 KHz, A_{max} = 0.1 dB, A_{min} = 42 dB. Realize the digital filter with as low filter order as possible using a Cauer T-type ladder wave digital filter. Use Richard's variable $\Psi = (z-1)/(z+1)$ and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients. (10 p)

3) Synthesize a digital Chebyshev-I filter that meets the specification below. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. Select the lowest possible filter order. (10 p)

$\omega_c T = 0.65\pi$, $\omega_s T = 0.4\pi$, A_{max} = 1 dB, A_{min} = 35 dB.

4) A filter structure is given according to Fig. 1.

a) Draw the signal-flow graph in precedence form. (6 p)

b) Determine the system of difference equations in computable order. (2 p)

c) Compute the minimal sample period T_{min} and critical path T_{CP}. Assume that $T_{mult} = 1\ \mu s$ and $T_{add} = 0.1\ \mu s$. (2 p)

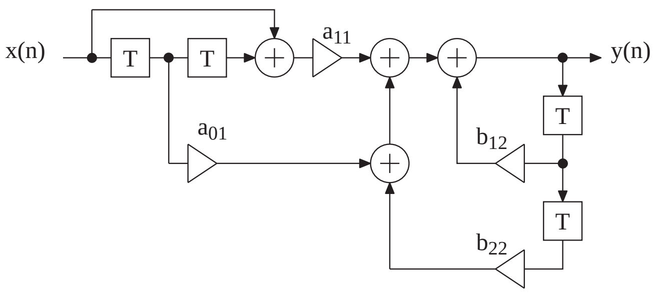

Fig. 1.

5 a) Compute the variance of the roundoff noise at the output of the filter in Fig. 2. Assume that quantization (rounding) is performed after each multiplication, and that the quantizations can be modeled with uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-14}$. Utilize that, when white noise propagates through an LTI-system with impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain (NG) of an LTI system with impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$NG = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b) Scale the filter using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constants and show where they should be inserted in the filter. (5 p)

L_2-norm of an LTI system with impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$\|F(e^{j\omega T})\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

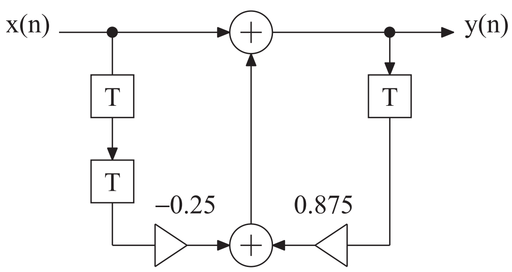

Fig. 2.

6) A symmetric bandpass filter $H(z)$, thus with passband symmetric around $\pi/2$, is to be realized in the form of $H(z) = G(z^M)F(z)$, where $G(z)$ is a lowpass filter and $F(z)$ is a bandpass filter, and M is an integer (frequency masking technique).

a) Which values of M are feasible? (5 p)

b) Given the passband and stopband edges of $H(z)$, express the passband and stopband edges of $G(z)$ and $F(z)$ in terms of the edges of $H(z)$ and M. (5 p)

7) Figure 3 shows two signals, $x_0(n)$ and $x_1(n)$, transmitted through a transmultiplexer to the received signals $w_0(n)$ and $w_1(n)$. Determine the passband and stopband edges of the lowpass filters $H_0(z)$ and $G_0(z)$, and highpass filters $G_1(z)$ and $H_1(z)$, so that $w_0(n) = x_0(n)$ and $w_1(n) = x_1(n)$. The transition bands should be as narrow as possible. Also determine the passband gains of the filters. For simplicity, the gains of the filters are assumed to be constant in the passband and zero in the stopband. Motivate the answer by sketching the spectra for all signals that are indicated in Fig. 3(a). That is, sketch $U_k(e^{j\omega T_2})$, $Y_k(e^{j\omega T_2})$, $V_k(e^{j\omega T_2})$, and $W_k(e^{j\omega T_1})$, for $k = 0$ and $k = 1$, as well as $Y(e^{j\omega T_2})$. Utilize the input-output relations for the upsampler and downsampler which, in the z-domain, are as given below. (10 p)

Upsampling by two: $\text{Out}(z) = \text{In}(z^2)$

Downsampling by two: $\text{Out}(z) = \dfrac{\text{In}(z^{1/2}) + \text{In}(-z^{1/2})}{2}$

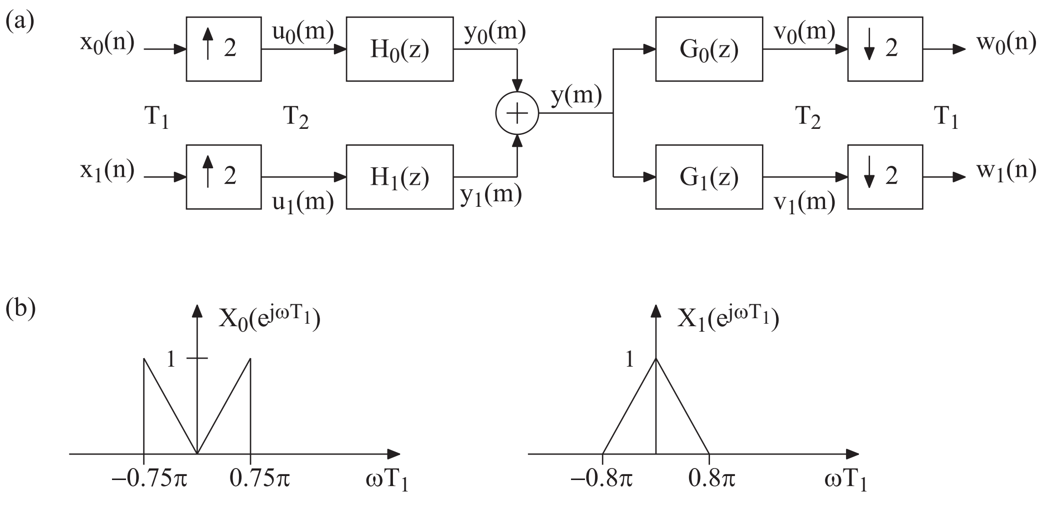

Fig. 3.