Tekniska Högskolan i Linköping
Institutionen för Systemteknik, Elektroniksystem

Tentamen i TSTE06/TEN1, Digitala Filter

Time:	2013-05-29, 8-12
Place:	U14
Teacher:	Håkan Johansson, 0703432897
Aid:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler & Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller & Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler & Lexikon Mathematical tables
Number of problems:	7
Instructions:	Maximum 70 points, 30 points required to pass the exam. **Note** that a motivation/solution is required to get the maximum number of points for a problem! **Note** that 10 points obtained at the **seminars** means that the first problem does not have to be solved. In case of 5 points, problems 1(a) and 1(b) do not have to be solved.
Results:	Available by 2013-06-12

1 a) Indicate in the z-plane typical zero locations for the following digital filters: 5th-order lowpass Cauer filter, 10th-order bandpass Butterworth filter, 7th-order highpass Chebyshev-II filter, 8th-order bandstop Chebyshev-I filter. (2.5 p)

b) An FIR filter is given as: $H(z) = a_0 z^{-3} + a_1 z^{-7} + a_0 z^{-11}$. Does the filter have a linear phase response? Can it be used as a highpass filter? Motivate your answers. (2.5 p)

c) State the typical features, advantages, and disadvantages of FIR filters and IIR filters and their realizations. (5 p)

2) A digital filter is used to filter an analog signal after sampling it with the sampling frequency $f_{sample} = 1/T = 11.2$ kHz. The specification of the filter is: $f_c = 1$ kHz, $f_s = 4$ kHz, $A_{max} = 0.5$ dB, $A_{min} = 70$ dB. Synthesize a digital Chebyshev-I filter that meets the specification. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. The filter order should not be higher than necessary. (10 p)

3) A filter structure is given according to Fig. 1.

a) Draw the signal-flow graph in precedence form. (6 p)

b) Compute the minimal sample period T_{min} and critical path T_{CP}. Assume that $T_{mult} = 0.06\mu s$ and $T_{add} = 0.04$ µs. (2 p)

c) Modify the filter structure so that the critical path contains one multiplication and at most two additions. You may use pipelining and change the order in which the additions are performed. The magnitude function of the filter must not be changed. (2 p)

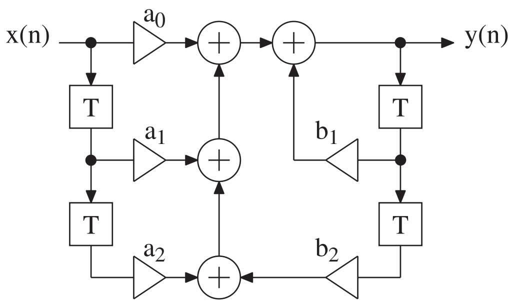

Fig. 1.

4) Realize a third-order Cauer wave digital filter satisfying the highpass filter requirements in Fig. 2. Start from a lowpass filter that is realized with a π-net with load and generator resistances of 1 Ω. Use Richard's variable $\Psi = (z-1)/(z+1)$ and

directly interconnected adaptors with reflection-free ports. Draw the wave-flow graph and compute the adaptor coefficients. (10 p)

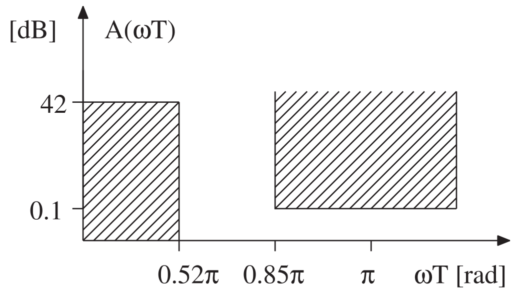

Fig. 2.

5) An FIR filter is realized according to Fig. 3 where h(0) = 0.01, h(1) = 0.05, h(2) = 0.12, and h(3) = 0.15.

a) Scale the filter using safe scaling. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. (5 p)

b) Compute the output noise variance as well as the change in SNR (signal-to-noise ratio) at the output that is obtained when the unscaled filter is replaced with the scaled filter. Assume that quantization (rounding) is performed after each multiplication, and that the quantization can be modeled as additive white noise with the variance $Q^2/12$ (in case of multiplications by 2^{-k}, k integer, these are for simplicity assumed not to contribute any noise). (5 p)

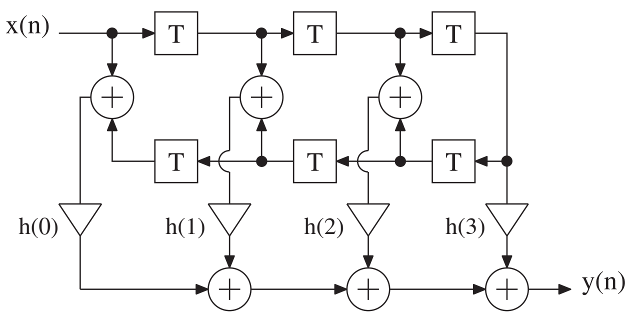

Fig. 3.

6) A digital narrow-band highpass linear-phase FIR filter $H(z)$ meeting the requirements given below is to be realized.

$$1 - 0.01 \leq |H(e^{j\omega T})| \leq 1 + 0.01, \quad \omega T \in [0.925\pi, \pi]$$
$$|H(e^{j\omega T})| \leq 0.0005, \quad \omega T \in [0, 0.9\pi]$$

a) Assume that the filter is realized in cascade form according to

$$H(z) = G(z^M)F(z)$$

where $G(z)$ and $F(z)$ are lowpass and higpass linear-phase FIR filters, respectively (that is, $H(z)$ is realized as a narrow-band frequency masking filter). Determine the passband and stopband edges as well as the passband and stopband ripples of $G(z)$ and $F(z)$ when $M = 6$ so that the specification of $H(z)$ is satisfied. The transition bands of $G(z)$ and $F(z)$ should not be more narrow than necessary. Motivate your answer by sketching the magnitude responses of the filters involved. (5 p)

b) Use the formula below to estimate the total number of multiplications required when $G(z)$ and $F(z)$ above are realized using Type I (thus even-order) linear-phase FIR direct-form structures. Compare the result with the number of multiplications required for a regular Type I linear-phase direct-form realization of $H(z)$. (3 p)

Filter order estimation: $N_{est} = -\frac{4\pi}{3}\frac{\log_{10}(10\delta_c\delta_s)}{\omega_s T - \omega_c T}$

where δ_c and δ_s denote the passband and stopband ripples, and $\omega_c T$ and $\omega_s T$ denote the passband and stopband edges, respectively.

c) What is the largest feasible M for which the narrow-band frequency masking technique above works? (2 p)

7) The normal-output transfer function H(z) of a lattice WDF, and its complementary-output transfer function $H_c(z)$, are given by $H(z) = 0.5[A_0(z)+A_1(z)]$ and $H_c(z) = 0.5[A_0(z)-A_1(z)]$ where $A_0(z)$ and $A_1(z)$ are unity-magnitude (magnitude = 1) allpass transfer functions.

a) Show that H(z) and $H_c(z)$ are power complementary, i.e., $|H(e^{j\omega T})|^2 + |H_c(e^{j\omega T})|^2 = 1$. (4 p)

b) Assume that H(z) and $H_c(z)$ simultaneously are to meet the specifications given below [that is, with the same pair of $A_0(z)$ and $A_1(z)$]. Determine the filter order required for the lattice WDF when H(z) is a Chebyshev I filter. (4 p)

$$1 - 0.01 \le |H(e^{j\omega T})| \le 1, \quad \omega T \in [0, 0.22\pi]$$
$$|H(e^{j\omega T})| \le 0.01, \quad \omega T \in [0.5\pi, \pi]$$

$$1 - 0.01 \le |H_c(e^{j\omega T})| \le 1, \quad \omega T \in [0.5\pi, \pi]$$
$$|H_c(e^{j\omega T})| \le 0.01, \quad \omega T \in [0, 0.22\pi]$$

c) Assume that z is replaced with $-z^2$ in H(z) and $H_c(z)$ in b). What types of filters (LP, HP, BP, or BS) do $H(-z^2)$ and $H_c(-z^2)$ correspond to? (2 p)