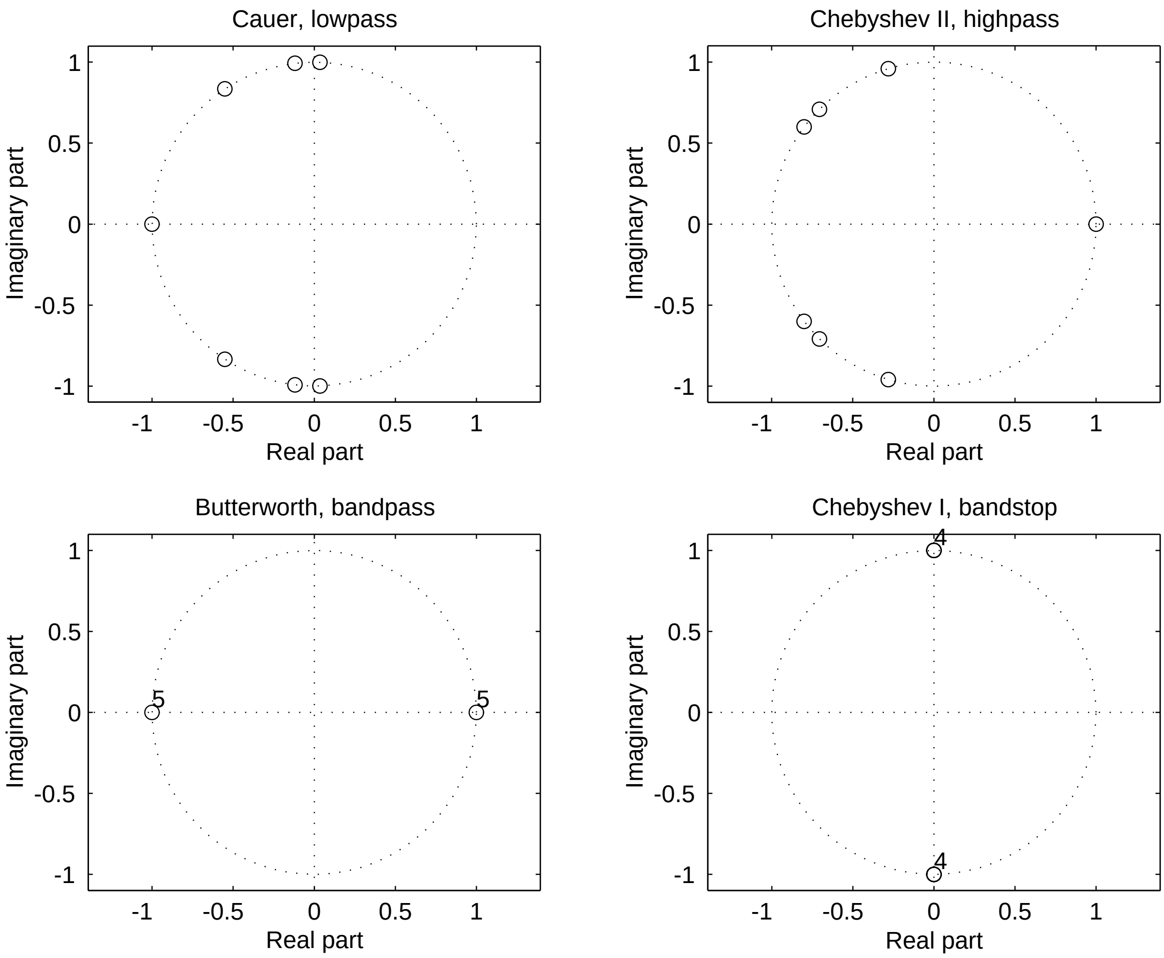
TSEI07, Digitala Filter 2013-05-29, Solutions

1 a)



b) Linear phase Type I FIR filter, can not be used as highpass filter, |H(1)| = |H(–1)|

c) FIR: + linear-phase, nonrecursive structures

– high order, long delays

IIR: + low order

– nonlinear-phase, recursive structures

2) Requirements for analog LP-filter:

$$\omega_{ac} = \frac{2}{T}\tan\left(\frac{\omega_c T}{2}\right) = \frac{2}{T}\tan\left(\frac{0{,}1786\pi}{2}\right) \approx \frac{2}{T}0{,}288095$$

$$\omega_{as} = \frac{2}{T}\tan\left(\frac{\omega_s T}{2}\right) = \frac{2}{T}\tan\left(\frac{0{,}7143\pi}{2}\right) \approx \frac{2}{T}2{,}076521$$

Filter order: nomogram => $N = 4$

Normalized poles:

$S'_{2,3} = -0{,}1753531 \pm j1{,}016253$

$S'_{3,4} = -0{,}4233398 \pm j0{,}4209457$

Zeros: 4 zeros i at S = ×

Denormalize with ω_{ac} =>

Poles:

$s_{2,3} = -0{,}0505183 \pm j0{,}2927774$

$s_{3,4} = -0{,}1219621 \pm j0{,}1212724$

Zeros: 4 zeros at s = ×

Transform into digital filter

$$z = \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}}$$

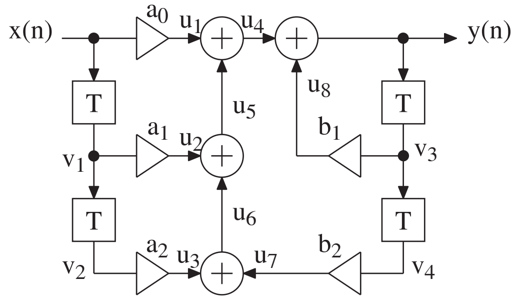
Poles:

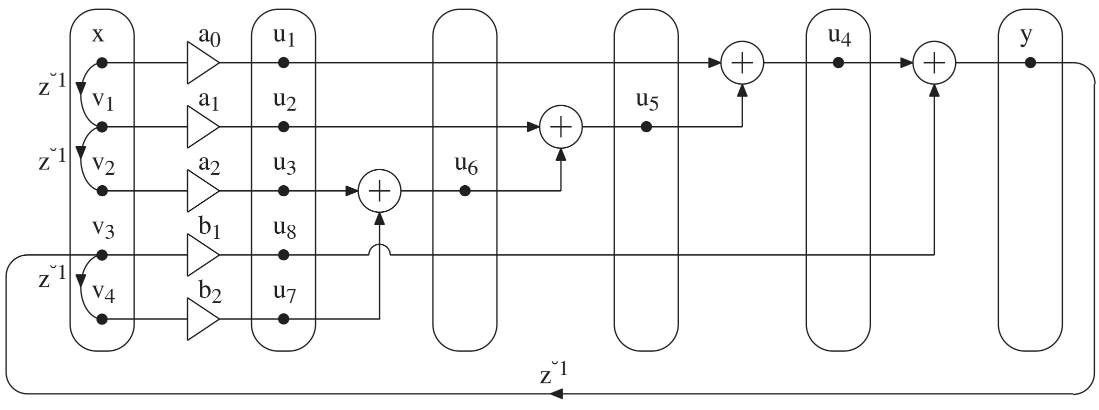
$z_{2,3} = 0{,}7666052 \pm j0{,}4923494$

$z_{3,4} = 0{,}7620053 \pm j0{,}1904544$

Zeros: 4 zeros at z = –1

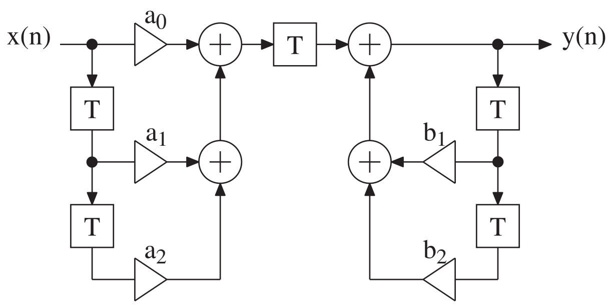
3 a)





b) $T_{CP} = T_{mult}+4T_{add} = 0.22\ \mu s$, $T_{min} = 0.5[T_{mult}+4T_{add}] = 0.11\ \mu s$

c)



4) Requirements for the analog reference filter:

$$\omega_{arc} = \frac{2}{T}\tan\left(\frac{\omega_c T}{2}\right) = \frac{2}{T}\tan\left(\frac{0,85\pi}{2}\right) \approx \frac{2}{T}4,165300$$

$$\omega_{ars} = \frac{2}{T}\tan\left(\frac{\omega_s T}{2}\right) = \frac{2}{T}\tan\left(\frac{0,52\pi}{2}\right) \approx \frac{2}{T}1,064882$$

LP specification

$$\Omega_c = \frac{\omega_I^2}{\omega_{arc}}, \quad \Omega_s = \frac{\omega_I^2}{\omega_{ars}}$$

Select $\Omega_c = 1 \Rightarrow \omega_I^2 = \omega_{arc}$, $\quad \Omega_s = \dfrac{\omega_{arc}}{\omega_{ars}}$

Requirements on ω_{ars} and A_{min} => $15 \le \Theta \le 15$

Normalized element values

C'₁ = C'₃ = 0.9944, C'₂ = 0.0463, L'₂ = 1.0941

Denormalize with R = 1, $\omega_0 = \Omega_c = 1$ =>

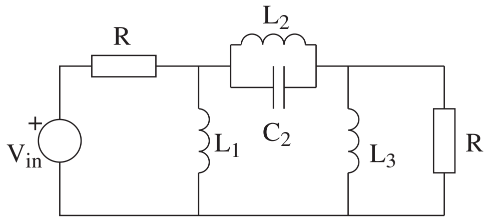
$C_1 = C_3 = 0.9944$, $C_2 = 0.0463$, $L_2 = 1.0941$

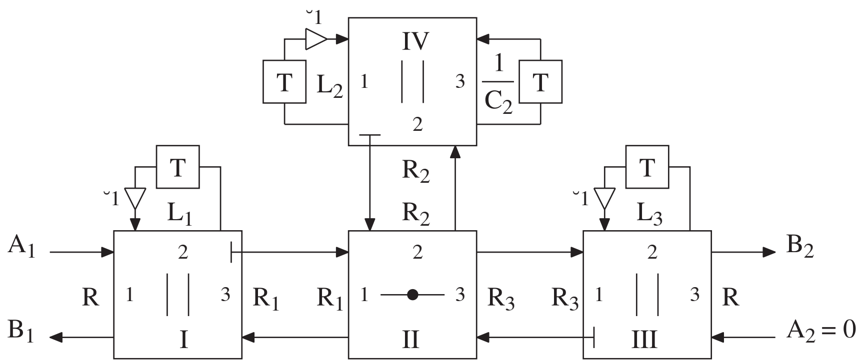
LP to HP transformation

$$C \rightarrow \frac{1}{\omega_I^2 L}, \quad L \rightarrow \frac{1}{\omega_I^2 C}$$

$L_1 = L_3 = 0.2414308$, $L_2 = 5.185286$, $C_2 = 0.2194304$

Reference highpass filter and corresponding wave flow graph





Adaptor coefficients

I: $R_1 = (1/L_1 + 1/R)^{-1} = 0{,}194478$

$$\alpha_1 = \frac{2/R}{1/R + 1/L_1 + 1/R_1} = 0{,}194478$$

$$\alpha_2 = 1 - \alpha_1 = 0{,}805522$$

$$\alpha_3 = 1$$

III: $R_3 = (1/L_3 + 1/R)^{-1} = 0{,}194478$

$$\alpha_1 = 1$$

$$\alpha_2 = \frac{2/L_3}{1/R + 1/L_3 + 1/R_3} = 0{,}805522$$

$$\alpha_3 = 1 - \alpha_2 = 0{,}194478$$

IV: $R_2 = (1/L_2 + C_2)^{-1} = 2{,}42551$

$$\alpha_1 = \frac{2/L_2}{1/R_2 + 1/L_2 + C_2} = 0{,}467769$$

$$\alpha_2 = 1$$

$$\alpha_3 = 1 - \alpha_1 = 0{,}532231$$

II:

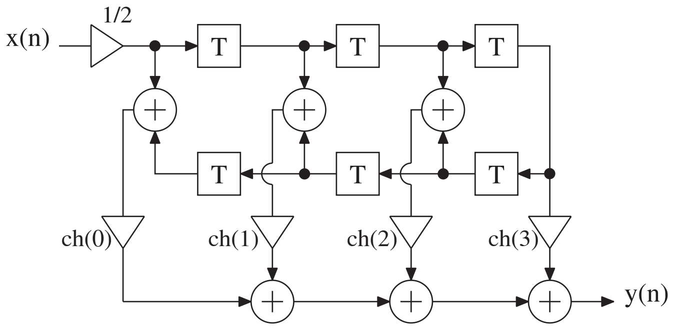
$$\alpha_1 = \frac{2R_1}{R_1 + R_2 + R_3} = 0{,}138199$$

$$\alpha_2 = \frac{2R_2}{R_1 + R_2 + R_3} = 1{,}723603$$

$$\alpha_3 = 1 - \alpha_1 - \alpha_2 = 0{,}138199$$

5 a) The filter is scaled according to the figure below where

$$c = \frac{1}{\sum_{n=0}^{6} \frac{1}{2}|h(n)|} = \frac{2}{0{,}51} \approx 3{,}92$$



b) SNR at the output before scaling:

$$\mathrm{SNR}_1 = 10\log_{10}\left(\frac{\sigma_x^2 \sum_{n=0} h^2(n)}{4\sigma_e^2}\right)$$

SNR at the output after scaling:

$$\mathrm{SNR}_2 = 10\log_{10}\left(\frac{\sigma_x^2 \sum_{n=0} \left(\frac{c}{2}\right)^2 h^2(n)}{4\sigma_e^2}\right)$$

Difference:

$$\mathrm{SNR}_2 - \mathrm{SNR}_1 = 10\log_{10}\left(\left(\frac{c}{2}\right)^2\right) = 20\log_{10}(3{,}92/2) \approx 5{,}84 \text{ dB}$$

6 a)

$$H(z) = \left(\frac{-\alpha_0 z + 1}{z - \alpha_0}\right) + \left(\frac{-\alpha_1 z^2 - \alpha_2(1-\alpha_1)z + 1}{z^2 - \alpha_2(1-\alpha_1)z - \alpha_1}\right)$$

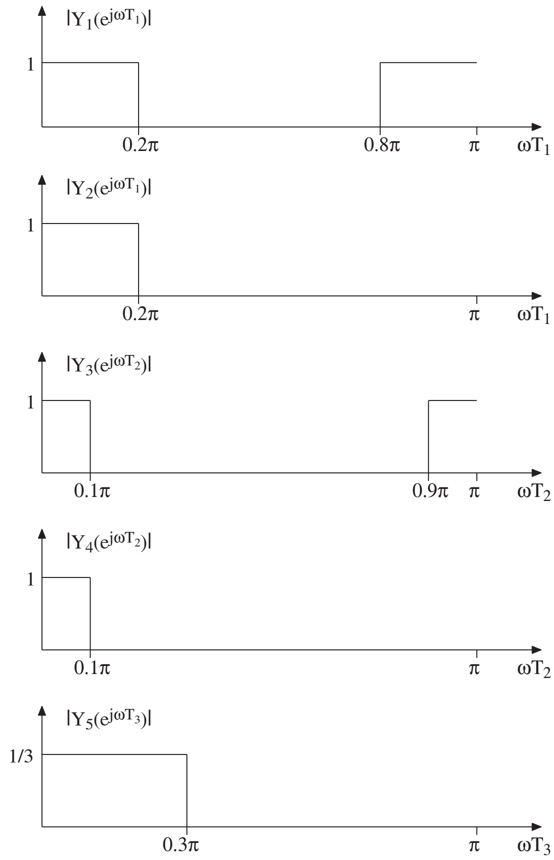
$$\alpha_0 = p_0 = -0{,}7149986$$

$$\alpha_1 = -p_1 p_2 = -0{,}7571239, \quad \alpha_2 = \frac{p_1 + p_2}{1 - \alpha_1} = -0{,}8088585$$

b)

|H(e^jωT)|
1
0.1π
0.25π
0.75π
0.9π
π
ωT

7)



|Y1(e^jωT1)|
1
0.2π
0.8π
π
ωT1
|Y2(e^jωT1)|
1
0.2π
π
ωT1
|Y3(e^jωT2)|
1
0.1π
0.9π
π
ωT2
|Y4(e^jωT2)|
1
0.1π
π
ωT2
|Y5(e^jωT3)|
1/3
0.3π
π
ωT3