LINKÖPING UNIVERSITY

Exam in TSEI07 Digital Filters

Exam code:	TEN1
Date:	2017-06-03 **Time:** 14–18
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/
Result:	Available by 2017-06-17

1 **a.** Why do some ports have to be reflection-free when realizing wave digital filters using directly interconnected adaptors? (2 p)

b. Sketch typical magnitude responses for lowpass filters designed in the minimax sense and least-squares sense, respectively. (2 p)

c. Draw a block diagram of a digital system, using upsampler(s), downsampler(s) and filter(s), that increases the sampling rate by a factor of 1.2. Also sketch the ideal frequency response of the filter(s). (2 p)

d. An overall linear-phase FIR filter is formed by cascading two Type-III linear-phase FIR filters. What type is the overall filter? (2 p)

e. Indicate in the z-plane typical zero locations for the following digital filters: 9th-order highpass Cauer filter, 7th-order lowpass Chebyshev-II filter, 8th-order bandstop Chebyshev-I filter, 6th-order bandpass Butterworth filter.

(2 p)

2 Synthesize a digital Chebyshev-I filter that meets the specification in Figure 1. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. Do not use a higher filter order than the specification requires. (10 p)

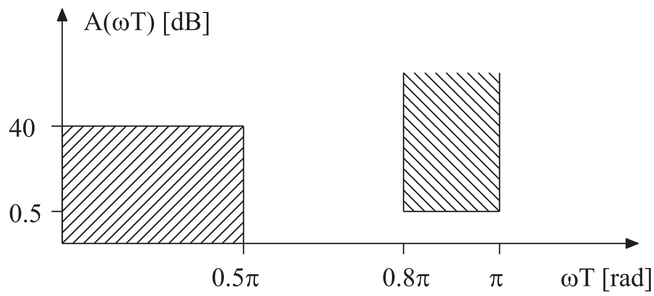

Figure 1:

3 A filter structure is given according to Figure 2.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Compute the minimal sample period $T_{\min}$ and critical path T_{CP}. Assume that $T_{\mathrm{mult}} = 1$ μs and $T_{\mathrm{add}} = 0.25$ μs. (2 p)

c. Use pipelining so that the critical path is reduced by one multiplier. The pipelined structure should not have more than five delay elements in total.

(2 p)

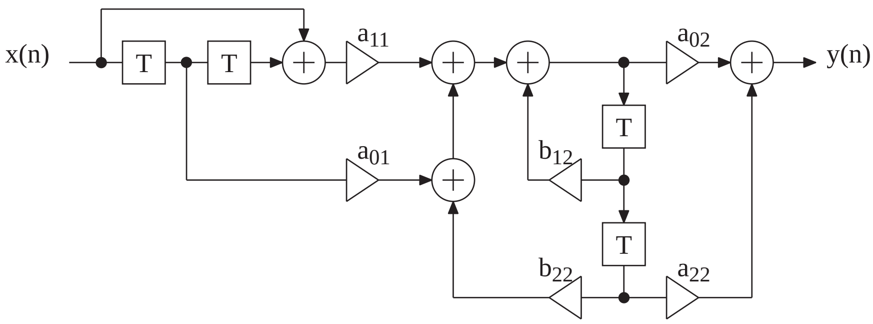

Figure 2:

4 A digital filter is used to filter an analog signal after sampling it with the sampling frequency $f_{sample} = 40$ MHz. The specification of the filter is: $f_c = 6$ MHz, $f_s = 15$ MHz, $A_{\max} = 0.1$ dB ($\rho = 15\%$), $A_{\min} = 40$ dB. Realize the digital filter with as low filter order as possible using a Cauer π-type ladder wave digital filter. Use Richards variable $\Psi = (z-1)/(z+1)$ and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients.

(10 p)

5 A filter is realized according to Figure 3.

a. Compute the variance of the roundoff noise at the output of the filter. Assume that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modeled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-14}$. Utilize that when white noise propagates through an LTI-system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. (5 p)

L_2-norm of an LTI system with the impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$||F(e^{j\omega T})||_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

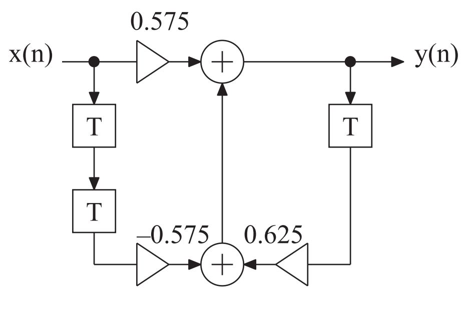

Figure 3:

6 Assume that the filter $G(z)$ is a lowpass filter with passband and stopband edges at 0.2π and 0.25π, respectively. For simplicity, also assume that the gain is unity in the passband and zero in the stopband.

a. Sketch the magnitude responses of the filter $G(z^M)$ when $M = 5$. (3 p)

b. Cascade $G(z^5)$ with a lowpass filter $F(z)$ so that the resulting filter $H(z) = G(z^5)F(z)$ becomes a lowpass filter. Determine the passband and stopband edges of $F(z)$ and $H(z)$. The transition band of $F(z)$ should be as wide as possible. Again, assume that the gain is unity in the passband and zero in the stopband. (3 p)

c. Assume now practical filters with passband and stopband ripples (δ_c, δ_s). How should the ripples of $G(z)$ and $F(z)$ be selected to ensure that the ripple specifications of $H(z)$ are satisfied? (2 p)

d. How should a realization of $G(z)$ be modified to obtain a realization of $G(z^M)$? (2 p)

7 Figure 4 shows a third-order Lattice WDF. The first-order allpass section in the upper branch has a transfer function according to

$$H_1(z) = \frac{-\alpha_0 z + 1}{z - \alpha_0}$$

a. Utilize the above transfer function to show that the transfer function of the second-order allpass section in the lower branch is $H_2(z)$ as given below. (5 p)

$$H_2(z) = \frac{-\alpha_1 z^2 - \alpha_2(1 - \alpha_1)z + 1}{z^2 - \alpha_2(1 - \alpha_1)z - \alpha_1}$$

b. Compute the adaptor coefficients $(\alpha_0, \alpha_1, \alpha_2)$ when the Lattice WDF realizes a Cauer filter with zeros $(z_{0,1,2})$ and poles $(p_{0,1,2})$ in the z-plane as given below. (5 p)

$z_0 = -1$
$z_{1,2} = -0.377748 \pm j0.925909$
$p_0 = 0.575585$
$p_{1,2} = 0.557214 \pm j0.590813$

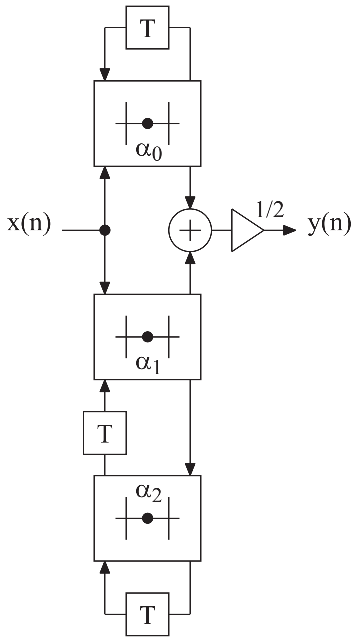

Figure 4: