LINKÖPING UNIVERSITY

Exam in TSEI07 Digital Filters

Exam code:	TEN1		
Date:	2016-06-01	**Time:**	14–18
Place:	T1		
Examiner:	Håkan Johansson		
Department:	ISY		
Allowed aids:	Pocket calculator Tables and formulas for analog and digital filters Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables		
Number of tasks:	7		
Grading:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.		
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSTE06/		
Result:	Available by 2016-06-15		

1

a. Explain what the zero-phase frequency response is for a linear-phase FIR filter. How is it related to the magnitude response? (2 p)

b. In terms of sensitivity, what is the difference between ladder wave digital filters and lattice wave digital filters? (2 p)

c. What is the difference between recursive and nonrecursive structures? How are they related to FIR and IIR filters? (2 p)

d. Sketch typical magnitude responses for lowpass filters designed in the minimax sense and least-squares sense, respectively. (2 p)

e. Draw a block diagram of a system, using upsampler(s), downsampler(s) and filter(s), that increases the sampling rate by a factor of 3.5. Also sketch the ideal frequency response of the filter(s). (2 p)

2 Synthesize a third-order digital high-pass Cauer filter that meets the specification below. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation $s = \frac{2}{T}\frac{z-1}{z+1}$. (10 p)

$\omega_c T = 0.8\pi$, $\omega_s T = 0.38\pi$, $A_{\text{max}} = 0.09883$ dB ($\rho = 15\%$), $A_{\text{min}} = 45$ dB.

3 Realize a minimum-order Chebyshev-I π-type ladder wave digital filter that meets the specification below. Use Richard's variable $\Psi = \frac{z-1}{z+1}$ and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients. (10 p)

$\omega_c T = 0.2\pi$, $\omega_s T = 0.45\pi$, $A_{\text{max}} = 0.1$ dB, $A_{\text{min}} = 40$ dB.

4 A filter structure is given according to Figure 1.

a. Draw the signal-flow graph in precedence form. (6 p)

b. Use pipelining so that the critical path (T_{CP}) equals the minimal sampling period (T_{min}) of the pipelined structure. The pipelined structure should not have more than three delay elements in total. (4 p)

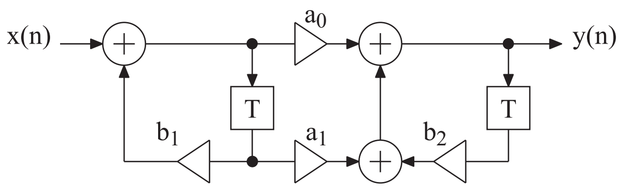

Figure 1:

5 **a.** Compute the variance of the roundoff noise at the output of the filter seen in Figure 2. Assume that quantization (rounding) is carried out after each multiplication, and that the quantizations can be modelled as uncorrelated zero-mean white-noise sources with the average power (variance) $Q^2/12$ where $Q = 2^{-10}$. Utilize that, when white noise propagates through an LTI-system with the impulse response $g(n)$, the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain of an LTI system with the impulse response $g(n)$ and frequency response $G(e^{j\omega T})$:

$$\text{Noise Gain} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

b. Scale the filter using the L_2-norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constant/constants and show where it/they should be inserted in the filter. Utilize that Branch 2 of the filter realizes a unity-gain allpass filter. (5 p)

L_2-norm of an LTI system with the impulse response $f(n)$ and frequency response $F(e^{j\omega T})$:

$$||F(e^{j\omega T})||_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

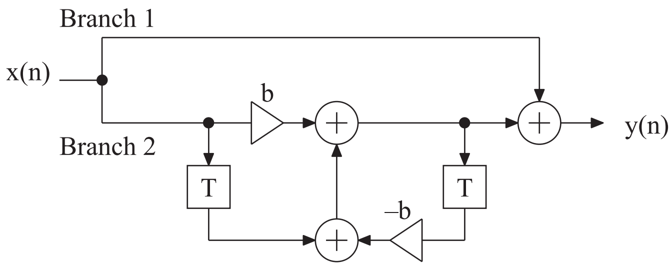

Figure 2:

6 Assume that the filter $H(z)$ is a lowpass filter with passband and stopband edges at 0.4π and 0.6π, respectively. For simplicity, also assume that the gain is unity in the passband and zero in the stopband. Sketch the magnitude responses of the three filters $H_1(z)$, $H_2(z)$, and $H_3(z)$, as given below. Also, for each filter, determine the filter order (expressed as a function of N), assuming that the filter order of $H(z)$ is N.

a. $H_1(z) = H(-z)$ (2 p)

b. $H_2(z) = H(z^2)H(z)$ (4 p)

c. $H_3(z) = H(z^4)H(z^2)H(z)$ (4 p)

7 There are four types of causal Nth-order linear-phase FIR filters as follows. Type I: symmetric impulse response and even N; Type II: symmetric impulse response and odd N; Type III: antisymmetric impulse response and even N; Type IV: antisymmetric impulse response and odd N.

a. For each of the four types, determine how many multipliers (expressed as a function of N) that are needed in a direct-form realization, when utilizing the impulse response symmetries and antisymmetries. (4 p)

b. What type of overall filter (Type I, II, III, or IV) is obtained if two Type IV filters are cascaded? (3 p)

c. What type of overall filter (Type I, II, III, or IV) is obtained if a Type III filter is cascaded with a Type I filter? (3 p)