

## Exam in TSEI07/TEN1, Digital Filters

- Time:** 2015-06-02, 14-18
- Place:** TB
- Examiner:** Håkan Johansson
- Aid:** Pocket calculator  
Tables and formulas for analog and digital filters  
Söderkvist: Formler & Tabeller  
Ingelstam, Rönngren, Sjöberg: Tefyma  
Ekbohm: Tabeller & Formler NT  
Nordling: Physics Handbook for Science and Engineering  
Strid: Formler & Lexikon  
Mathematical tables
- Number of problems:** 7
- Instructions:** Maximum 70 points, 30 points required to pass the exam.  
**Note** that a **motivation/solution** is required to get the maximal number of points for a problem!
- Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
- Results:** Available 2015-06-16

- 1
- An FIR filter has the transfer function  $H(z) = z^{-2} + 2z^{-3} + z^{-4}$ . Is it a lowpass or highpass filter? (2 p)
  - Explain which nodes need to be scaled, and why, in digital filters when two's-complement arithmetic is used. What is the difference between safe scaling and  $L_p$ -norm scaling? (2 p)
  - State and explain two methods of avoiding delay-free loops when realizing ladder wave digital filters. (2 p)
  - A linear-phase FIR filter transfer function is given as  $H(z) = z^{-10}(z + z^{-1})^{10}$ . What is the group delay of the filter? (2 p)
  - Indicate in the z-plane typical zero locations for the following digital filters: (2 p)  
 5th-order lowpass Butterworth filter, 7th-order highpass Chebyshev-I filter  
 10th-order bandpass Caueer filter, 12th-order bandstop Chebyshev-II filter
- 2) A digital filter is used to filter an analog signal after sampling it with the sampling frequency  $f_{\text{sample}} = 13.6$  KHz. The specification of the filter is:  $f_c = 3.4$  KHz,  $f_s = 5.75$  KHz,  $A_{\text{max}} = 0.1$  dB,  $A_{\text{min}} = 42$  dB. Realize the digital filter with as low filter order as possible using a Caueer T-type ladder wave digital filter. Use Richard's variable  $\Psi = (z - 1)/(z + 1)$  and directly interconnected adaptors with reflection-free ports. Draw the block diagram and compute the adaptor coefficients. (10 p)
- 3) Synthesize a digital Chebyshev-I filter that meets the specification below. Determine the poles and zeros, and indicate their locations in the z-plane. Use the bilinear transformation  $s = \frac{2z - 1}{Tz + 1}$ . Select the lowest possible filter order. (10 p)  
 $\omega_c T = 0.65\pi$ ,  $\omega_s T = 0.4\pi$ ,  $A_{\text{max}} = 1$  dB,  $A_{\text{min}} = 35$  dB.
- 4) A filter structure is given according to Fig. 1.
- Draw the signal-flow graph in precedence form. (6 p)
  - Determine the system of difference equations in computable order. (2 p)
  - Compute the minimal sample period  $T_{\text{min}}$  and critical path  $T_{\text{CP}}$ . Assume that  $T_{\text{mult}} = 1 \mu\text{s}$  and  $T_{\text{add}} = 0.1 \mu\text{s}$ . (2 p)

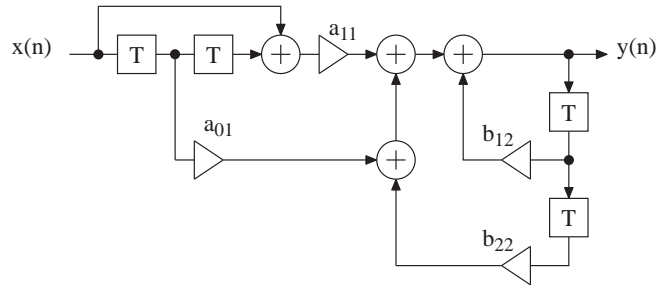


Fig. 1.

- 5 a) Compute the variance of the roundoff noise at the output of the filter in Fig. 2. Assume that quantization (rounding) is performed after each multiplication, and that the quantizations can be modeled with uncorrelated zero-mean white-noise sources with the average power (variance)  $Q^2/12$  where  $Q = 2^{-14}$ . Utilize that, when white noise propagates through an LTI-system with impulse response  $g(n)$ , the noise power is amplified/attenuated by the corresponding noise gain given below. (5 p)

Noise gain ( $NG$ ) of an LTI system with impulse response  $g(n)$  and frequency response  $G(e^{j\omega T})$ :

$$NG = \frac{1}{2\pi} \int_{-\pi}^{\pi} |G(e^{j\omega T})|^2 d(\omega T) = \sum_{n=-\infty}^{\infty} |g(n)|^2$$

- b) Scale the filter using the  $L_2$ -norm defined below. Assume that the input signal is scaled and that two's-complement arithmetic is used. Compute the scaling constants and show where they should be inserted in the filter. (5 p)

$L_2$ -norm of an LTI system with impulse response  $f(n)$  and frequency response  $F(e^{j\omega T})$ :

$$\|F(e^{j\omega T})\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} |F(e^{j\omega T})|^2 d(\omega T)} = \sqrt{\sum_{n=-\infty}^{\infty} |f(n)|^2}$$

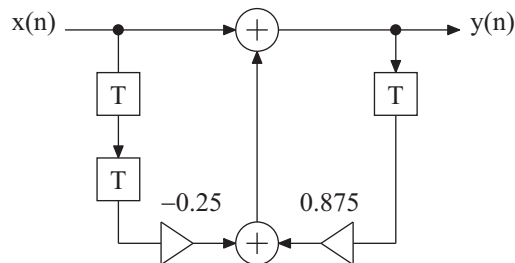


Fig. 2.

- 6) A sequence  $x(n_1)$  with spectrum according to Fig. 3(a) has been obtained by sampling an analog signal  $x_a(t)$  with the sampling period  $T_1$ , i.e.,  $x(n_1) = x_a(n_1 T_1)$ . The system in Fig. 3(b) is used to form a new sequence  $y(n_3)$  which should equal the sequence that is obtained when  $x_a(t)$  is sampled with the sampling period  $T_3 = 2T_1/3$ , i.e.,  $y(n_3) = x_a(n_3 T_3)$ . Determine the gain constant as well as passband and stopband edges of the filter  $H(z)$  so that  $y(n_3) = x_a(n_3 T_3)$ . The transition band should be as wide as possible. The frequency response of  $H(z)$  is assumed to be real and, further, constant in the passband and zero in the stopband. Also sketch the spectra for the sequences  $u(n_2)$ ,  $v(n_3)$ , and  $y(n_3)$ .

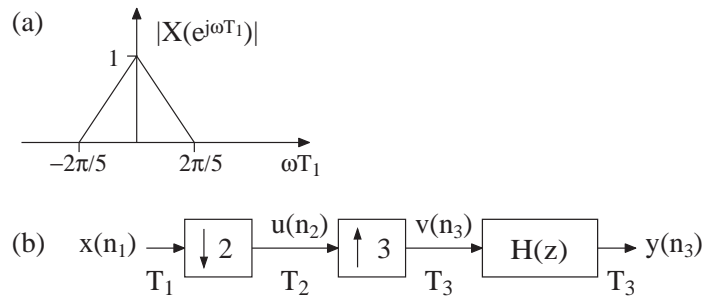


Fig. 3.

- 7) Compute the adaptor coefficients for the filter in Fig. 4. The filter realizes a Causer filter with poles in the  $\Psi$ -domain reference filter, as given below, where  $\Psi$  is Richard's variable  $\Psi = (z - 1)/(z + 1)$ . (10 p)

$$\Psi_0 = -0.4695435$$

$$\Psi_{1,2} = -0.07469961 \pm j0.7447807$$

$$\Psi_{3,4} = -0.2884280 \pm j0.5555904$$

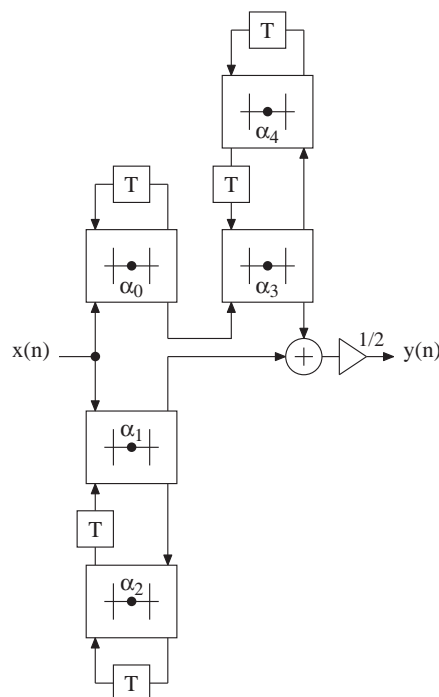


Fig. 4.