$\boxed{8.8}$ The BER for DBPSK over Rayleigh channel is $\frac{1}{2(1+\bar{\gamma}_b)}$.

The BER for the coded bits is then

$$p = \frac{1}{2(1+R\bar{\gamma}_b)} = \frac{1}{2(1+R.100)} = \frac{1}{2(1+\frac{26}{31}\cdot 100)} \approx 5{,}89.10^{-3}$$

a) We use the upper bound on P_u to get

$$P_u < 1 - \sum_{i=0}^{2} p^i (1-p)^{31-i} \approx 8{,}12.10^{-4}$$

The acceptance probability can be approximated as

$$P_a = P_c + P_u \approx (1-p)^{31} + P_u \approx 0{,}833$$

This gives the efficiency

$$\eta = \frac{k}{n} \cdot P_a \approx \frac{26}{31}\, 0{,}833 \approx 0{,}699$$

b) We have 4 branch MRC. We can maximum ratio combine every bit so the total BER for the coded bits is

$$p \approx \binom{2M-1}{M}\left(\frac{1}{2R\bar{\gamma}_b}\right)^M = \binom{7}{4}\left(\frac{1}{2\frac{26}{31}\cdot 100}\right)^4 \approx 4{,}42.10^{-8}$$

The acceptance probability is approximated as

$$P_a = P_c + P_u \approx (1-p)^{31} + 1 - \sum_{i=0}^{2}\binom{31}{i} p^i (1-p)^{31-i} = 1 - \sum_{i=1}^{2}\binom{31}{i} p^i (1-p)^{31-i} \approx 1$$

8.9 According to eq. (8.33) the messsge arrival rate for the "power capture" model is

$$\lambda = t e^{-t}\left(1 + \frac{t}{\beta^{1/2}}\right) = t e^{-t}\left(1 + \frac{t}{10^{\frac{1}{2}}}\right) = t e^{-t}\left(1 + \frac{t}{\sqrt{10}}\right)$$

Analyzing this function of t we can conclude the the througput λ has its maximum in $t \approx 1{,}2896$ and $\lambda_{max} \approx 0{,}5$

If we use Direct Sequence modulated system, we have increasing of the power of the signal by a factor of 10 at the receiver. For correct reception when collision occurs we require

$$\Gamma = \frac{P_i}{P_j} > 10$$

At the receiver we observe $\Gamma = \frac{10P_i}{P_j}$.
The requirement is now converted to

$$\frac{10P_i}{P_j} > 10 \iff \frac{P_i}{P_j} > 1$$

This corresponds to "power capture" with $\beta = 1$. Thus

$$\lambda = t e^{-t}\left(1 + \frac{t}{1}\right) = t e^{-t}(1+t)$$

The maximal throughput is $\lambda_{max} \approx 0{,}84$ in this case

$\boxed{8.10}$ a) The maximal total throughput for the slotted-ALOHA system is $\lambda_{max} = 1 \cdot e^{-1} \approx 0{,}368$

Since the transmission rate is 100 kbits/s, we obtain maximal throughput 36,8 kbits/s

b) For a stable system we reqiure a packet arrival rate at most the maximal throughput, i.e.

$\Lambda < 36{,}8\ \text{kbit/s} = 36{,}8$ packets/s

For a symmetric system we have $\Lambda_i = \frac{\Lambda}{M} = \frac{\Lambda}{100} = 0{,}368 \frac{\text{packets}}{s}$ for each of the users.

c) Eq. (8.29) gives the ~~average~~ expected packet delay as

$E[D] = 1 + \xi(e^{t} - 1)$, where t is such that $\lambda = t e^{-t}$.

For λ we have $\lambda = \Lambda \cdot T_e = M \cdot \Lambda_i T_e = 100 \cdot (0{,}05 \cdot 1000) \frac{\text{bits}}{s} \cdot 10^{-5} s = 0{,}05$

This gives $t = 0{,}051$ and $E[D] \approx 3{,}77$ if $\xi = 50$ and $E[D] \approx 2{,}39$ if $\xi = 25$.

d) The expected delay of a fixed assignement scheme can be calculated as

$$E[D_{FA}] = \frac{M}{2} + 1 + \frac{\lambda M}{2(1-\lambda)} = 1 + \frac{M}{2(1-\lambda)} = 1 + \frac{100}{2 \cdot 0{,}95} \approx 53{,}7$$

e) For a perfect scheduling system we have (eq.(8.18))

$$E[D_{PS}] = 1 + \frac{\lambda}{2(1-\lambda)} \approx 1{,}03$$

8.12 Assume that we have 2 colliding packets ~~of~~ with received power P_1 and P_2

a) The probability that the collision is resolved is:

$$P_{cap} = Pr\left\{\frac{P_1}{P_2} \geq 10\right\} + Pr\left\{\frac{P_2}{P_1} \geq 10\right\} = 2\,Pr\left\{\frac{P_1}{P_2} \geq 10\right\}$$

P_1 and P_2 are exponentially distributed and independent.

Now we have

$$P_{cap} = 2\int_0^\infty\left(\int_{10t_1}^\infty \frac{1}{t_0^2} e^{-\frac{t_1}{t_0}} e^{-\frac{t_2}{t_0}}\,dt_2\right)dt_1 = \frac{2}{t_0^2}\int_0^\infty e^{-\frac{t_1}{t_0}}\left(\int_{10t_1}^\infty e^{-\frac{t_2}{t_0}}\,dt_2\right)dt_1 =$$

$$= \frac{2}{t_0}\int_0^\infty e^{-\frac{t_1}{t_0}}\left[-e^{-\frac{t_2}{t_0}}\right]_{10t_1}^\infty dt_1 = \frac{2}{t_0}\int_0^\infty e^{-\frac{t_1}{t_0}} e^{-\frac{10t_1}{t_0}}\,dt_1 =$$

$$= \frac{2}{t_0}\int_0^\infty e^{-\frac{11t_1}{t_0}}\,dt_1 = \frac{2}{11}\left[-e^{-\frac{11t_1}{t_0}}\right]_0^\infty = \frac{2}{11}$$

b) $t_0 = 20\,dB = 100$

$$P_{PL} = Pr\{P \leq 10\} = \int_0^{10} \frac{1}{t_0} e^{-\frac{t}{t_0}}\,dt = 1 - e^{-0,1} \approx 0,095$$

c) As in the calculations in (8.33) we have

$$\lambda = t \cdot Pr\{\text{successful transmission}\} = t \cdot \left((1-P_{PL}) \cdot e^{-t} + t \cdot e^{-t} \cdot P_{cap}\right) =$$

$$= te^{-t}\left(0,905 + \frac{2}{11}t\right)$$

$$\frac{d\lambda}{dt} = e^{-t}\left(0,905 + \frac{2}{11}t\right) - te^{-t}\left(0,905 + \frac{2}{11}t\right) + te^{-t} \cdot \frac{2}{11} =$$

$$= e^{-t}\left(0,905 + \frac{2}{11}t - 0,905t - \frac{2}{11}t^2 + \frac{2}{11}t\right) \approx$$

$$\approx e^{-t}\left(-0,182t^2 - 0,54t + 0,905\right) = 0 \quad \Rightarrow \quad t = -4,16 \text{ or } t = 1,1948$$

$$\lambda_{max} = \lambda(1,1948) \approx 0,406$$