

8.2 The efficiency of the stop-and-wait ARQ is calculated according to the formula:

$$\eta = \frac{K}{n} \cdot \frac{P_a}{1 + \frac{4r_c}{n}}$$

Here we have the block length $n = \frac{200}{K} + 16$ since in every block we have $\frac{200}{K}$ information bits and 16 bits introduced by the error-detecting code. Obviously $K = \frac{200}{K}$. The probability of acceptance P_a can be approximated by the probability for correctly received codeword $P_c = (1-p)^n = 0,98^n$. For the idle time t_i we have $t_i r_c = 20$ bits, where r_c is the transmission rate in bits/s. Finally the efficiency η as a function of K is

$$\eta = \frac{\frac{200}{K}}{\frac{200}{K} + 16} \cdot \frac{0,98^{\frac{200}{K} + 16}}{1 + \frac{20}{\frac{200}{K} + 16}} = \frac{50 \cdot 0,98^{\frac{200}{K} + 16}}{50 + 9K}$$

Checking the values of η for small integer K 's we have:

$$K = 5 : \eta \approx 0,1698$$

$$K = 6 : \eta \approx 0,1775$$

$$K = 7 : \eta \approx 0,1798$$

$$K = 8 : \eta \approx 0,179$$

This gives the maximum of η for $K = 7$.

8.4

For the ARQ scheme we have:

$$\text{The coded BER: } P_e = \frac{1}{2(1+p_b)} = \frac{1}{2\left(1 + \frac{45}{56} 10^{-2,3}\right)} \approx 3,1 \cdot 10^{-3}$$

$$\text{Probability of acceptance: } P_a \approx P_c = (1 - P_e)^{56} \approx 0,84$$

$$\text{Efficiency: } \eta = \frac{k}{n} \cdot \frac{P_a}{1 + \frac{t_{rec}}{n}} \approx 0,49737$$

$$\text{The message error probability (FER): } P_u \approx 1 - \sum_{i=0}^4 \binom{4}{i} p^i (1-p)^{4-i} =$$

$$= 1 - \sum_{i=0}^4 \binom{56}{i} P_e^i (1 - P_e)^{56-i} \approx 9,6 \cdot 10^{-7}$$

according to eq. (8.12)

For the FEC system we have,

$$\text{Efficiency: } \eta = \frac{k}{n} = \frac{15}{35} = \frac{3}{7} \approx 0,43$$

$$\text{The coded BER: } P_e = \frac{1}{2(1+p_b)} = \frac{1}{2\left(1 + \frac{3}{7} 10^{-2,3}\right)} \approx 5,8 \cdot 10^{-3}$$

The codeword error rate P_c :

$$P_c = \Pr\{\text{wt}(e) \geq 5\} = 1 - \sum_{i=0}^4 \binom{35}{i} P_e^i (1 - P_e)^{35-i} \approx 1,8 \cdot 10^{-6}$$

Since a message consists of 3 codewords ($\frac{45}{15} = 3$) we have the message error probability

$$P_{\text{block}} = 1 - (1 - P_c)^3 \approx 5,4 \cdot 10^{-6}$$

8.5 The BER for DPSK in Rayleigh fading is

$$P_B = \frac{1}{2(1+\beta)} = \frac{1}{2(1+100)} = \frac{1}{202} \approx 5 \cdot 10^{-3}$$

The probability for being in "bad" state is $\frac{g_1}{g_1+g_2}$, where (eq. 6.16)

$$P_B = \frac{g_1}{g_1+g_2} \cdot P_e + \frac{g_2}{g_1+g_2} \cdot P_g = \frac{g_1}{g_1+g_2} \cdot 0,05 \Rightarrow \frac{g_1}{g_1+g_2} = 10^{-1}$$

a) The probability of acceptance is:

$$\begin{aligned} P_a \approx P_c &= \text{Prob}\{\text{"bad state"}\} \cdot (1-p_e)^{100} + \text{Prob}\{\text{"good state"}\} \cdot (1-p_g)^{100} = \\ &= 0,1 (1-0,005)^{100} + 0,9 \cdot 1 \approx 0,9 \end{aligned}$$

The efficiency for the selective-repeat scheme is

$$\eta = \frac{k}{n} \cdot P_a \approx \frac{100-16}{100} \cdot 0,9 = 0,756$$

b) The probability of acceptance in this case is

$$P_a \approx P_c = (1-p_e)^{100} \approx 0,6058$$

The efficiency is:

$$\eta = \frac{k}{n} \cdot P_c \approx 0,51$$

8.6 The BER for the Rayleigh channel is (see (6.16)):

$$p = \frac{q_1}{q_1 + q_2} P_e + \frac{q_2}{q_1 + q_2} P_g = \frac{10^{-3}}{10^{-3} + 10^{-2}} \cdot 0,5 \approx 0,04545$$

a) If interleaving is used we can assume uncorrelated errors with probability $p \approx 0,04545$. We have

$$P_a \approx P_e = (1-p)^n = (1-p)^{k+20}$$

The efficiency η as a function of k is:

$$\eta = \frac{k}{k+20} \cdot P_a \approx \frac{k}{k+20} \cdot (1-p)^{k+20} = \frac{k}{k+20} \cdot 0,95455^{k+20}$$

The maximum of η is achieved for $k=13$ and $\eta_{\max} \approx 0,0849$

This give information data rate $\eta_{\max} \cdot r = 0,0849 \cdot 10^5 \approx 8,5 \cdot 10^3$ bits/s

b) The probability of acceptance without using interleaving is (see Example 8.2 on page 606)

$$P_a \approx \frac{\bar{t}_g}{\bar{t}_g + \bar{t}_e} \left(1 - \frac{n}{\bar{t}_g}\right) = \frac{q_2}{q_1 + q_2} \left(1 - q_1 (k+20)\right) =$$

$$= \frac{10}{11} \cdot \left(1 - \frac{k+20}{10^3}\right) = \frac{980-k}{11 \cdot 10^2}$$

The efficiency is

$$\eta = \frac{k}{k+20} P_a \approx \frac{k}{k+20} \cdot \frac{980-k}{11 \cdot 10^2} = \frac{(980-k)k}{1100(k+20)}$$

The maximum is achieved for $k=122$ and $\eta_{\max} \approx 0,67$

If we use the exact expression for the approximation of P_a from example 8.2 we get

$$\eta = \frac{k}{k+20} \cdot \frac{10}{11} \cdot \left(1 - \frac{1}{10^3}\right)^{k+20} = \frac{10k}{11(k+20)} \cdot 0,999^{k+20}$$

and the maximum is achieved for $k=132$ and $\eta_{\max} \approx 0,678$