6.14 The BER used is $P_b = 10^{-6}$. Let the mean SNR on each of the branches is γ_0.

a) The BER for non-coherent BFSK and maximal ratio combining with M branches is given by (5.44). In the case of $M=3$ we have

$P_b \approx \binom{2M-1}{M}\left(\frac{1}{\gamma_0}\right)^M = \binom{5}{3}\gamma_0^{-3} = 10\gamma_0^{-3}$.

The requirement $P_b = 10^{-6}$ gives $\gamma_0 \approx 10^{\frac{7}{3}} \approx 215{,}4 \approx 23{,}3\,dB$

Without diversity (i.e. $M=1$) we have

$P_b \approx \binom{2M-1}{M}\left(\frac{1}{\gamma_0}\right)^M = \gamma_0^{-1} \implies \gamma_0 \approx 10^6 = 60\,dB$

Thus the diversity gain is $60-23{,}3 = 36{,}7\,dB$

b) Without coding the BER is $P_b \approx \frac{1}{\gamma_0}$. If coding is used the SNR is decreased to $R\gamma_0 = \frac{21}{31}\gamma_0$. Thus BER for the coded bits is $P_{b,c} \approx \frac{31}{21\gamma_0} = p$. The BER after decoding can be approximated by $P_b \approx \frac{d}{n} P_{cw} = \frac{5}{31} P_{cw}$, where the codeword error probability can be bounded as

$$P_{cw} \le 1 - \sum_{e=0}^{\lfloor\frac{d-1}{2}\rfloor} p^e (1-p)^{n-e} = 1-(1-p)^{31} - 31p(1-p)^{30} - 465p^2(1-p)^{29}.$$

Applying the requirement $P_b \le 10^{-6}$ we get

$\left(1-(1-p)^{31} - 31p(1-p)^{30} - 465p^2(1-p)^{29}\right)\cdot\frac{5}{31} \le 10^{-6}$. Numerically this gives $p \approx 1{,}086.10^{-3}$ and $\gamma_0 = \frac{31}{21p} \approx 1359{,}3 \approx 31{,}3\,dB$

The coding gain is then $60-31{,}3 \approx 28{,}7\,dB$

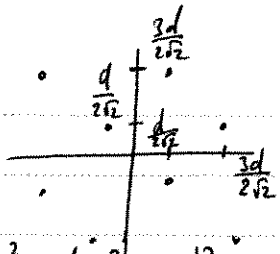
c) For MRC we have $P_b \approx 10\,(SNR)^{-3} = p$ (from a)). If coding is used the SNR is decreased to $R\gamma_0 = \frac{21}{31}\gamma_0$.

Thus $p \approx \frac{10}{(\frac{21}{31}\gamma_0)^3}$. From b), the requirement gives

$p \approx 1{,}086.10^{-3}$, which results in $\gamma_0 \approx \frac{31}{21}\sqrt[3]{\frac{10}{p}} \approx 30{,}941 \approx 14{,}9\,dB$

This gives combined gain of $60 - 14{,}9 \approx 45{,}1\,dB$.

6.17 The signal constellation is



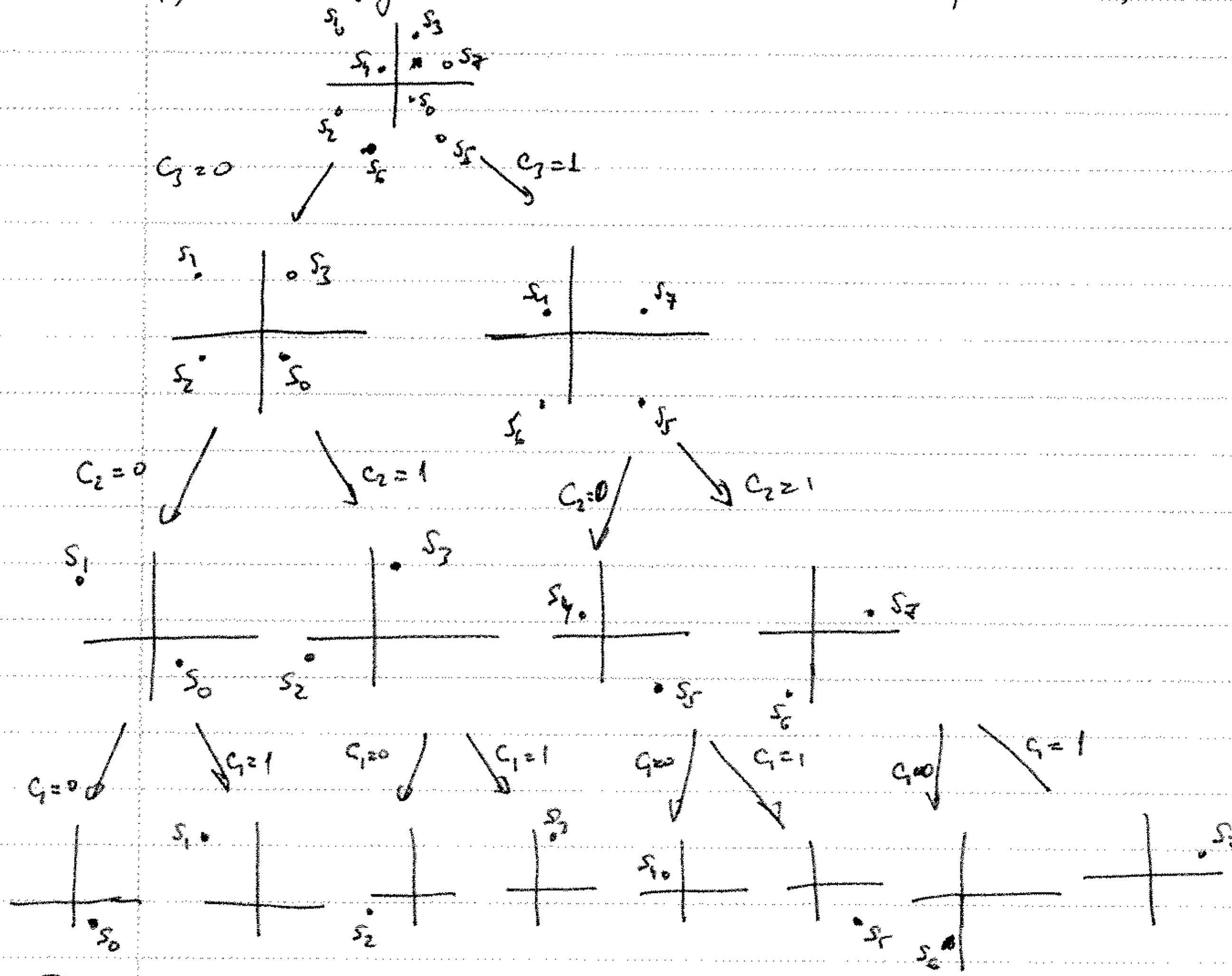
a) We have 2 points of energy $\left(\frac{d}{2\sqrt{2}}\right)^2 + \left(\frac{d}{2\sqrt{2}}\right)^2 = \frac{d^2}{4}$,

2 points of energy $\left(\frac{3d}{2\sqrt{2}}\right)^2 + \left(\frac{3d}{2\sqrt{2}}\right)^2 = \frac{9d^2}{4}$ and 4 points of energy

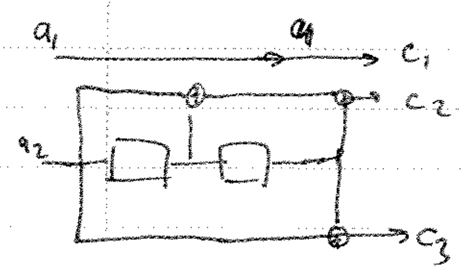
$\left(\frac{d}{2\sqrt{2}}\right)^2 + \left(\frac{3d}{2\sqrt{2}}\right)^2 = \frac{10d^2}{8} = \frac{5d^2}{4}$. Thus the average energy per symbol is

$$E_s = \frac{1}{8}\left(2\cdot\frac{d^2}{4} + 2\cdot\frac{9d^2}{4} + 4\cdot\frac{5d^2}{4}\right) = \frac{5}{4}d^2$$

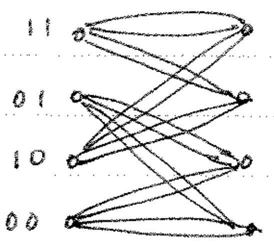
b) As in figure 6.42 we can make the partitioning



The encoder can be for example.



The trellis diagram is

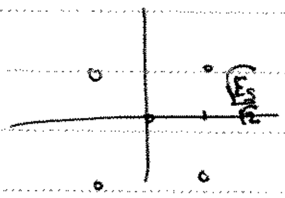


$\boxed{6.17}$ c) The minimum free distance for the TCM scheme is

$$D^2_{free} = \min\{|s_0-s_1|^2, |s_0-s_6|^2+|s_0-s_2|^2+|s_0-s_6|^2\} =$$

$$= \min\{4d^2, d^2+2d^2+d^2\} = 4d^2 = 4\cdot\frac{4}{5}E_s = \frac{16}{5}E_s$$

In the uncoded QPSK case we have the constellation

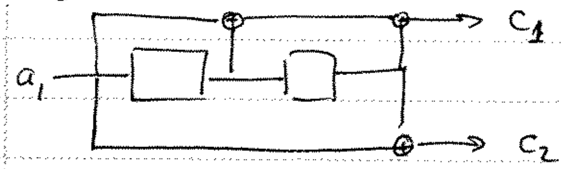


~~and~~ The minimum distance between the points is $D^2 = (\sqrt{2}\sqrt{E_s})^2 = 2E_s$

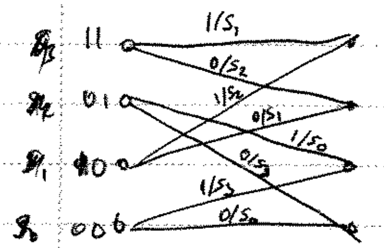
The assymptotic coding gain is the ration between those minimum squared distances, i.e.

$$\frac{D^2_{free}}{D^2} = \frac{\frac{16}{5}E_s}{2E_s} = \frac{8}{5} = 1{,}6 \approx 2\,dB.$$

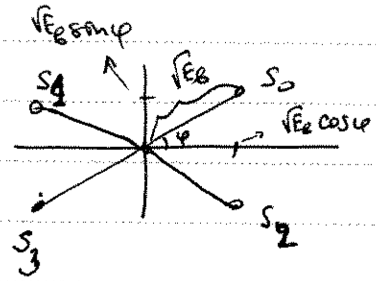
$\boxed{6.21}$ a) Let the average bit energy be E_b. We have to map one bit to a symbol point by using a rate $\frac{1}{2}$ convolutional encoder. We can use the encoder



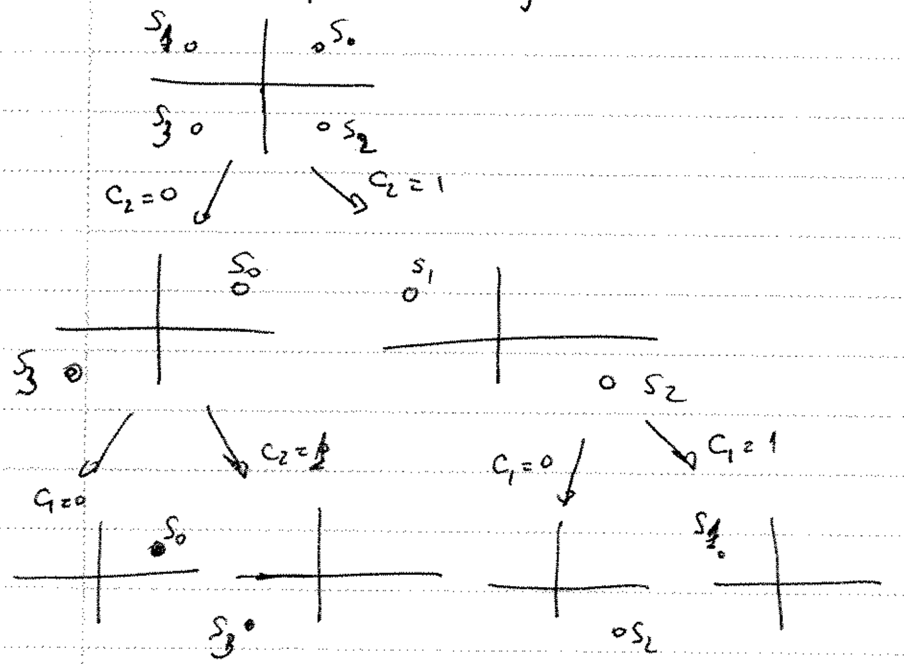
This has the trellis diagram as follows:



The constellation is



The set partitioning is



b) ~~The~~ Let us assume that $0\le\varphi\le 45°$ and that the zero message has been transmitted. To calculate the minimum squared Euclidean distance we observe that

$|s_0-s_3|^2 = 4E_b^2$, $|s_0-s_2|^2 = 4E_b\sin^2\varphi$, $|s_0-s_1|^2 = 4E_b\cos^2\varphi$ and

that the "closest" paths is either (s_3, s_1, s_3) or (s_3, s_2, s_2, s_3)

This gives

$$D_1^2 = 2|s_0 - s_3|^2 + |s_0 - s_2|^2 = E_b\,(8 + 4\sin^2\varphi) \quad \text{and}$$

$$D_2^2 = 2|s_0 - s_3|^2 + 2|s_0 - s_1|^2 = E_b\,(8 + 8\cos^2\varphi)$$

$$D_{min}^2 = \min\{D_1^2, D_2^2\}$$

c) If we consider $\varphi \in [0, \frac{\pi}{2}]$ we see that D_1^2 is increasing function, while D_2^2 is decreasing. We can maximize D_{min}^2 by finding the "crossing" point for these functions, i.e. $D_1^2 = D_2^2$.

This is equivalent to $2\cos^2\varphi = \sin^2\varphi \Rightarrow \tan\varphi = \sqrt{2} \Rightarrow$
$\Rightarrow \varphi \approx 0{,}95$ rad

<u>Remark:</u> Observe that we have denoted by φ the ~~arg~~ argument of s_0, i.e. the angle between the positive x axis and the vector s_0.