5.12 a) The probability density function of an exponentially distributed stochastic variable Γ of mean t_0 is given as:

$$p_\Gamma(t) = \frac{1}{t_0} e^{-\frac{t}{t_0}}, \quad t \geq 0, \quad \text{and } 0 \text{ otherwise}.$$

The distribution function is:

$$P_r\{\Gamma \leq t\} = P_\Gamma(t) = 1 - e^{-\frac{t}{t_0}}, \quad t \geq 0$$

Thus $P_r\{\Gamma < t_0\} = P_\Gamma(t_0) = 1 - e^{-1} \approx 0{,}63$

b) The effective rate R_{eff} is

$$R_{eff} = R \cdot (1 - P_r\{\Gamma < t_0\}) = R \cdot P_r\{\Gamma \geq t_0\} \approx 3{,}7 \text{ kbps}$$

c) If maximum ratio combining is used, the probability density function and the distribution functions of the SNR Γ are:

$$p_\Gamma(t) = \frac{1}{1!} \frac{t}{t_0^2} e^{-\frac{t}{t_0}} = \frac{t}{t_0^2} e^{-\frac{t}{t_0}}, \quad t \geq 0 \quad \text{and}$$

$$P_\Gamma(t) = 1 - \left(1 + \frac{t}{t_0}\right) e^{-\frac{t}{t_0}}, \quad t \geq 0,$$

respectively.

The effective rate is now

$$R_{eff} = R \cdot P_r\{\Gamma \geq t_0\} = 10 \cdot (1 - 2e^{-1}) \approx 7{,}36 \text{ kbps}$$

5.17 a) The BER for DBPSK modulation over AWGN channel of SNR γ is given by:

$$P_b(\gamma) = \frac{1}{2}e^{-\gamma}$$

The channel "seen" by the first antenna is an AWGN channel of SNR γ_0. Therefore the signal from antenna 1 is used whenever the instantaneous SNR γ on antenna 2 is less than γ_0. Thus

$$P_b = P_b(\gamma_0)\, P_r\{\gamma \le \gamma_0\} + \int_{\gamma_0}^{\infty} P_b(\gamma) \cdot p_r(\gamma)\, d\gamma =$$

$$= \frac{1}{2}e^{-\gamma_0} \cdot P_r(\gamma_0) + \int_{\gamma_0}^{\infty} P_b(\gamma) \cdot p_r(\gamma)\, d\gamma =$$

$$= \frac{1}{2}e^{-\gamma_0} \cdot \left(1 - e^{-\frac{\gamma_0}{\gamma_0}}\right) + \int_{\gamma_0}^{\infty} \frac{1}{2}e^{-\gamma} \cdot \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}}\, d\gamma =$$

$$= \frac{1}{2}\left(e^{-\gamma_0} - e^{-(\gamma_0+1)}\right) - \frac{1}{2\gamma_0\left(1+\frac{1}{\gamma_0}\right)} e^{-\left(1+\frac{1}{\gamma_0}\right)\gamma} \Big|_{\gamma_0}^{\infty} =$$

$$= \frac{1}{2}\left(e^{-\gamma_0} - e^{-(\gamma_0+1)}\right) + \frac{1}{2(\gamma_0+1)} \cdot e^{-(\gamma_0+1)} = \frac{1}{2e^{\gamma_0}}\left(1 - \frac{\gamma_0 e^{-1}}{\gamma_0+1}\right)$$

b) For maximum ratio combining the SNR Γ is distributed as $\Gamma = \Gamma_1 + \Gamma_2$. In this situation $\Gamma_1 = \text{const} = \gamma_0$ and Γ_2 is exponenticlly distributed with mean γ_0. Thus

$$p_r(\gamma) = \begin{cases} \frac{1}{\gamma_0} e^{-\frac{(\gamma-\gamma_0)}{\gamma_0}} & , \gamma \ge \gamma_0 \\ 0 & , \gamma < \gamma_0 \end{cases}$$

The BER in this case is

$$P_b = \int_{-\infty}^{\infty} P_b(\gamma) \cdot p_r(\gamma)\, d\gamma = \int_{\gamma_0}^{\infty} \frac{1}{2}e^{-\gamma} \frac{e}{\gamma_0} \cdot e^{-\frac{\gamma}{\gamma_0}}\, d\gamma = -\frac{e}{2\gamma_0\left(1+\frac{1}{\gamma_0}\right)} \cdot e^{-\left(1+\frac{1}{\gamma_0}\right)\gamma}\Big|_{\gamma_0}^{\infty} =$$

$$= \frac{e^{-\gamma_0}}{2(\gamma_0+1)} = \frac{1}{2e^{\gamma_0}(\gamma_0+1)}$$

c) When $\gamma_0 = 10\,dB = 10$, we have

$$P_{b,SC} = \frac{1}{2 \cdot e^{10}}\left(1 - \frac{10e^{-1}}{11}\right) \approx 1{,}51 \cdot 10^{-5}$$

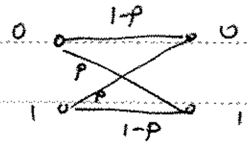
$$P_{b,MRC} = \frac{1}{2 \cdot e^{10} \cdot 11} \approx 2{,}0636 \cdot 10^{-6}$$

6.8 The rate of the code used is $R = \frac{7}{15}$. The average SNR per transmitted bit is then

$$\gamma_b = R \cdot \gamma_b' = \frac{7}{15} \cdot 10^{1,6} \approx 18{,}578 \approx 12{,}69\,\text{dB}$$

For coherent BPSK, the BER is (eq. 4.138)

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right) \approx 0{,}01294 = p$$



The code is able to correct all possible combinations of at most 2 errors. Thus

$$P_{cw} \leq 1 - \sum_{k=0}^{2} \binom{15}{k} p^k (1-p)^{15-k} = 1-(1-p)^{15} - 15p(1-p)^{14} - 105p^2(1-p)^{13} \approx$$

$$\approx 8{,}7673 \cdot 10^{-4}$$

Using equation (6.51) we can ~~~~ approximate the average BER after coding as

$$P_b \approx \frac{d}{n} P_{cw} = \frac{1}{3} \cdot P_{cw} \approx 2{,}9224 \cdot 10^{-4}$$

$\boxed{6.12}$ a) We have to calculate the transition probabilities q_1 and q_2. According to (6.16) the BER is

$$p = \frac{q_1}{q_1+q_2} p_B + \frac{q_2}{q_1+q_2} p_G = \frac{q_1}{2(q_1+q_2)} = 10^{-2}$$

According to (6.14) the average time spent in bad state (burst error) is

$$\tau_B = \frac{1}{q_2} = 50 \implies q_2 = \frac{1}{50} = 0{,}02$$

Thus $\frac{q_1}{2q_1+0{,}04} = 10^{-2} \implies 0{,}98 q_1 = 4 \cdot 10^{-4} \implies q_1 \approx 4{,}08 \cdot 10^{-4}$

b) A (31, 21, 5) code can correct upp to 2 errors. If the code is to be "effective" no more than 2 positions of every "burst" should be from the same codeword. If a block interleaver of depth ℓ is to be used, that is

$$2 \geq \frac{\tau_B}{\ell} \implies \ell \geq \frac{\tau_B}{2} = 25$$

c) When ℓ is large we can "assume "full interleaving" conditions. The transition probability p for the BSC model is $p = 10^{-2}$. As in Problem 6.8 we calculated the BER with coding as

$$P_B \approx \frac{d}{n} P_{cw} \leq \frac{5}{31}\left(1 - \sum_{k=0}^{2} \binom{31}{k} p^k (1-p)^{31-k}\right) =$$

$$= \frac{5}{31}\left(1 - (1-p)^{31} - 31p(1-p)^{30} - 465p^2(1-p)^{29}\right) \approx$$

$$\approx \frac{5}{31} \cdot 3{,}65 \cdot 10^{-3} \approx 5{,}88 \cdot 10^{-4}$$