4.28 The baseband (LP) OFDM signal is given as

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{N-1} \tilde{s}_k[n] \cdot g_k(t - nT_t) \quad , \quad T_t = T_g + T$$

Assuming this representation, the PDS of the baseband signal is given as

$$\Phi_{\tilde{s}}(f) = \sum_{k=0}^{N-1} \Phi_{\tilde{s}_k}(f) = \sum_{k=0}^{N-1} \frac{E_S}{T_t} |G_k(f)|^2 ,$$

where $G_k(f) = \mathcal{F}\{g_k(t)\}$. In the case of BPSK modulation and the presence of guard intherval we have:

$$g_k(t) = \frac{1}{\sqrt{T_t}} \left(u(t+T_g) - u(t-T) \right) e^{j2\pi \frac{k}{T} t}$$

Thus

$$G_k(f) = \sqrt{T_t} \cdot e^{j\pi (T-T_g) f} \cdot \operatorname{sinc}\left(\left(f - \frac{k}{T}\right) T_t \right)$$

and the PDS is

$$\Phi_{\tilde{s}}(f) = \sum_{k=0}^{N-1} E_S \operatorname{sinc}^2 \left(f(T+T_g) - k\left(1 + \frac{T_g}{T}\right) \right)$$

4.31 The OFDM symbol for the described scheme can be written (for $0 \le t \le T$) as:

$$\sum_{k=0}^{\frac{N}{2}-1} \tilde{S}_k[0]\, g_k(t) = \sum_{k=0}^{\frac{N}{2}-1} \tilde{S}_k[0]\, A \sin\left(\frac{\pi t}{T}\right) e^{j2\pi\left(\frac{1}{2T}+\frac{2k}{T}\right)t},$$

since $g_0(t) = A \sin\left(\frac{2\pi t}{2T}\right) e^{j2\pi\left(\frac{1}{2T}\right)t} = A\sin(2\pi f_0 t).\, e^{j2\pi f_0 t}$

and the subcarrier separation is $\Delta f = \frac{2}{T}$, thus $f_k = f_0 + k\Delta f$.

The expression above can be further developed as:

$$\sum_{k=0}^{\frac{N}{2}-1} \tilde{S}_k[0]\, g_k(t) = \sum_{k=0}^{\frac{N}{2}-1} \tilde{S}_k[0]\, A \left(\frac{e^{j\frac{\pi t}{T}} - e^{-j\frac{\pi t}{T}}}{2j}\right). e^{j2\pi\left(\frac{1}{2T}+\frac{2k}{T}\right)t} =$$

$$= \sum_{k=0}^{\frac{N}{2}-1} \frac{A}{2j} \tilde{S}_k[0] \left(e^{j2\pi\left(\frac{1}{T}+\frac{2k}{T}\right)t} - e^{j2\pi\frac{2k}{T}t}\right) =$$

$$= \sum_{k=0}^{N-1} \tilde{\ell}_k[0].\, e^{j2\pi\frac{k}{T}t}$$

where $$\tilde{\ell}_k[0] = \begin{cases} \frac{A}{2j} \tilde{S}_{\frac{k-1}{2}}[0], & \text{if } k \text{ is odd} \\ -\frac{A}{2j} \tilde{S}_{\frac{k}{2}}[0], & \text{if } k \text{ is even} \end{cases}$$

This shows that the described system is equivalent to OFDM modulation with N subcarriers and underlying PSK or QAM modulation, which is given in 4.28.

$\boxed{4.33}$ a) ISI is avoided if there is no overlap between the received signals from the two different paths. Thus the GI T_{GI} should be:

$$T_{GI} \geq \tau_1 - \tau_1 = 200\ ns = 2.10^{-7} s.$$

b) The wasted power is $\frac{T_{GI}}{T_t} < 0{,}01$.

The transmission rate is

$$R = \frac{1}{T_t} \frac{\text{OFDM symbols}}{s} = \frac{2N}{T_t}\ bits/s = 10^8\ \frac{bits}{s}$$

Thus

$$2.10^{-7} \leq 10^{-2} \cdot T_t = 2.10^{-10} N \Rightarrow N \geq 10^3.$$

c) For coherent QPSK on fading channels we have,

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right)$$

The mean SNR γ_0 is calculated as:

$$\gamma_0 = 2\sigma^2 \alpha_g \frac{E_b}{N_0}, \text{ where}$$

$$2\sigma^2 = \sum_{i=0}^{1} \overline{|h_i|^2} = 1 \quad \text{and} \quad \alpha_g = \frac{T}{T+T_{GI}} = \frac{T}{T_t}$$

4.34 a) As in 4.33 we must have $T_{GI} \geq 1{,}5 . T_b - 0 = 1{,}5 T_b$

The OFDM symbol duration (without GI) is

$T = N . T_b = 32 T_b$

The power loss in dB is

$$10 \log_{10} \left(\frac{T + T_{GI}}{T} \right) = 10 \log_{10} \left(\frac{33{,}5}{32} \right) \approx 0{,}19895 \text{ dB}$$

b) The average BER calculated for one OFDM symbol is

$$P_b = \frac{1}{32} \cdot \sum_{k=0}^{31} Q \left(\sqrt{\frac{2 E_b^{(k)}}{N_0}} \right),$$

where $E_b^{(k)}$ is the received signal energy for the k-th subcarrier (on frequency $f_k = \frac{k}{T}$)

The channels transfer function is

$$H_e(f) = \mathcal{F}\{h_e(t)\} = 0{,}7 - 0{,}25\, e^{-j 2\pi f \frac{T_b}{2}} + 0{,}669\, e^{-j 2\pi f \frac{3 T_b}{2}}$$

The energy gain due to the introduction of the GI is

$$\alpha_g = \frac{T}{T + T_{GI}} = \frac{32 T_b}{33{,}5 T_b} \approx 0{,}95522$$

The energy gain on the k-th subcarrier is

$$\left| H_e\left(\frac{k}{T}\right) \right|^2 = \left| 0{,}7 - 0{,}25 . e^{-j\pi \frac{k}{32}} + 0{,}669 e^{-j\pi \frac{3k}{32}} \right|^2 = H_k$$

Thus $E_b^{(k)} = \alpha_g . H_k . E_b$ and

$$P_b = \frac{1}{32} \sum_{k=0}^{31} Q \left(\sqrt{\frac{2 \alpha_g H_k . E_b}{N_0}} \right)$$

c) For maximum ratio combining we add the received signal energy and thus the average BER is

$$P_b = \frac{1}{16} \sum_{k=0}^{16} Q \left(\sqrt{\frac{2 \alpha_g (H_k + H_{k+16}) E_b}{N_0}} \right)$$

$\boxed{4.36}$ For the SNR at the receiver we have

$$SNR = \begin{cases} \frac{E_b}{N_0}, & \text{on non-jammed channel (probability } 1-\frac{K}{N}) \\ \frac{E_b}{N_0 + \frac{10}{N}\frac{E_b}{K/N}}, & \text{on jammed channel (prob. } \frac{K}{N}) \end{cases}$$

Define $q = \frac{K}{N}$. The BER for DPSK modulation is $P_{b,DPSK} = \frac{1}{2} e^{-SNR}$. Thus the expected BER with jamming is:

$$P_b(q) = \left(1 - \frac{K}{N}\right) \cdot \frac{1}{2} e^{-\frac{E_b}{N_0}} + \frac{K}{N}\frac{1}{2} e^{-\frac{E_b}{N_0 + \frac{10}{N}\frac{E_b}{K/N}}} =$$

$$= \frac{1}{2}(1-q) e^{-\frac{E_b}{N_0}} + \frac{1}{2} q e^{-\frac{E_b/N_0}{1 + \frac{10 E_b/N_0}{Nq}}}$$

For the derivative we have

$$\frac{dP_b(q)}{dq} = \frac{1}{2} e^{-\frac{E_b}{N_0}} + \frac{1}{2} e^{-\frac{E_b/N_0}{1+\frac{10 E_b/N_0}{Nq}}} + \frac{1}{2} q e^{\left(-\frac{E_b/N_0}{1 + \frac{10 E_b/N_0}{Nq}}\right)} \cdot \frac{-10(E_b/N_0)^2}{(Nq + 10 E_b/N_0)^2}$$

However exact solution to $\frac{dP_b(q)}{dq} = 0$ is difficult to find in this case. Therefore we approximate the $P_b(q)$ be neglecting the thermal noise (N_0) in the "jamming" term, i.e.

$$P_b(q) = \frac{1}{2}(1-q)e^{-t} + \frac{1}{2} q e^{-\frac{Nq}{10}} \approx \frac{1}{2} q e^{-10q}$$

The maximum value is achieved for $q = \frac{1}{10}$, i.e. $K = 10$, and

$$P_{b,max} = P_b\left(\frac{1}{10}\right) \approx \frac{1}{20} e^{-1} \approx 1{,}8 \cdot 10^{-2}.$$

4.37 a) For the received signal we have:

$$r(t) = \alpha_1 s(t-\tau_1) + \alpha_2 s(t-\tau_2) = \alpha_1 \operatorname{Re}\{\tilde{s}(t-\tau_1) e^{j2\pi f_c (t-\tau_1)}\} + \alpha_2 \operatorname{Re}\{\tilde{s}(t-\tau_2) e^{j2\pi f_c (t-\tau_2)}\} =$$

$$= \operatorname{Re}\{(\alpha_1 \tilde{s}(t-\tau_1) e^{-j2\pi f_c \tau_1} + \alpha_2 \tilde{s}(t-\tau_2) e^{-j2\pi f_c \tau_2}) \cdot e^{j2\pi f_c t}\}$$

Thus

$$\tilde{r}(t) = \alpha_1 \tilde{s}(t-\tau_1) e^{-j2\pi f_c \tau_1} + \alpha_2 \tilde{s}(t-\tau_2) e^{-j2\pi f_c \tau_2}$$

Here $\tau_1 = \frac{2 \cdot 10^3}{3 \cdot 10^8}$ s and $\tau_2 = \frac{R + \sqrt{2000^2 + R^2}}{3 \cdot 10^8}$ s.

b) The chip duration time is $T_c = \frac{1}{3{,}84 \cdot 10^6}$ s

The condition is equivalent to:

$$T_c \leq \tau_2 - \tau_1$$

Thus

$$\frac{1}{3{,}84 \cdot 10^6} \leq -\frac{2}{3} \cdot 10^{-5} + \frac{R}{3 \cdot 10^8} + \frac{\sqrt{2000^2 + R^2}}{3 \cdot 10^8} \approx \frac{R}{3 \cdot 10^8}$$

if $R \ll 2000$m

This gives

$$R \gtrsim \frac{3 \cdot 10^8}{3{,}84 \cdot 10^6} \approx 84 \text{ m}$$