$\boxed{3.18}$ The parameters for the link-budget calculations are:

- distance $R = 7{,}5 . 10^9 \text{ km} = 7{,}5 . 10^{12} \text{ m}$
- frequency $f = 2 \text{ GHz} = 2 . 10^9 \text{ Hz}$
- transmitter's power $P_T = 10 \text{ W}$
- receiver's antenna diameter $D_R = 64 \text{ m}$
- receiver's noise temperature $T_R = 16 \text{ K}$
- Both antennas' efficiency $\eta = 0{,}5$
- data rate of the transmission $r = 300 \frac{\text{bits}}{\text{s}}$
- required SNR : $E_b/N_0 \geq 9{,}88 \text{ dB}$

Combining equations (3.13) and (3.18) we obtain the antenna gain for a parabolic antenna to be,

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi}{\lambda^2} \cdot \eta \cdot \frac{\pi D^2}{4} = \frac{\eta \pi^2 D^2}{\lambda^2} = \frac{\eta \pi^2 D^2 f^2}{c^2} = \frac{\eta \cdot \pi^2 D^2 f^2}{9 . 10^{16}}$$

Thus we obtain the gains:

$$G_R = \frac{\eta \pi^2 D_R^2 f^2}{c^2} = \frac{0.5 . \pi^2 . 64^2 . 4 . 10^{18}}{9 . 10^{16}} \approx 8{,}9835 . 10^5 \Rightarrow [G_R]_{dB} \approx 59{,}5 \text{ dB}$$

$$G_T = \frac{\eta \pi^2 D_T^2 f^2}{c^2} = \frac{0{,}5 \pi^2 D_p^2 . 4 . 10^{18}}{9 . 10^{16}} \approx 219{,}32 . D_p^2 \Rightarrow [G_T]_{dB} \approx 23{,}4 \text{ dB} + 2 [D_p]_{dB}$$

The received carrier energy is $C = r . E_b = 300 . E_b$

The link-budget equation can be written as

$$[C/N_0]_{dB} = [P_T]_{dB} + [G_T]_{dB} + [G_R]_{dB} - [T_R]_{dB} - [L_{FS}]_{dB} - [K]_{dB}$$

Here we need also:

By eq. (2.11): - the free space loss : $L_{FS} = \frac{(4\pi R)^2}{\lambda^2} = \frac{9\pi^2 . 10^{26} . 4 . 10^{18}}{9 . 10^{16}} \approx 3{,}9478 . 10^{29} \Rightarrow [L_{FS}]_{dB} \approx 295{,}96 \text{ dB}$

- the Boltzmanns constant : $k = 1{,}38 \times 10^{-23} \Rightarrow [k]_{dB} = -228{,}6 \text{ dB}$
- receiver's noise temperature : $T_R = 16 \text{K} \Rightarrow [T_R]_{dB} \approx 12{,}04 \text{ dB}$
- $[C/N_0]_{dB} = [300 E_b/N_0]_{dB} \approx 24{,}77 + [E_b/N_0]_{dB}$; $P_T = 10 \text{W} \Rightarrow [P_T]_{dB} = 10$

Thus:

$$[E_b/N_0]_{dB} = -24{,}77 + 10 + 23{,}4 + 2 [D_p]_{dB} + 59{,}5 - 12{,}04 - 295{,}96 + 228{,}6$$

$$\Rightarrow [D_p]_{dB} = \frac{1}{2} \left([E_b/N_0]_{dB} + 11{,}27 \right) \geq \frac{1}{2} (9{,}88 + 11{,}27) = 10{,}575$$

$$\Rightarrow D_p \geq 10^{\frac{10{,}575}{10}} \approx \underline{11{,}416 \text{ m}}$$

3.19 The parameters are:

- frequency $f = 2 \cdot 10^9 Hz$, thus wavelength $\lambda = \frac{c}{f} = 1{,}5 \cdot 10^{-1} m$
- transmit power $P_T = 1W \Rightarrow [P_T]_{dB} = 0\, dB$
- transmit antenna gain (dipole) $[G_T]_{dB} = 2{,}1\, dB$ (see eq. (3.14))
- receive antenna diameter $D_R = 1m$ (parabolic)
- receiver noise factor $[F_R]_{dB} = 8_{dB} \Rightarrow F_R \approx 6{,}3096$
- receiver's feeder: 25m cable with attenuation 20dB/100m
- required signal-to-noise ratio: $[C/N]_{dB} \geq 20dB$
- transmission's bandwidth: $B = 0{,}5 MHz = 5 \cdot 10^5 Hz \Rightarrow [B]_{dB} \approx 57 dB$
- receive antenna efficiency: $\eta_R = 0{,}55$
- Boltzmanns constant: $k = 1{,}38 \cdot 10^{-23} J/K \Rightarrow [k]_{dB} = -228{,}6\, dB$

We need to calculate:

- the receiver's feeder loss: $[L_{RF}]_{dB} = \frac{25}{100} \cdot 20\, dB = 5dB$
- free space loss: $[L_{FS}]_{dB} = [(\frac{4\pi R}{\lambda})^2]_{dB} \approx 38{,}46 + 2[R]_{dB}$
- receive antenna gain: $G_R = \frac{\eta \pi^2 D_R^2}{\lambda^2} \approx 241{,}26 \Rightarrow [G_R]_{dB} \approx 23{,}83$
- receiver's noise temperature: $T_R = F_R \cdot T_0 = 290 \cdot F_R \approx 1829{,}8 \Rightarrow [T_R]_{dB} \approx 32{,}62\, dB$

Now the link budget equation can be written as:

$$[C/N]_{dB} = [P_T]_{dB} + [G_T]_{dB} + [G_R]_{dB} - [T_R]_{dB} - [L_{FS}]_{dB} - [L_{RF}]_{dB} - [B]_{dB} - [k]_{dB} =$$

$$= 0 + 2{,}1 + 23{,}83 - 32{,}62 - 38{,}46 - 2[R]_{dB} - 5 - 57 + 228{,}6 =$$

$$= 121{,}45 - 2[R]_{dB}$$

Thus

$$[R]_{dB} = \frac{1}{2}(121{,}45 - [C/N]_{dB}) \leq 50{,}725 \Rightarrow$$

$$\Rightarrow R \leq 10^{5{,}0725} m \approx 1{,}1817 \cdot 10^5 m \approx 118\, km$$

$\boxed{3.20}$ The link-budget calculation parameters are:

- distance $R = 50\,km = 5.10^4\,m$
- signal bandwidth $B = 25\,kHz = 2{,}5.10^4\,Hz \Rightarrow [B]_{dB} \approx 43{,}98$
- frequency $f = 4\,GHz = 4.10^9\,Hz$
- receive antenna gain $[G_R]_{dB} = 2{,}2\,dB$
- receiver noise factor $[F_R]_{dB} = 11\,dB$
- required SNR : $[C/N]_{dB} \geq 25\,dB$
- transmit antenna gain $[G_T]_{dB} = 8\,dB$
- receiver's feeder loss: $[L_{RF}]_{dB} = 9\,dB$

We need to calculate:

- receiver's noise temperature: $[T_R]_{dB} = [F_R \cdot T_0]_{dB} = [F_R]_{dB} + [T_0]_{dB} =$
  $\approx 11 + 24{,}62 = 35{,}62$
- free space loss: $[L_{FS}]_{dB} = \left[\left(\frac{4\pi R f}{c}\right)^2\right]_{dB} = 2\left[\frac{4\pi 5.10^4 . 4.10^9}{3.10^8}\right]_{dB} =$
  $= 2\left[\frac{8.10^6 \pi}{3}\right]_{dB} \approx 138{,}46$

The link-budget equation is now written as:

$$[C/N]_{dB} = [P_T]_{dB} + [G_T]_{dB} + [G_R]_{dB} - [T_R]_{dB} - [L_{FS}]_{dB} - [L_{RF}]_{dB} - [B]_{dB} - [k]_{dB} =$$

$$= [P_T]_{dB} + 8 + 2{,}2 - 35{,}62 - 138{,}46 - 9 - 43{,}98 + 228{,}6 =$$

$$= [P_T]_{dB} + 11{,}74$$

Thus

$$[P_T]_{dB} = [C/N]_{dB} - 11{,}74 \geq 13{,}26 \Rightarrow$$

$$\Rightarrow P_T \geq 10^{1{,}326} \approx 21{,}18\,W$$

3.21 We use the plane-earth model equations. The parameters are:

- transmit power $P_T = 25W \Rightarrow [P_T]_{dB} \approx 13{,}98 dB$
- frequency $f = 900 MHz = 9.10^8 Hz$ (wavelength $\lambda = \frac{1}{3} m$)
- transmit antenna height : $h_T = 50m$
- transmit antenna gain $[G_T]_{dB} = 8dBd = (8+2{,}1)\, dB = 10{,}1 dB$
- distance : $R = 10 km = 10^4 m$
- receiver's noise factor : $[F_R]_{dB} = 5dB$
- receive antenna height : $h_R = 3m$
- transmission bandwidth : $25 kHz = 2{,}5.10^4 Hz \Rightarrow [B]_{dB} \approx 43{,}98 dB$
- receive antenna gain (dipole) : $[G_R]_{dB} = 2{,}1 dB$

a) Since $2\pi h_1 h_2 \approx 942 << \frac{10^4}{3} = \lambda . R$, we can calculate the loss as

$$L_{PE} \approx \frac{R^4}{(h_1 h_2)^2} = \frac{10^{16}}{2{,}25.10^4} \approx 4{,}44.10^{11} \Rightarrow [L_{PE}]_{dB} \approx 116{,}48 dB$$

The receiver's noise temperature is $[T_R]_{dB} = [F_R]_{dB} + [T_0]_{dB} = 29{,}62 dB$

The link-budget equation takes the form:

$$[C/N]_{dB} = [P_T]_{dB} + [G_T]_{dB} + [G_R]_{dB} - [T_R]_{dB} - [L_{PE}]_{dB} - [B]_{dB} - [K]_{dB} =$$

$$= 13{,}98 + 10{,}1 + 2{,}1 - 29{,}62 - 116{,}48 - 43{,}98 + 228{,}6 = 64{,}7 dB$$

b) The loss is influenced and we can again use the approximation to get

$$L_{PE} \approx \frac{R^4}{(h_1 h_2)^2} = \frac{10^{16}}{10^4} = 10^{12} \Rightarrow [L_{PE}]_{dB} = 120 dB$$

Thus $[C/N]_{dB} = \dots = 64{,}7 - 3{,}52 = 61{,}18 dB$

c) Again

$$L_{PE} \approx \frac{R^4}{(h_1 h_2)^2} = \frac{10^{16}}{75^2} \approx 1{,}78.10^{12} \Rightarrow [L_{PE}]_{dB} \approx 122{,}5 dB$$

Thus $[C/N]_{dB} = \dots = 61{,}18 - 2{,}5 = 58{,}68 dB$

$\boxed{3.24}$ From the graph we can read out the coordinates of two points on the curve (line) to be
$(x_1, y_1) = (5, 46)$ and $(x_2, y_2) = (40, 68)$.
Denote by d the distance in meter and by L the path loss (linear)

a) The scale on the d axis is logarithmic, while the scale on the path loss axis is linear but for the logarithmic path loss ($[L]_{dB} = 10\log_{10}(L)$). The curve is actually a line and can be represented by the function $f(x) = ax + b$. To find a and b we observe that
$10\log_{10}(L_1) = y_1 = 46 = f(x_1) = ax_1 + b = a\,10\log_{10}(d_1) + b = 10a\log_{10}(5) + b$
~~and~~ and $68 = 10a\log_{10}(40) + b$. From this system of linear equations we obtain
$$a = \frac{2{,}2}{\log_{10}(8)} \approx 2{,}44 \quad \text{and} \quad b = 46 - 24{,}4\log_{10}(5) \approx 28{,}95$$
Thus the path loss is
$[L]_{dB} \approx 28{,}95 + 2{,}44 \cdot [d]_{dB}$ and in linear units this is
$\underline{L \approx 785 \cdot d^{2{,}44}}$
The free-space propagation loss for the frequency 900 MHz is
$[L]_{dB} = \left[\left(\frac{4\pi d}{\lambda}\right)^2\right]_{dB} = \left[\left(\frac{4\pi f}{c}\right)^2\right]_{dB} + 2[d]_{dB} \approx 31{,}53 + 2[d]_{dB}$
The slopes are 2,44 and 2, respectively

b) The power density S_R is on one hand
$S_R = \frac{|E|^2}{Z_0} = \frac{|E|^2}{120\pi}$, where $|E|$ is the field strength, and
$S_R = \frac{EIRP}{L}$, where L is representing the losses.
Thus $\frac{|E|^2}{120\pi} = \frac{EIRP}{L}$, which in logarithmic units is
$[L]_{dB} = [EIRP]_{dB} + 10\log_{10}(120\pi) - 2[|E|]_{dB} \leq -20 + 25{,}763 + 82{,}5 \approx 88{,}26$
Thus $[L]_{dB} = 28{,}95 + 2{,}44[d]_{dB} \leq 88{,}26 \implies [d]_{dB} \leq 24{,}3 \implies \boxed{d \lesssim 269{,}15\,m}$
We used that $[EIRP]_{dB} = 10\log_{10}(0{,}01) = -20$ and
$[|E|]_{dB} = 10\log(75 \cdot 10^{-6}) \approx -41{,}25$.

$\boxed{3.29}$ The noise power at the receiver is

$N = KT_R B = KF_R T_0 B \Rightarrow [N]_{dB} = [KT_0]_{dB} + [F_R]_{dB} + [B]_{dB}$

Here we have $[KT_0]_{dB} = [1.38 \cdot 10^{-23} \cdot 290]_{dB} \approx -204 dB$

$[F_R]_{dB} = 6dB$ and $[B]_{dB} = [2{,}5 \cdot 10^4]_{dB} \approx 44 dB$

Thus $[N]_{dB} \approx -154 dB$

The SNR is $[C/N]_{dB} = [C]_{dB} - [N]_{dB}$ and thus the required received signal power is

$[P_R]_{dB} = [C]_{dB} = [C/N]_{dB} + [N]_{dB} \geq -139 dB$

a) The received power at distance $d \leq 1km$ is

$[P_R]_{dB} = [P_0]_{dB} - 3[d/d_0]_{dB} = -30 + 3[d]_{dB}$

In particular the received power at distance 1km $[P_1]_{dB}$ is

$[P_1]_{dB} = -30 - 90 = -120 dB$

The received power at distance $d \geq 1km$ is

$[P_R]_{dB} = -120 - 4[d/1000]_{dB}$. Now the requirement can be written as

$-120 - 4[d/1000]_{dB} \geq -139 \Rightarrow [d]_{dB} - 30 \leq \frac{19}{4} \Rightarrow [d]_{dB} \leq 34{,}75 \Rightarrow \boxed{d \leq 2985m}$

b) The distribution of the logarithmic power $[P]_{dB}$ at the receiver is normal with deviation $\sigma = 8$ and mean $[P_R]$ as calculated above. The stochastic variable $X = \frac{[P]_{dB} - [P_R]_{dB}}{\sigma}$ is $N(0,1)$ distributed. The probability the $X \geq x$ is

$P_r\{X \geq x\} = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt = Q(x)$

It is tabulated the $Q(-2{,}33) \approx 0{,}99 = 99\%$

Thus we must hav

$\frac{[P]_{dB} - [P_R]_{dB}}{\sigma} \leq -2{,}33 \Rightarrow [P]_{dB} \leq [P_R]_{dB} - 2{,}33\sigma$

We have the requirement $[P]_{dB} \geq -139dB$ and thus

$[P_R]_{dB} \geq -139 + 2{,}33\sigma = -120{,}36$

In this case we have $d \geq 1km$ and

$[P_R]_{dB} = -120 - 4[d/1000]_{dB} \geq -120{,}36 \Rightarrow [d]_{dB} - 30 \leq 0{,}09 \Rightarrow [d]_{dB} \leq 30{,}09 \Rightarrow$

$\Rightarrow \boxed{d \leq 1020{,}9 m}$