$\boxed{3.5}$ Assumptions: $S_h(\tau;\psi) = \begin{cases} K, & \text{if } |\tau| \le 10^{-3} \text{ and } |\psi| \le 0.1 \\ 0, & \text{otherwise} \end{cases}$

a) Delay spread (T_m): The shortest interval where $\varphi_h(\tau)$ is nonzero.

$$\varphi_h(\tau) = \varphi_h(\tau;0) = \int_{-\infty}^{\infty} S_h(\tau;\psi)\, d\psi = \begin{cases} 0{,}2K, & \text{if } |\tau| \le 10^{-3} \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow T_m = 2 \cdot 10^{-3}\,s = 2ms$$

Coherence bandwidth (B_m): $B_m \approx \frac{1}{T_m} = 500\,Hz$

Doppler spread (B_d): The shortest interval where $S_H(\psi) = S_H(0;\psi)$ is nonzero.

$$S_H(\psi) = S_H(0;\psi) = \int_{-\infty}^{\infty} S_h(\tau;\psi)\, d\tau = \begin{cases} 2 \cdot 10^{-3} K, & \text{if } |\psi| \le 10^{-1} \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow B_d = 2 \cdot 10^{-1}\,Hz = 0{,}2\,Hz$$

Coherence time (T_d): $T_d \approx \frac{1}{B_d} = 5s$

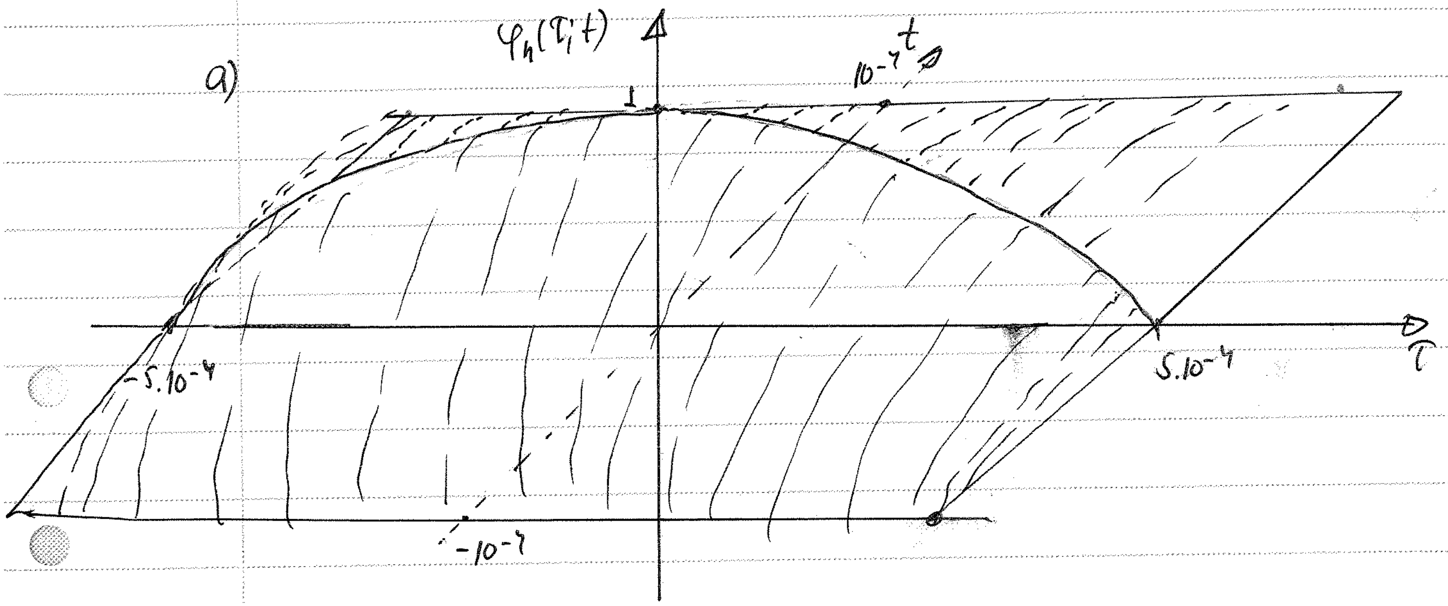
Scattering factor ($T_m B_d$): $T_m B_d = 2 \cdot 10^{-3} \cdot 2 \cdot 10^{-1}\,Hz \cdot s = 4 \cdot 10^{-4}$

b) The channel is not frequency selective in a bandwidth W if $W < B_m = 500\,Hz$. This means that signals of bandwidth less than $0{,}5\,kHz$ will pass through a frequency nonselective channel.

The channel is slowly fading with a coherence time $T_d = 5s$. This means that if the symbol duration of a signal is less than $5s$ then it will pass through a time-invariant channel.

3.6 The impulse response is $\varphi_h(\tau;t) = \begin{cases} \cos(10^3\pi\tau)(1-10^4|t|), & \text{if } |\tau|<5\cdot10^{-4}, |t|<10 \\ 0, & \text{otherwise} \end{cases}$

a)



b) $$\Phi_h(f;t) = \mathcal{F}_\tau\{\varphi_h(\tau;t)\} = \int_{-\infty}^{\infty} \varphi_h(\tau;t)\, e^{-j2\pi\tau f}\, d\tau =$$

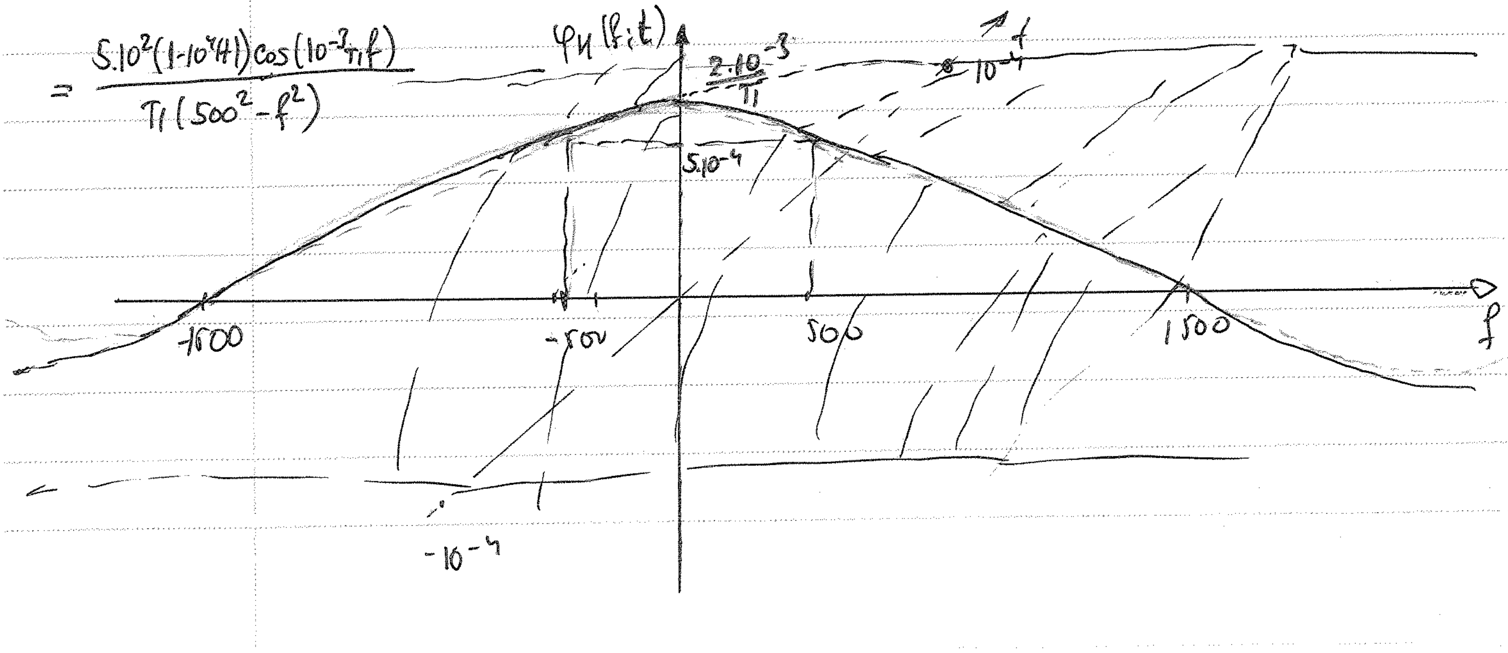
$$= \int_{-5\cdot10^{-4}}^{5\cdot10^{-4}} (1-10^4|t|)\cos(10^3\pi\tau)\cdot e^{-j2\pi\tau f}\, d\tau = (1-10^4|t|)\int_{-5\cdot10^{-4}}^{5\cdot10^{-4}} \frac{e^{-j2\pi\tau(f-500)}+e^{-j2\pi\tau(f+500)}}{2}\, d\tau$$

$$= (1-10^4|t|)\left(\frac{e^{-j2\pi\tau(f-500)}}{-j4\pi(f-500)} + \frac{e^{-j2\pi\tau(f+500)}}{-j4\pi(f+500)}\right)\Bigg|_{-5\cdot10^{-4}}^{5\cdot10^{-4}} =$$

$$= (1-10^4|t|)\cdot\left(\frac{\sin(\pi10^{-3}(f-500))}{2\pi(f-500)} + \frac{\sin(\pi10^{-3}(f+500))}{2\pi(f+500)}\right) =$$

$$= 5\cdot10^{-4}(1-10^4|t|)\left(\text{sinc}(10^{-3}(f-500)) + \text{sinc}(10^{-3}(f+500))\right)$$

$$= \frac{5\cdot10^{2}(1-10^4|t|)\cos(10^{-3}\pi f)}{\pi(500^2-f^2)}$$

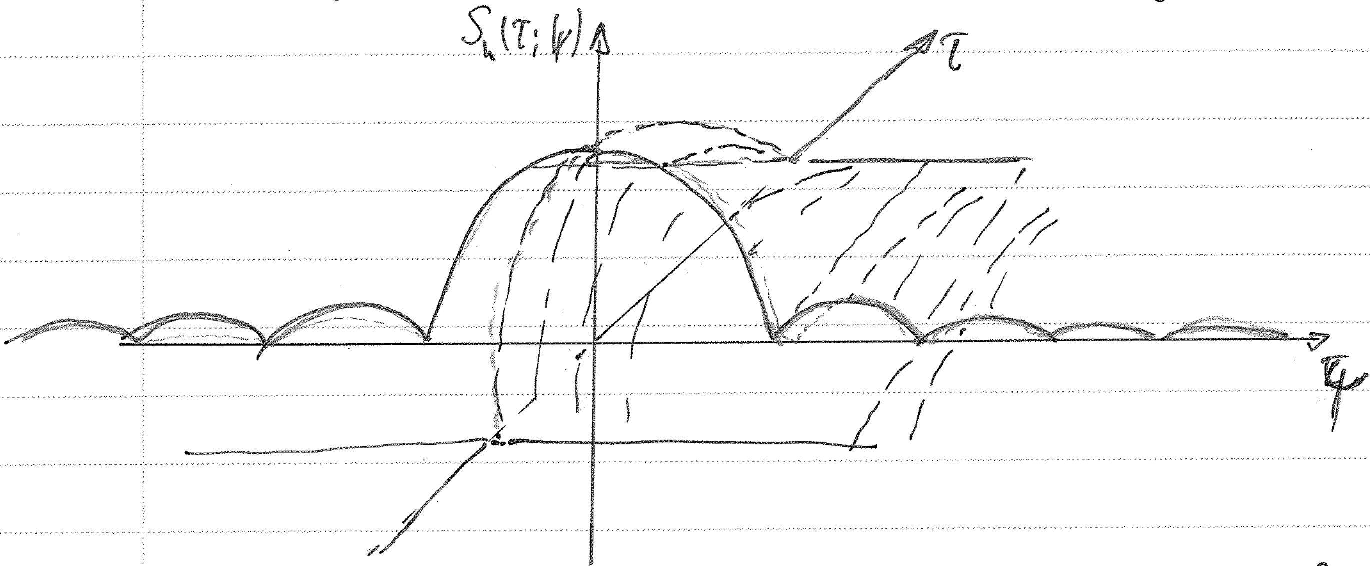


$\boxed{3.6}$ c) $S_h(\tau;\psi) = \mathcal{F}_t\{\varphi_h(\tau;t)\} = \begin{cases} \cos(10^3\pi\tau)\cdot \mathcal{F}\{(1-10^4|t|),\ |t|\le 10^{-4}\},\ \text{if } |\tau|\le 5\cdot 10^{-4} \\ 0,\ \text{otherwise} \end{cases} =$

cont.

$$= \begin{cases} \cos(10^3\pi\tau)\cdot 10^{-4}\cdot \operatorname{sinc}^2(10^{-4}\psi),\ \text{if } |\tau|\le 5\cdot 10^{-4} \\ 0,\ \text{otherwise} \end{cases}$$

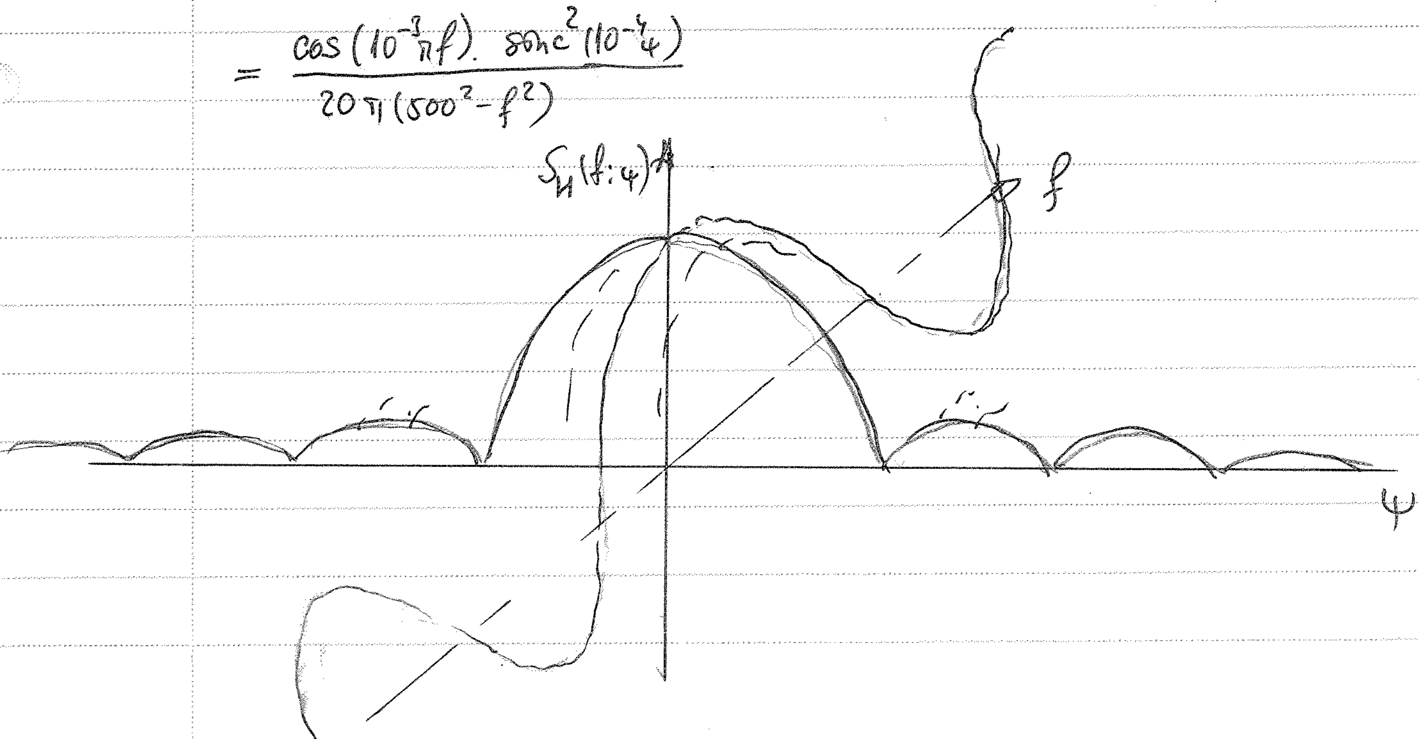
according to the tables for Fourier transform of a "triangle" function.



d) $S_H(f;\psi) = \mathcal{F}_\tau\{S_h(\tau;\psi)\} = \{10^{-4}\operatorname{sinc}^2(10^{-4}\psi)\cdot \mathcal{F}\{\cos(10^3\pi t),\ |\tau|\le 5\cdot 10^{-4}\} =$

$$= 5\cdot 10^{-8}\left(\operatorname{sinc}(10^{-3}(f-500)) + \operatorname{sinc}(10^{-3}(f+500))\right)\cdot \operatorname{sinc}^2(10^{-4}\psi) =$$

$$= \frac{\cos(10^{-3}\pi f)\cdot \operatorname{sinc}^2(10^{-4}\psi)}{20\pi(500^2-f^2)}$$



3.7 a) Delay spread (T_m):

$$\varphi_h(\tau) = \varphi_h(\tau;0) = \begin{cases} \cos(10^3 \pi \tau), & \text{if } |\tau| \leq 5.10^{-4} \\ 0, & \text{otherwise} \end{cases}$$

$\Rightarrow T_m = 2.5.10^{-4} s = 10^{-3} s = 1 ms$

b) Coherence bandwidth (B_m):

$$\varphi_H(f) = \varphi_H(f;0) = \frac{5.10^2 \cos(10^{-3}\pi f)}{\pi(500^2 - f^2)}$$

$\Rightarrow$ the "first nulls" of this function are in ± 1500 Hz

$\Rightarrow$ "first null" ~~ban~~ coherence bandwidth $B_m = 3$ kHz

c) $B_m T_m = 3 \approx 1$

d) Coherence time (T_d):

$$\varphi_H(t) = \varphi_H(0;t) = \begin{cases} \frac{2.10^{-3}}{\pi}(1 - 10^4|t|), & \text{if } |t| \leq 10^{-4} \\ 0, & \text{otherwise} \end{cases}$$

$\Rightarrow T_d = 2.10^{-4} s = 0{,}2 ms$

e) Doppler spread (B_d):

$$S_H(\psi) = S_H(0;\psi) = \frac{\operatorname{sinc}^2(10^{-4}\psi)}{5.10^6 \pi}$$

$\Rightarrow$ the "first nulls" of this function are in $\pm 10^4$ Hz

$\Rightarrow$ "first null" ~~cohe~~ doppler spread is $B_d = 2.10^4$ Hz $= 20$ kHz

f) $B_d T_d \approx 4 \approx 1$

3.11 a) First we simplify the expression for P as

$$P = P_0 \left| 1 - e^{-j2\pi f \tau} \right|^2 = P_0 \left| e^{-j\pi f \tau} \left(e^{j\pi f \tau} - e^{-j\pi f \tau} \right) \right|^2 = 4P_0 \sin^2(\pi f \tau)$$

Now we express τ as a function of P:

$$\sin^2(\pi f \tau) = \frac{P}{4P_0} \Rightarrow \sin(\pi f \tau) = \sqrt{\frac{P}{4P_0}} \Rightarrow \pi f \tau = \arcsin\left(\sqrt{\frac{P}{4P_0}}\right) \Rightarrow$$

$$\Rightarrow \tau = \frac{1}{\pi f} \cdot \arcsin\sqrt{\frac{P}{4P_0}}$$

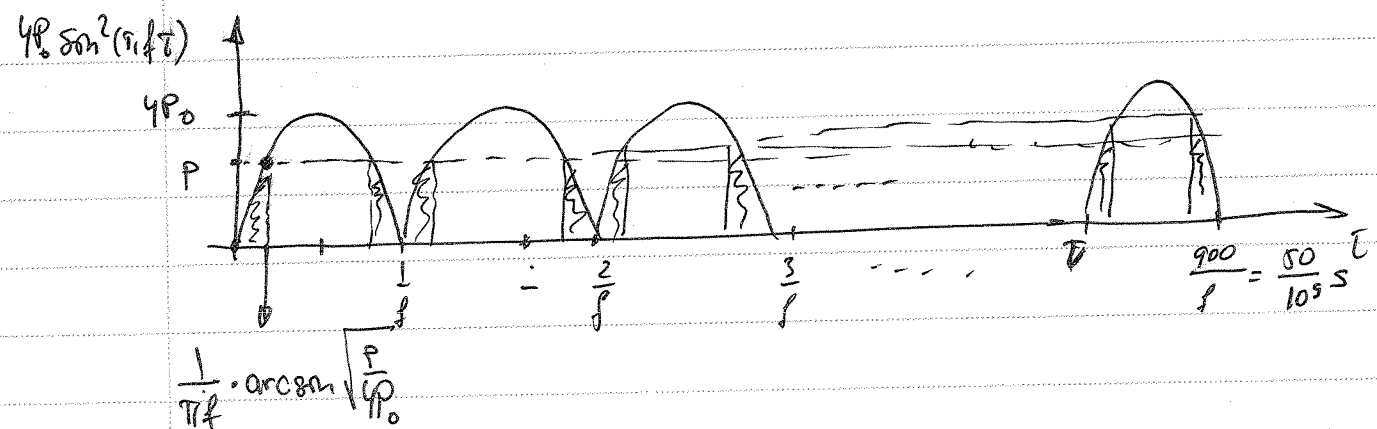
The probability density function of τ is $f_T(\tau) = \begin{cases} \dfrac{1}{50 \cdot 10^{-9}}, & 0 \le \tau \le 50 ns \\ 0, & \text{otherwise} \end{cases}$

The distribution function of P is defined as

$$F_P(p) = P_r\{P \le p\} = P_r\{4P_0 \sin^2(\pi f \tau) \le p\}$$

Obviously $F_P(p) = 0$ if $p \le 0$ and $F_P(p) = 1$ if $p \ge 4P_0$.

We have the following graph:



$$\Rightarrow F_p(p) = P_r\{4P_0 \sin^2(\pi f \tau) \le p\} = 1800 \cdot \frac{1}{\pi f} \arcsin\sqrt{\frac{p}{4P_0}} \cdot \frac{1}{50 \cdot 10^{-9}} =$$

$$= \frac{18 \cdot 10^2 \cdot \arcsin\sqrt{\frac{p}{4P_0}}}{\pi \cdot 18 \cdot 10^9 \cdot 50 \cdot 10^{-9}} = \frac{2}{\pi} \arcsin\sqrt{\frac{p}{4P_0}}, \quad \text{if } 0 \le p \le 4P_0$$

Finally we get

$$F_p(p) = \begin{cases} 0, & \text{if } p \le 0 \\ \dfrac{2}{\pi} \arcsin\sqrt{\dfrac{p}{4P_0}}, & \text{if } 0 \le p \le 4P_0 \\ 1, & \text{if } p \ge 4P_0 \end{cases}$$

3.11 b) (cont.) The threshold power $\wp_t$ for 95% availability satisfies:

$$F_p(\wp_t) = 1 - 0{,}95 \iff \frac{2}{\pi}\arcsin\sqrt{\frac{\wp_t}{4P_0}} = 0{,}05$$

$$\iff \wp_t = 4P_0 \sin^2\frac{0{,}05\pi}{2} \approx 2{,}46233 \cdot 10^{-2} \cdot P_0 \approx \frac{P_0}{40}$$

We need also to calculate the mean received power which is evaluated as follows:

$$\wp_0 = E\{p\} = \int_{-\infty}^{\infty} p \cdot dF_p(p) = \int_0^{4P_0} p \cdot d\left(\frac{2}{\pi}\arcsin\sqrt{\frac{p}{4P_0}}\right) =$$

$$= \frac{2}{\pi}\int_0^{4P_0} \frac{p}{\sqrt{1-\frac{p}{4P_0}}\,\sqrt{4P_0}\cdot 2\sqrt{p}}\,dp = \frac{1}{\pi}\int_0^{4P_0}\sqrt{\frac{\frac{p}{4P_0}}{1-\frac{p}{4P_0}}}\,dp =$$

$$= \frac{4P_0}{\pi}\int_0^1\sqrt{\frac{p}{1-p}}\,dp = \frac{4P_0}{\pi}\left(-\sqrt{p(1-p)} - \arcsin\sqrt{1-p}\right)\Big|_0^1 = 2P_0$$

Finally the fading margin is defined as the ratio between the expected power and the threshold power which in this case is

$$\frac{\wp_0}{\wp_t} = \frac{2P_0}{\frac{P_0}{40}} = 80 \quad \text{and in dB it is} \quad 10\log_{10} 80 \approx 19\,dB$$

c) The threshold power $\wp_t$ for 99% availability is calculated in the same way as in b) to be

$$\wp_t = 4P_0\sin^2\frac{0{,}01\pi}{2} \approx \frac{P_0}{1000}$$

The fading margin is in this case

$$\frac{\wp_0}{\wp_t} = \frac{2P_0}{\frac{P_0}{1000}} = 2000 \quad \text{and in dB it is} \quad 10\log_{10} 2000 \approx 33\,dB$$

$\boxed{3.12}$ a) The coherence bandwidth can be estimated as

$$B_m \approx \frac{1}{T_m} = 10^3 \text{ Hz} = 1 \text{ kHz}$$

The coherence time can be estimated as

$$T_d \approx \frac{1}{B_d} = 10^{-1} \text{ s} = 0{,}1 \text{ s}$$

b) The signal bandwidth, 25 kHz, is much bigger than the coherence bandwidth and thus we have a frequency selective channel.

c) The symbol length of the transmitted signal is approximately $T_s \approx \frac{1}{W_s} = \frac{1}{25 \cdot 10^3} = 4 \cdot 10^{-5} \text{ s}$

This symbol duration is much less than the coherence time of the channel and thus we experienc slowly fading.