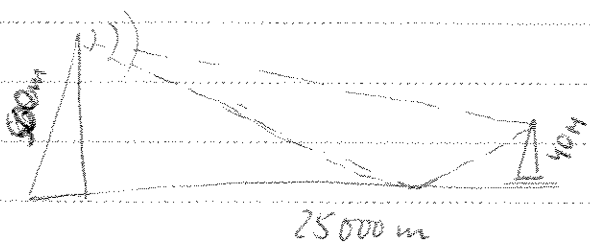
2.1 We use the Plain Earth Model to calculate the field strength:



$h_T = 500$
$h_R = 40$
$R = 25000$

If we assume the the field strength for free-space propagation is E_0, then the received field strength (E) is obtained according to the formula (2.17)

$$E^2 = P_R = 4E_0^2 \sin^2\left(\frac{2\pi h_T h_R}{\lambda R}\right) = 4E_0^2 \sin^2\left(\frac{2\pi h_T h_R f}{c \cdot R}\right)$$

This gives

$$\frac{E}{E_0} = 2\left|\sin\frac{2\pi h_T h_R f}{cR}\right| \approx 2\left|\sin(1{,}6755 \cdot 10^{-8} \cdot f)\right|$$

For the different values of f we obtain:

$f = 475\ \text{MHz}$: $\frac{E}{E_0} \approx 1{,}989 \approx 2{,}9864\ dB$

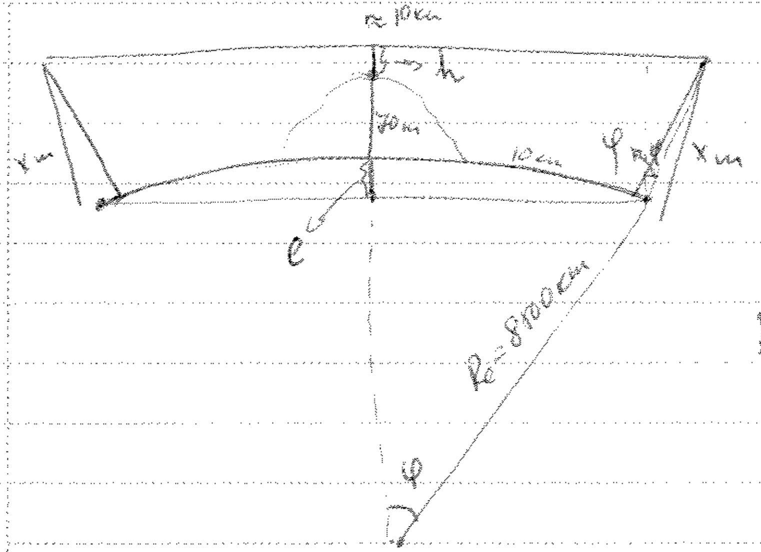
$f = 500\ \text{MHz}$: $\frac{E}{E_0} \approx 1{,}7321 \approx 2{,}3856\ dB$

$f = 525\ \text{MHz}$: $\frac{E}{E_0} \approx 1{,}1756 \approx 0{,}70249\ dB$

$f = 550\ \text{MHz}$: $\frac{E}{E_0} \approx 0{,}41582 \approx -3{,}8109\ dB$

Here we use $[a]_{dB} = 10\log_{10}(a)$!

$\boxed{23}$ We have the following situation:



$d_1 = d_2 = 5000\,m$

$\lambda = \frac{c}{f} \approx 1{,}667 \cdot 10^{-2}\,m$

$R_e \rightarrow$ the effective Earth radius

The first Fresnel zone is defined by the equation

$$h = \sqrt{\frac{K\lambda d_1 d_2}{d_1 + d_2}} \overset{(K=1)}{\approx} 6{,}455\,m$$

The height l caused by the Earth's curvature can be calculated as:

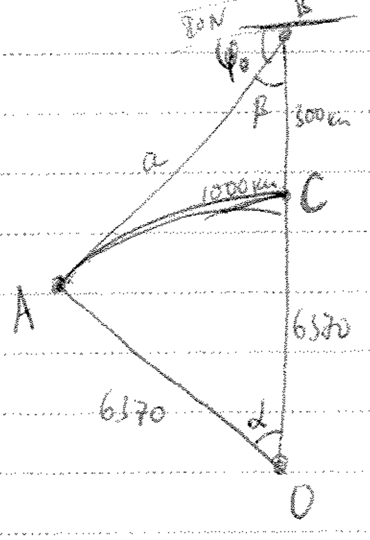
$$l = R_e(1 - \cos\varphi) \approx 1{,}47\,m$$

where $\varphi = \frac{5000\,m}{R_e\,(m)}\,rad = \frac{5}{8500} \approx 5{,}88 \cdot 10^{-4}\,rad$

Finally we can calculate the required antenna height (x) as:

$$x = \frac{(h + 70 + l)}{\cos\varphi} \approx 77{,}925\,m$$

2.4 We have the following situation.



We want to calculate the angle φ_0. For the angle α we have

$$\alpha = \frac{1000}{6370} \text{ rad} \approx 0{,}157 \text{ rad}$$

The cosine theorem gives the length (a) of the segment AB

$$a^2 = 6370^2 + 6670^2 - 2 \cdot 6370 \cdot 6670 \cdot \cos\alpha \approx 1{,}1349 \cdot 10^6$$

$$\Rightarrow a \approx 1065{,}3 \text{ km}$$

Again the cosine theorem gives

$$\cos\beta = \frac{a^2 + 6670^2 - 6370^2}{2 \cdot a \cdot 6670} \approx 0{,}35513$$

Thus $\varphi_0 = \frac{\pi}{2} - \beta = \frac{\pi}{2} - \arccos(0{,}35513) \approx 0{,}36305$ rad

The MUF is obtained by the inequality

$$n \approx \sqrt{1 - \frac{81 \cdot N_e}{f^2}} \leq \cos\varphi_0$$

This gives $f \leq \frac{9\sqrt{N_e}}{\sin\varphi_0} = \frac{9 \cdot 10^6}{\cos\beta} \approx \frac{9 \cdot 10^6}{0{,}35513} \approx 25{,}343$ MHz

$\boxed{2.5}$ The critical frequency 9MHz gives the electron density according to (2.66) as

$$9\cdot 10^6 = 9\cdot\sqrt{N_e} \implies N_e = 10^{12} \text{ electrons}/m^3$$

We have the same situation as in Problem 2.4. With the notations there we have to find the angle α.

The equation:

$$n \approx \sqrt{1-\frac{81 N_e}{f^2}} = \cos\varphi_0$$

for $f = 12$ MHz gives the angle φ_0 as

$$\cos\varphi_0 \approx 0{,}66144 \implies \cos\beta = \cos\left(\frac{\pi}{2}-\varphi_0\right) = \sin\varphi_0 = \sqrt{1-\cos^2\varphi} \approx \frac{3}{4}$$

The cosine theorem gives

$$6370^2 = a^2 + 6670^2 - 2\cdot a\cdot 6670\cdot\cos\beta \implies a^2 - 10005\cdot a + 3912000 = 0$$

This quadratic equation has the solutions:

$$a_{1,2} \approx \frac{10005 \pm 9189{,}8}{2} \quad \text{, i.e.} \quad a_1 \approx 9597{,}4 \text{ and } a_2 \approx 407{,}6$$

It is easily checked that a_1 corresponds to a situation where the EM-wave passes through the Earth and thus is not valid.

Thus the length of the segment AB is 407,6.

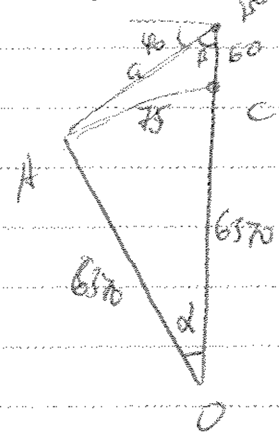
Again the cosine theorem gives

$$\cos\alpha = \frac{6370^2+6670^2-407{,}6^2}{2\cdot 6370\cdot 6670} \approx 0{,}9991 \implies \alpha \approx \arccos(0{,}9991) \approx 0{,}04238 \text{ rad}$$

Now the surface distance between A and C is

$6370\cdot\alpha \approx 269{,}96$ km, which is the half of the skip distance. Thus the required distance is approx. 540 km.

2.7 The following picture describes the situation



The angle α is computed as

$$\alpha = \frac{75}{6370} \text{ rad} \approx 0{,}011811 \text{ rad}$$

The cosine theorem gives

$$a = \sqrt{6370^2 + 6430^2 - 2 \cdot 6370 \cdot 6430 \cdot \cos\alpha} \approx 96{,}322 \text{ km}$$

The angle β is calculated as

$$\cos\beta = \frac{a^2 + 6430^2 - 6370^2}{2 \cdot a \cdot 6430} \approx 0{,}62749$$

The electron density of the E-layer can be computed from the critical frequency and is

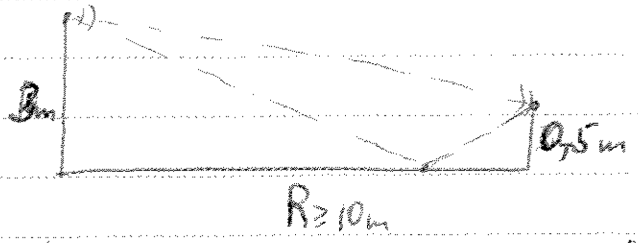
$$N_e = \frac{f^2}{81} = \frac{10^6}{81} \text{ electrons/m}^3$$

As in Problem 2.4. the MUF for the distence 150 km is calculated as

$$f \leq \frac{9\sqrt{N_e}}{\cos\beta} = \frac{10^6}{\cos\beta} \approx 1{,}5936 \cdot 10^6 \text{ Hz} \approx 1{,}6 \text{ MHz}$$

Thus frequencies above 1,6 MHz can not reach the shore station via the skywave. ~~and~~ Those frequencies are avoiding multipath propagation completely.

2.10 We assume that the Plane Earth model is valid. The following picture is valid.



According to the PE formula

$$P_R = 4E_0^2 \sin^2\left(\frac{2\pi h_T h_R}{\lambda R}\right)$$

the deep fades occur when $\frac{2h_T h_R}{\lambda R}$ is an integer number.

Thus $\frac{2h_T h_R}{\lambda R} = k \in \mathbb{Z}$. This gives distances

$$R_k = \frac{h_T h_R}{2\lambda k} = \frac{2f\, h_T h_R}{c\, k} = \frac{24}{k} \quad \text{(meters)}$$

The fading dips for distances down to 10m are at distances 24 and 12 meters from the sign. If the car travels more than 12 meters for the duration of the transmission of the packet than it can experience 2 fading dips. For ~~distances~~ travel distances less than 12 meters this is impossible. The ~~[illegible]~~ data rate (r) should satisfy

$$\underbrace{\left(\frac{500}{r}\right)}_{\text{time in seconds}} \cdot \underbrace{200 \cdot \frac{1000}{3600}}_{\text{speed in } \frac{m}{s}} \leq \underbrace{12}_{\text{distance in meters}} \Rightarrow r \gtrsim 2315 \ \frac{bits}{s}$$