Equalization and Diversity

- Frequency selective fading and ISI
- Transversal filter/equalizer
- Other equalization techniques
- Diversity techniques
- Summary

Frequency selective fading and ISI

Coherence BW

\[ B_C < B_s \] and \[ T_s < \sigma_s \]  "frequency selective fading"

Inter-symbol Interference

Equalization on Rx side
Equalization concept

Channel → Equalizer

\[ H_{ch}(s)H_{eq}(s) = Ke^{-st_0} \]

Possibly flat frequency ch-cs

\[ H_{eq}(s) = \frac{Ke^{-st_0}}{H_{ch}(s)} \]

Usually discrete-time approach

\[ H_{eq}(z) = \sum_{n=0}^{N} w_n z^{-n} \]

FIR filter

Linear transversal filter /equalizer

Received training sequence

Adaptively adjusted weights

\[ \hat{x}(k) = \sum_{n=0}^{N} w_n(k)y(k-n) \]

\[ \hat{x}(k) = \mathbf{w}^T \mathbf{y}_k \]

\[ e(k) = x(k) - \hat{x}(k) \]

\[ e^2(k) = \mathbb{E}[e(k)^2] = \mathbb{E}[x^2(k)] + w_i^2 \mathbb{E}[y_i y_i^T]w_i - 2 \mathbb{E}[x_i y_i^T]w_i \]

\[ \mathbb{E}[e^2] = \mathbb{E}[x^2(k)] + w_i^2 \mathbb{R}_i w_i - 2p_i^2 w_i \]

MSE
Transversal filter (cont’d)

\[ E_k = E[x^2] + w^T_k R w_k - 2p^T w_k \]

Tune adaptively the filter for minimum MSE
Achieve flat frequency response, i.e. reduce ISI
When the process stationary then Min(\(e\)) can be found

Gradient approach:
\[ \nabla E = \frac{\partial E}{\partial w} = 2R w - 2p \]
\[ w_{opt} = R^{-1} p \]
As the channel changes in time, \(w_{opt}\) must be recalculated regularly

\[ E_{min} = E[x^2] + (R^{-1} p)^T R R^{-1} p - 2p^T R^{-1} p \]

But noise at the equalizer input can be an issue

Transversal filter (cont’d)

\[ w_{opt} = R^{-1} p \]
Inverting NxN matrix is computer intensive while time is limited

LMS iterative procedure:
\[ e_i^2 = (x_i - w_i^T y_i)^2 \]
\[ \nabla E = \frac{\partial e_i^2}{\partial w_i} = -2(x_i - w_i^T y_i)y_i \]
\[ \nabla E = -2e_i y_i \]
\[ w_i(n+1) = w_i(n) - \alpha \nabla e(n) \]
Slowly convergent – not sufficient

Recursive LMS:
\[ w_i(n+1) = w_i(n) + K(n)e_i(n,n+1) \]
Makes use of actual averages - faster
Other equalization techniques

Nonlinear equalizers - useful when the spectral “nulls” are deep

- Decision feedback equalizer
- Maximum likelihood symbol detection
- Maximum likelihood sequence estimation

Diversity techniques

Enabling extra signal paths to combat flat fading

- Antenna diversity (small/large-scale fading)
- Polarization diversity
- Frequency diversity  //OFDM
- Time diversity  //Rake receiver (CDMA)
- Interleaving  //coding
Summary

• Equalization necessary in frequency selective fading channels suffering from ISI
• Equalization should be adaptive because of channel variations
• TDMA well suited for adaptive equalization because of latency
• Diversity helpful to combat “flat fading” – various techniques available
• Equalization and diversity – two powerful approaches to counteract fading.