Solutions to TSE150 Linear systems 170107

1) a) Linear system: $x(n) = a_1 x_1(n) + a_2 x_2(n) \rightarrow y(n) = a_1 y_1(n) + a_2 y_2(n)$
if $x_1(n) \rightarrow y_1(n)$ and $x_2(n) \rightarrow y_2(n)$

Time-invariant system: $x_1(n) = x(n-k) \rightarrow y_1(n) = y(n-k)$
if $x(n) \rightarrow y(n)$

b) BIBO stable system: Bounded Input $\rightarrow$ Bounded Output
$|x(n)| \leq M_1 < \infty \qquad |y(n)| \leq M_2 < \infty \quad \forall n$

Causal system: Output $y(n)$ depends only on earlier input sample values $x(n), x(n-1), x(n-2), \ldots$

c) Transform expression and ROC $\Leftrightarrow$ unique sequence

d) ROC includes the unit circle $\Leftrightarrow$ stable system
(causal syst. poles inside the unit circle)

e) Lowpass since $\underset{\omega T=0}{H(1)} = 1$ and $\underset{\omega T=\pi}{H(-1)} = 0$

2) a) $H(z) = \dfrac{1 - z^{-1} + z^{-2}}{1 + b_1 z^{-1}} = \dfrac{z^2 - z + 1}{z(z + b_1)}$

$\Rightarrow$ zeros: $z_{1,2} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 1} = \frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

b) Poles at 0 and $-b_1 \Rightarrow$ stable iff $|b_1| < 1$

c) $H(z) = \dfrac{1}{1 + b_1 z^{-1}} \left[1 - z^{-1} + z^{-2}\right]$

$\Rightarrow h(n) = (-0,75)^n u(n) - (-0,75)^{n-1} u(n-1) + (-0,75)^{n-2} u(n-2)$

$$= \begin{cases} 0 & n < 0 \\ 1, & n = 0 \\ -1,75, & n = 1 \\ \frac{37}{9} \cdot (-0,75)^n, & n \geq 2 \end{cases}$$

3) $h(n) = 0{,}5^n u(n) \Leftrightarrow H(z) = \dfrac{z}{z-0{,}4}$

$x(n) = x_1(n) + x_2(n-2)$ where

$x_1(n) = 0{,}2^n u(n) \Leftrightarrow X_1(z) = \dfrac{z}{z-0{,}2}$

$x_2(n) = 0{,}8^n u(n) \Leftrightarrow X_2(z) = \dfrac{z}{z-0{,}8}$

$Y_1(z) = H(z)X_1(z) = \dfrac{z}{z-0{,}4} \cdot \dfrac{z}{z-0{,}2} = \dfrac{2z}{z-0{,}4} - \dfrac{z}{z-0{,}2}$

$\Rightarrow y_1(n) = 2 \cdot 0{,}4^n u(n) - 0{,}2^n u(n)$

$Y_2(z) = H(z)X_2(z) = \dfrac{z}{z-0{,}4} \cdot \dfrac{z}{z-0{,}8} = \dfrac{-z}{z-0{,}4} + \dfrac{2z}{z-0{,}8}$

$\Rightarrow y_2(n) = -0{,}4^n u(n) + 2 \cdot 0{,}8^n u(n)$

$y(n) = y_1(n) + y_2(n-2)$ (since LTI)

$$= 2 \cdot 0{,}4^n u(n) - 0{,}2^n u(n) - 0{,}4^{n-2} u(n-2) + 2 \cdot 0{,}8^{n-2} u(n-2)$$

$$= \begin{cases} 0 & n < 0 \\ 1 & n = 1 \\ 0{,}6 & n = 2 \\ -\frac{17}{4} \cdot 0{,}4^n - 0{,}2^n + \frac{25}{8} \cdot 0{,}8^n, & n \geq 2 \end{cases}$$

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4) $H(e^{j\omega T}) = a_0 + a_1 e^{-j\omega T} + a_0 e^{-j2\omega T} = e^{-j\omega T}(2a_0 \cos(\omega T) + a_1)$

Desired: $\begin{cases} |H(e^{j0{,}25\pi})| = 0 \\ |H(e^{j0{,}75\pi})| = 2 \end{cases} \Leftrightarrow \begin{cases} 2a_0 \cos(0{,}25\pi) + a_1 = 0 & (1) \\ 2a_0 \cos(0{,}75\pi) + a_1 = \pm 2 & (2) \end{cases}$

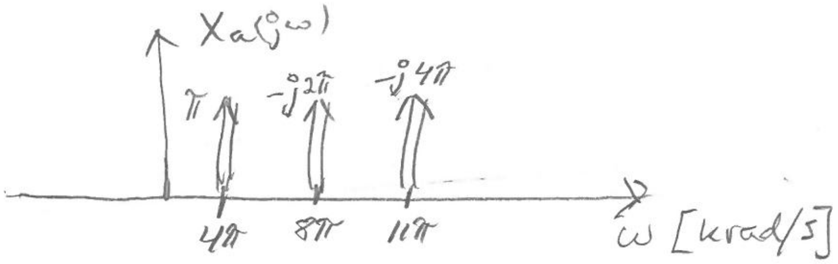
$(1) \Rightarrow a_1 = -2a_0 \cos(0{,}25\pi)$

(1) in $(2) \Rightarrow 2a_0(\cos(0{,}75\pi) - \cos(0{,}25\pi)) = \pm 2$

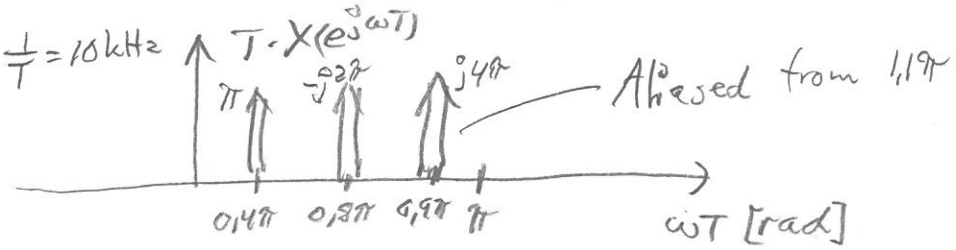
$\Leftrightarrow a_0 = \dfrac{\pm 1}{\cos(0{,}75\pi) - \cos(0{,}25\pi)} = \pm$

$a_1 = \dfrac{\mp 2\cos(0{,}25\pi)}{\cos(0{,}75\pi) - \cos(0{,}25\pi)} = \mp$

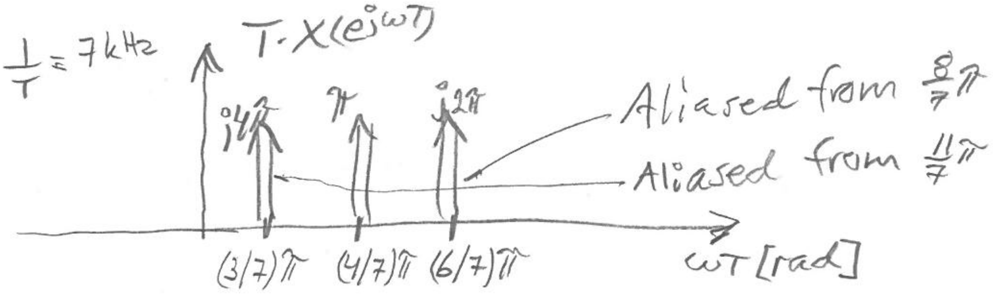
5a)



$\frac{1}{T} = 10\,kHz$

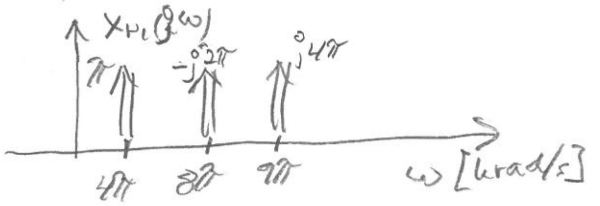


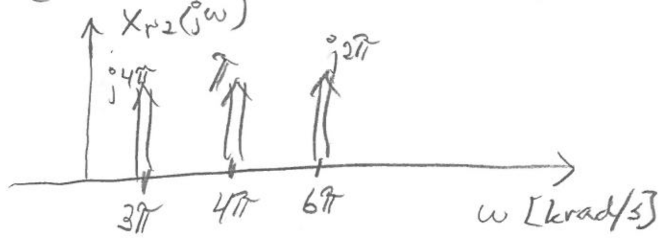
$\frac{1}{T} = 7\,kHz$



b) $x_{r1}(t) = \cos(4000\pi t) + 2\sin(8000\pi t) - 4\sin(9000\pi t)$

$x_{r2}(t) = -4\sin(3000\pi t) + \cos(4000\pi t) - 2\sin(6000\pi t)$





6a) $x(n) = 2\sin(0{,}125\pi n) + 4\cos(0{,}75\pi n + 0{,}5\pi)$

$$= -j\,e^{j0{,}125\pi} + j\,\underbrace{e^{-j0{,}125\pi}}_{=e^{j1{,}875\pi}} + 2e^{j0{,}5\pi}\cdot e^{j0{,}75\pi n} + 2e^{-j0{,}5\pi}\cdot \underbrace{e^{-j0{,}75\pi n}}_{=e^{j1{,}25\pi n}}$$

$$= \Big/\ 0{,}125\pi = 2\pi\cdot\tfrac{1}{16},\ 1{,}875\pi = 2\pi\cdot\tfrac{15}{16},\ 0{,}75\pi = 2\pi\cdot\tfrac{6}{16},\ 1{,}25\pi = 2\pi\cdot\tfrac{10}{16}\ \Big/$$

$$= \sum_{k=0}^{15} c_k\cdot e^{j\frac{2\pi k}{16}n} \text{ with}$$

$c_1 = -j$

$c_6 = 2e^{j0{,}5\pi} =$

$c_{10} = 2e^{-j0{,}5\pi} =$

$c_{15} = j$

All other $c_k = 0$

b) $X_1(k) = 16\cdot c_k$

7a) $L = \frac{5}{4}$ (Interpol. factor)

b)