Solutions to Exam in TSE150 Linear systems 161028

1a) $x(n)$ applied at $n=0$

$y(n) = y_s(n) + y_t(n)$ — transient $\to 0$ when $n \to \infty$ for stable syst.

steady-state, same type as $x(n)$

b) Stable iff $\sum_n |h(n)| < \infty$

c) $f_s > 2f_0$, f_0 highest freq. in $x_a(t)$

d) $H(z) = z^{-1} + z \leftrightarrow y(n) = x(n-1) + \underbrace{x(n+1)}_{>n} \Rightarrow$ noncausal

e) $\overset{\omega T = 0}{H(1)} = 0$, $\overset{\omega T = \pi}{H(-1)} = 1 \Rightarrow$ HP filter

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2a) $y(n) + b_1 y(n-1) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2)$

Here $b_1 = -0,5$, $a_0 = a_2 = 0,125$, $a_1 = 0,25$

b) $$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1}} = 0,125 \frac{1 + 2z^{-1} + z^{-2}}{1 - 0,5z^{-1}}$$

$$= 0,125 \cdot \frac{z^2 + 2z + 1}{z(z - 0,5)} = 0,125 \cdot \frac{(z+1)^2}{z(z-0,5)}$$

2 zeros at $z = -1$, poles at $z = 0$ and $z = 0,5$

Stable since $|\text{poles}| < 1$

c) Let $H_1(z) = \dfrac{1}{1 - 0,5z^{-1}} \leftrightarrow h_1(n) = 0,5^n \cdot u(n)$

$$h(n) = 0,125 h_1(n) + 0,25 h_1(n-1) + 0,125 h_1(n-2)$$

$$= 0,125 \cdot 0,5^n u(n) + 0,25 \cdot 0,5^{n-1} u(n-1) + 0,125 \cdot 0,5^{n-2} u(n-2)$$

$$= \begin{cases} 0,125, & n = 0 \\ 0,3125, & n = 1 \\ 1,125 \cdot 0,5^n, & n = 2 \end{cases}$$

3) $H(z) = \frac{1}{1-0,5z^{-1}} = \frac{z}{z-0,5}$

$x(n) = x_1(n) + x_2(n)$, $x_1(n) = u(n)$, $x_2(n) = \sin(0,5\pi n)$

$X_1(z) = \frac{z}{z-1}$

$Y_1(z) = H(z) X_1(z) = \frac{z}{z-0,5} \cdot \frac{z}{z-1} = \frac{-z}{z-0,5} + \frac{2z}{z-1}$

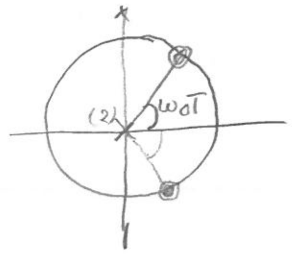
$\leftrightarrow y_1(n) = -0,5^n u(n) + 2u(n)$

$y_2(n) \approx |H(e^{j0,5\pi})| \cdot \sin(0,5\pi n + \arg\{H(e^{j0,5\pi})\})$

$\approx 0,894 \cdot \sin(0,5\pi n - 0,4636)$

$y(n) = y_1(n) + y_2(n)$

4a)



zeros at $e^{\pm j\omega_0 T} \Rightarrow y(n) = 0$

when $x(n) = A \cdot \sin(\omega_0 T n)$

Here $\omega_0 T = 0,25\pi$

$\Rightarrow \alpha = \pm 0,25\pi$, $r = 1$

b) $H(z) = G \cdot \frac{z^2 - 2\cos(0,25\pi) z + 1}{z^2} = G \cdot (1 - 2\cos(0,25\pi) z^{-1} + z^{-2})$

$$\leftrightarrow h(n) = \begin{cases} G & n=0,\ n=2 \\ -G \cdot 2\cos(0,25\pi), & n=1 \\ 0, & \text{otherwise} \end{cases}$$

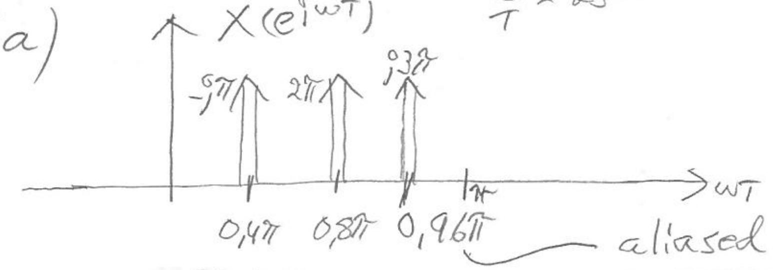
given

$$\Rightarrow \sum h^2(n) = G^2 \underbrace{\left(1 + 4\overbrace{\cos^2(0,25\pi)}^{=\left(\frac{1}{\sqrt{2}}\right)^2} + 1\right)}_{=4} = 1$$

$\Rightarrow G = \pm\frac{1}{2}$

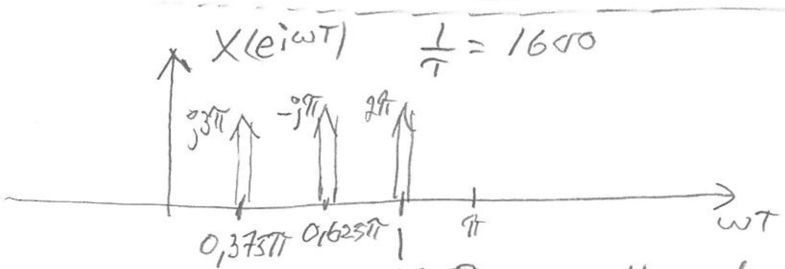
5a)

$\frac{1}{T} = 2500$



aliased from 1.04π

$\frac{1}{T} = 1600$

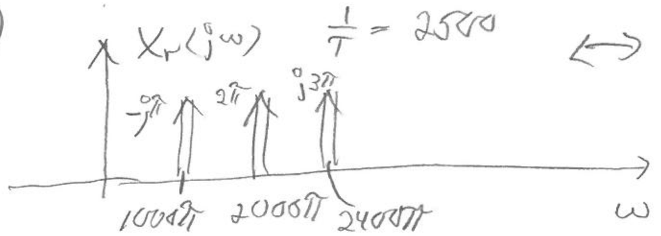


aliased from 1.25π

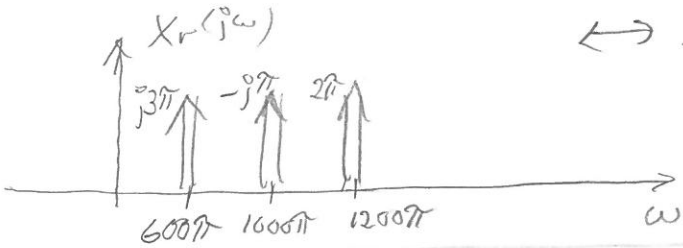
aliased from 1.625π

b)

$\frac{1}{T} = 2500$



$$\longleftrightarrow \quad x_r(t) = \sin(1000\pi t) + 2\cos(2000\pi t) - 3\sin(2400\pi t) \neq x_a(t)$$



$$\longleftrightarrow \quad x_r(t) = -3\sin(600\pi t) + \sin(1000\pi t) + 2\cos(1200\pi t) \neq x_a(t)$$

6a) $x(n+16) = 1 + \cos(0{,}875\pi(n+16) + 0{,}25\pi) =$

$= 1 + \cos(0{,}875\pi n + 14\pi + 0{,}25\pi) = \Big/ \begin{matrix} \cos(a + 2\pi \cdot \text{Integer}) \\ = \cos(a) \end{matrix} \Big/$

$= 1 + \cos(0{,}875\pi n + 0{,}25\pi) = x(n)$

b) $$x(n) = 1 + \frac{e^{j(0{,}875\pi n + 0{,}25\pi)} + e^{-j(0{,}875\pi n + 0{,}25\pi)}}{2}$$

$$= \Big/ e^{j0{,}875\pi n} = e^{j\frac{2\pi \cdot 7}{16}n},\quad e^{-j0{,}875\pi n} = e^{j1.125\pi n} = e^{j\frac{2\pi \cdot 9}{16}n} \Big/$$

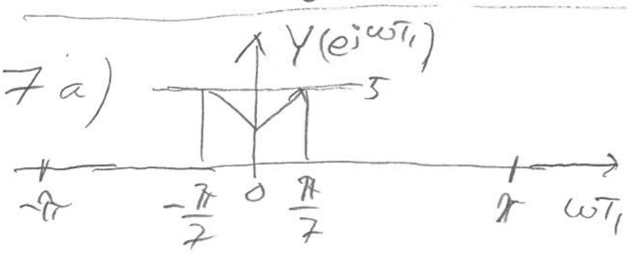
$$= 1 \cdot e^{j\frac{2\pi \cdot 0}{16}n} + \frac{1}{2} e^{j0{,}25\pi} \cdot e^{j\frac{2\pi \cdot 7}{16}n} + \frac{1}{2} e^{-j0{,}25\pi} \cdot e^{j\frac{2\pi \cdot 9}{16}n} \qquad (1)$$

IDFT: $$x(n) = \frac{1}{16} \sum_{k=0}^{15} X_1(k) e^{j\frac{2\pi k}{16}n} \qquad (2)$$
$N = 16$

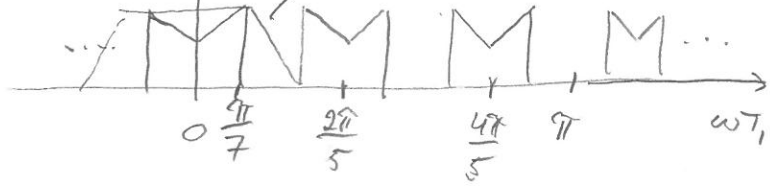
$$(1) = (2) \Rightarrow X_1(k) = \begin{cases} 16 & k=0 \quad |X_1| = 16 \quad \arg\{X\} = 0 \\ 8e^{j0{,}25\pi}, & k=7 \quad |X_1| = 8 \quad \arg\{X\} = 0{,}25\pi \\ 8e^{-j0{,}25\pi}, & k=9 \quad |X_1| = 8 \quad \arg\{X\} = -0{,}25\pi \\ 0 & \text{all other } k \quad |X_1| = 0 \quad \arg\{X\} \text{ undefined} \end{cases}$$

$8 \cdot e^{\pm j0{,}25\pi} \approx 5.6569(1 \pm j)$

7 a)



b)



$q = 5$

$\omega_c T_1 = \frac{\pi}{7} \approx 0{,}1429\pi$

$\omega_s T_1 = \frac{2\pi}{5} - \frac{\pi}{7} = \frac{9\pi}{35} \approx 0{,}2571\pi$