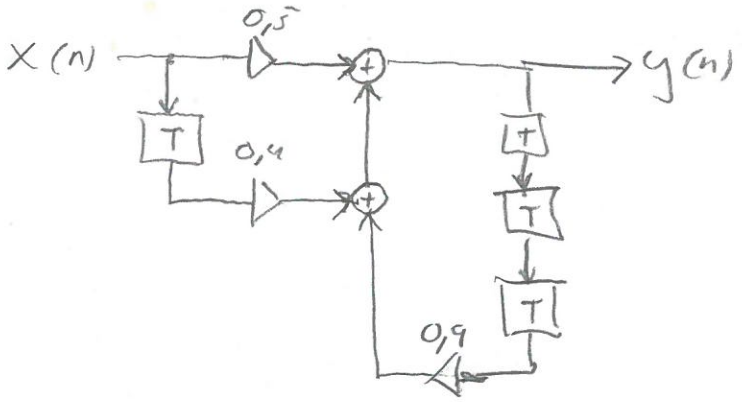
Linear systems exam 151029 - SOLUTIONS

1a) FIR, causal: $H(z) = \sum_{n=0}^{N} h(n) z^{-n} = \dfrac{\sum_{n=0}^{N} h(n) z^{N-n}}{z^{N}}$

$\Rightarrow$ N poles at $z=0$

b) Periodic sequence and DFT over one or several periods

c) $H(z) = 1+z \Leftrightarrow y(n) = x(n) + x(n+1)$ ∴ Noncausal

d) 

e) $x(n) = \delta(n) \Rightarrow y(n) = \delta(n)$ ∴ Time-varying syst.

$x(n-1) = \delta(n-1) \Rightarrow y(n) = 0$

2a) $H(z) = \dfrac{z}{z+0,4} + \dfrac{2,5}{z-0,5} = \dfrac{z^2+2z+1}{(z+0,4)(z-0,5)} = \dfrac{(z+1)^2}{(z+0,4)(z-0,5)}$

2 zeros at $z=-1$, poles at $z=-0,4$ & $z=0,5$ ∴ stable $|-0,4|<1$, $|0,5|<1$

b) ROC $|z| > \max(|-0,4|, |0,5|) = 0,5$

c) $H(z) = \dfrac{1+2z^{-1}+z^{-2}}{1-0,1z^{-1}-0,2z^{-2}} \Leftrightarrow y(n) - 0,1\,y(n-1) - 0,2\,y(n-2) = x(n) + 2x(n-1) + x(n-2)$

d) $h(n) = h_1(n) + h_2(n)$

$h_1(n) = (-0,4)^n u(n)$, $h_2(n) = 2,5 \cdot 0,5^{n-1} u(n-1)$

3) $y(n) = y_1(n) + y_2(n) + y_3(n)$

$y_1(n)$: $x_1(n) = 2\delta(n) \rightarrow y_1(n) = 2\cdot h(n) = 2\cdot(-0{,}1)^n u(n)$

$y_2(n)$: $x_2(n) = \delta(n-1) \rightarrow y_2(n) = h(n-1) = 2\cdot(-0{,}1)^{n-1} u(n-1)$

$y_3(n)$: $Y_3(z) = H(z)X_3(z) = \dfrac{z}{z+0{,}1}\cdot z^{-2}\cdot\dfrac{z}{z-0{,}4}$

$$= z^{-2}\cdot\left(\frac{0{,}2z}{z+0{,}1} + \frac{0{,}8z}{z-0{,}4}\right)$$

$$\Longrightarrow y_3(n) = \left(0{,}2\cdot(-0{,}1)^{n-2} + 0{,}8\cdot(0{,}4)^{n-2}\right) u(n-2)$$

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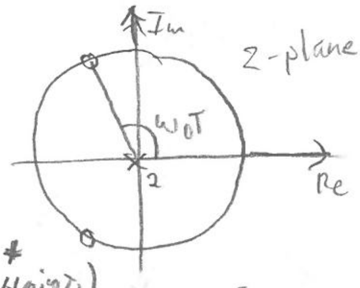
4) $H(z) = G\cdot(1 - 2\cos(\omega_0 T)z^{-1} + z^{-2})$

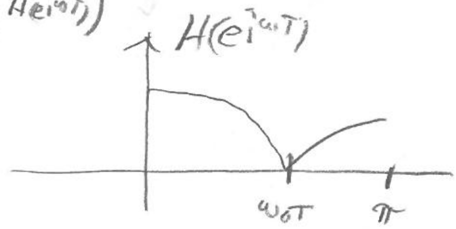
$$= G\cdot\frac{(z^2 - 2\cos(\omega_0 T)z + 1)}{z^2} = G\cdot\frac{(z-e^{j\omega_0 T})(z-e^{-j\omega_0 T})}{z^2}$$

zeros at $\omega T = \pm\omega_0 T \Longleftrightarrow z = e^{\pm j\omega_0 T}$

$\Longrightarrow$ the term $3\cdot\sin(0{,}8\pi n)$ eliminated if $\omega_0 T = 0{,}8\pi$ since $\sin(\omega T n) \rightarrow |H(e^{j\omega_0 T})|\cdot\sin(\omega T n + \arg H(e^{j\omega T}))$



$\omega T = 0{,}2\pi$:



$$|H(e^{j\omega T})| = G\cdot|1 - 2\cos(\omega_0 T)e^{-j\omega T} + e^{-j2\omega T}|$$

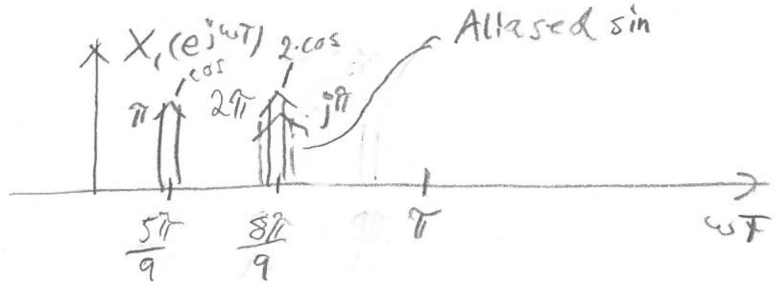
$$= G\cdot|2\cdot\cos(\omega T) - 2\cos(\omega_0 T)|$$

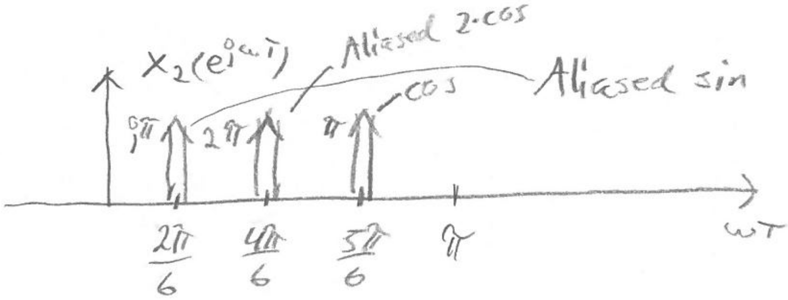
$$= |\omega T = 0{,}2\pi,\ \omega_0 T = 0{,}8\pi| = 2\cdot G\cdot|\cos(0{,}2\pi) - \cos(0{,}8\pi)|$$

$$= \pm\frac{1}{2} \Longrightarrow G = \pm\frac{1}{4(\cos(0{,}2\pi) - \cos(0{,}8\pi))} \approx \pm 0{,}1545$$

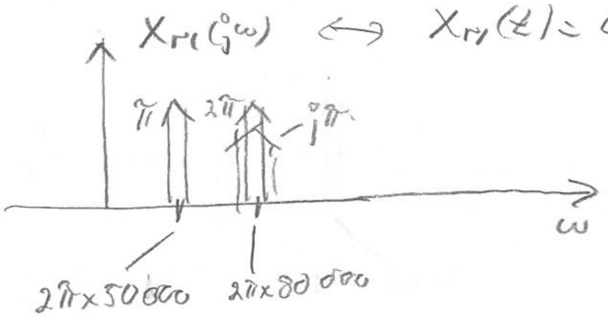
↑ Desired

5a)

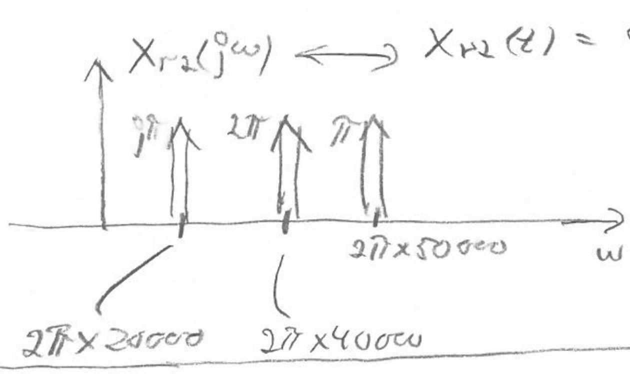




b)



$$X_{r1}(j\omega) \longleftrightarrow x_{r1}(t) = \cos(2\pi \times 50000 t) + 2\cos(2\pi \times 80000 t) \underbrace{- \sin(2\pi \times 80000 t)}_{\text{Due to aliasing}}$$



$$X_{r2}(j\omega) \longleftrightarrow x_{r2}(t) = \cos(2\pi \times 50000 t) + 2\cos(2\pi \times 40000 t) - \sin(2\pi \times 20000 t)$$

6) $x(n) = 2 + \cos(0{,}625\pi n + 0{,}2)$

$$= 2 + \frac{e^{j0{,}2}}{2} \cdot e^{j\frac{2\pi \cdot 10}{32} \cdot n} + \frac{e^{-j0{,}2}}{2} \cdot \underbrace{e^{j\frac{2\pi \cdot 22}{32} n}}_{= e^{-j\frac{2\pi \cdot 10}{32} \cdot n}}$$

$$= \frac{1}{32} \cdot \sum_{k=0}^{31} X_1(k) e^{j\frac{2\pi k}{32} \cdot n}$$

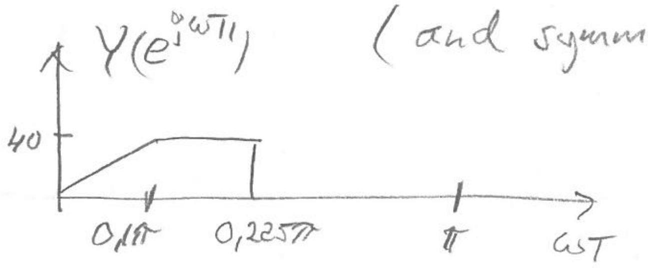
$$\Longrightarrow X_1(k) = \begin{cases} 64 & k=0 \\ 16 e^{j0{,}2} & k=10 \\ 16 e^{-j0{,}2} & k=22 \\ 0 & \text{otherwise } 0 \le k \le 31 \end{cases} \Rightarrow$$

	$k=0$	10	22
$\lvert X_1(k)\rvert =$	64	16	16
$\arg\{X_1(k)\} =$	0	0,2	−0,2

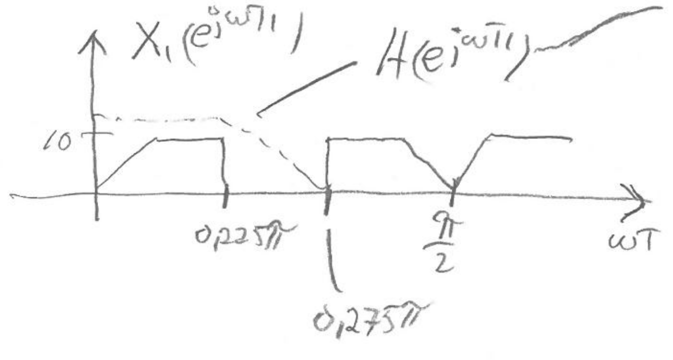
$|X_1(k)|$ zero otherwise

$\arg\{X_1(k)\}$ undefined otherwise

7a) (and symmetric around $\omega T = 0$)



b)



$L = 4$

$\omega_c T_1 = 0{,}225\pi$

$\omega_s T_1 = 0{,}275\pi$