Solutions to TSEI 50 Linear Systems, 2013-01-09

1 a) Linear system: $x(n) = ax_1(n) + bx_2(n)$ gives $y(n) = ay_1(n) + by_2(n)$ if $x_1(n)$ gives $y_1(n)$ and $x_2(n)$ gives $y_2(n)$. Time-invariant system: $x_1(n) = x(n–k)$ gives $y_1(n) = y(n–k)$ if $x(n)$ gives $y(n)$.

2 a) A (BIBO-)stable system: bounded input gives a bounded output. Causal system: the output $y(n)$ depends only on "earlier" sample values of the input, i.e., $x(n)$, $x(n–1)$, $x(n–2)$, etc.

b) Periodic sequences.

c) Lowpass because $H(1) = 2$ and $H(–1) = 0$.

3)
$$y(n) = \sum_{p=-\infty}^{\infty} x(p)h(n-p) = \sum_{p=-\infty}^{\infty} 0{,}9^p u(p) 0{,}1^{n-p} u(n-p)$$

$$= \sum_{p=0}^{n} 0{,}9^p 0{,}1^{n-p} = 0{,}1^n \sum_{p=0}^{n} \left(\frac{0{,}9}{0{,}1}\right)^p$$

$$= 0{,}1^n \frac{1-\left(\frac{0{,}9}{0{,}1}\right)^{n+1}}{1-\left(\frac{0{,}9}{0{,}1}\right)} = \frac{9 \times 0{,}9^n - 0{,}1^n}{8}$$

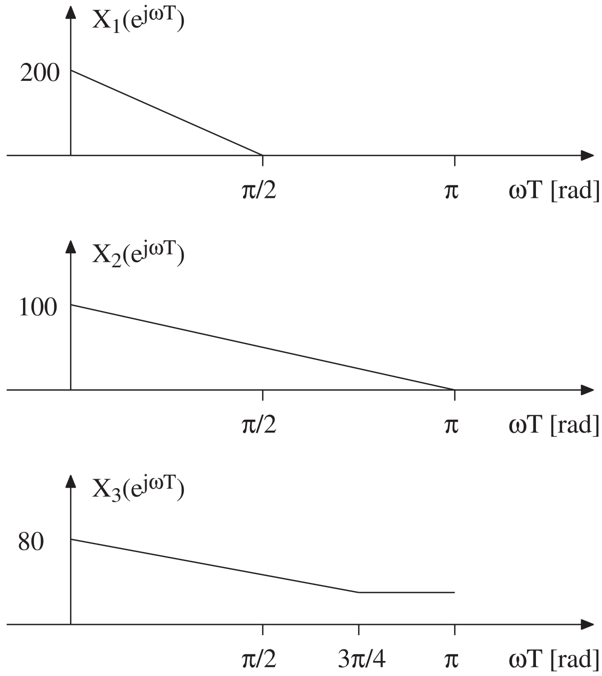
4 a) Transfer function: $H(z) = \dfrac{1 + 2z^{-1}}{1 + 0.5z^{-1}} = \dfrac{z+2}{z+0.5}$

Zero: $n = -2$, Pole: $p = -0.5$

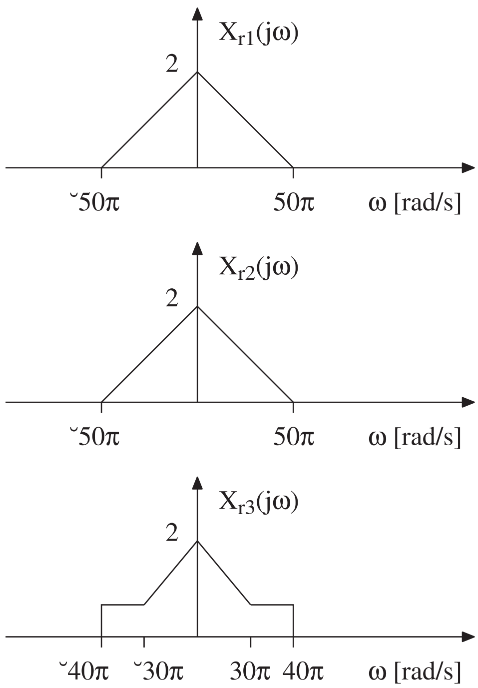
b) Impulse response: $h(n) = (-0.5)^n u(n) + 2(-0.5)^{n-1} u(n-1)$

c) Magnitude response: $|H(e^{j\omega T})| = |H(z)|\big|_{z = e^{j\Omega}} = \dfrac{|e^{j\omega T} + 2|}{|e^{j\omega T} + 0.5|} = 2$ (allpass)

5 a)



b)



6) $H(z) = \dfrac{z}{z-0.1}$, $X_1(z) = \dfrac{z}{z-1}$, $X_2(z) = -z^{-5}X_1(z)$

$$Y_1(z) = H(z)X_1(z) = \frac{z}{z-0.1}\frac{z}{z-1} = \frac{-(1/9)z}{z-0.1} + \frac{(10/9)z}{z-1}$$

$$Y_2(z) = -H(z)z^{-5}X_1(z) = -z^{-5}\frac{-(1/9)z}{z-0.1} + \frac{(10/9)z}{z-1}$$

$$Y(z) = Y_1(z) + Y_2(z)$$

Invers transformation =>

$$y_1(n) = \left(\frac{10}{9} - \frac{1}{9}(0.1)^n\right) u(n), \quad y_2(n) = -\left(\frac{10}{9} - \frac{1}{9}(0.1)^{n-5}\right) u(n-5)$$

$$y(n) = y_1(n) + y_2(n) = \left(\frac{10}{9} - \frac{1}{9}(0.1)^n\right) u(n) - \left(\frac{10}{9} - \frac{1}{9}(0.1)^{n-5}\right) u(n-5)$$

7)

$$\begin{aligned} X(e^{j\omega T}) &= 1 + e^{-j\omega T} + e^{-j2\omega T} + e^{-j3\omega T} \\ &= e^{-j1.5\omega T}(e^{j1.5\omega T} + e^{j0.5\omega T} + e^{-j0.5\omega T} + e^{-j1.5\omega T}) \\ &= e^{-j1.5\omega T}(2\cos(1.5\omega T) + 2\cos(0.5\omega T)) \end{aligned}$$

$$X(k) = X(e^{j\omega T})\Big|_{\omega T = \frac{2\pi k}{8} = \frac{\pi k}{4}}, \quad k = 0, 1, 2, \ldots, 7$$

$$|X(k)| = |2\cos(1.5\omega T) + 2\cos(0.5\omega T)|\Big|_{\omega T = \frac{\pi k}{4}}, \quad k = 0, 1, 2, \ldots, 7$$

$$Y(e^{j\omega T}) = X(e^{j\omega T})X(e^{j\omega T}) \quad =>$$

$$|Y(k)| = |X(k)|^2\Big|_{\Omega = \frac{\pi k}{4}}, \quad k = 0, 1, 2, \ldots, 7$$

$$|Y(0)| = 16, |Y(1)| = |Y(7)| = 6.828427, |Y(3)| = |Y(5)| = 1.171573$$

$$|Y(2)| = |Y(4)| = |Y(6)| = 0$$

8)

$$y_s(n) = |H(e^{j\omega_0 T})| \sin(\omega_0 Tn + \arg H(e^{j\omega_0 T}))u(n)$$

$$y_s(n) = 0 \text{ om } |H(e^{j\omega_0 T})| = 0$$

$$|H(e^{j\omega_0 T})| = |a_0 + a_1 e^{-j\omega_0 T} + a_0 e^{-j2\omega_0 T}| = a_1 + 2a_0\cos(\omega_0 T)$$

$$H(1) = 2a_0 + a_1 = 1 \Leftrightarrow 2a_0 = 1 - a_1$$

$$a_1 + (1 - a_1)\cos(0.28\pi) = 0 \Leftrightarrow a_1 = \frac{-\cos(0.28\pi)}{1 - \cos(0.28\pi)} \approx -1.75804$$

$$a_0 = \frac{1 - a_1}{2} \approx 1.37902$$