LINKÖPING UNIVERSITY

Exam in TSEI50 Linear Systems

Exam code:	TEN1
Date:	2017-01-07 **Time:** 8–12
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSEI50/
Result:	Available by 2017-01-21

1 **a.** Explain what a linear and time-invariant system is. (2 p)

b. Explain what a causal and BIBO stable system is. (2 p)

c. The expression $z/(z-1)$ can be obtained by z-transforming either the sequence $u(n)$ or the sequence $-u(-n-1)$. How do we separate these two cases? (2 p)

d. Given the transfer function $H(z)$ of an LTI system (causal or noncausal), how can we determine whether it is a stable system or not? (2 p)

e. A filter (LTI system) has the transfer function $H(z) = 0.5 + 0.5z^{-1}$. Is it a lowpass or highpass filter? (2 p)

2 A causal discrete-time LTI system is described by the following difference equation:

$$y(n) + b_1 y(n-1) = x(n) - x(n-1) + x(n-2)$$

a. Determine the zeros of the system's transfer function $H(z)$ and indicate them in the z-plane. (4 p)

b. Determine the values of b_1 for which the system is stable. (2 p)

c. Determine the system's impulse response $h(n)$ when $b_1 = 0.75$. (4 p)

3 Determine the output sequence $y(n)$ of a causal discrete-time LTI system with the impulse response $h(n) = 0.4^n u(n)$ when the input sequence is $x(n) = 0.2^n u(n) + 0.8^{n-2} u(n-2)$, where $u(n)$ is the unit step sequence. (10 p)

4 A causal discrete-time LTI system has the following transfer function: $H(z) = a_0 + a_1 z^{-1} + a_0 z^{-2}$. Assume that the input sequence is $x(n) = \sin(0.25\pi n) + \cos(0.75\pi n)$ for all n. Determine the values of a_0 and a_1 so that the output sequence becomes $y(n) = 2\cos(0.75\pi n + \phi)$.The value of ϕ does not have to be determined. (10 p)

5 A continuous-time signal $x_a(t) = \cos(4000\pi t) + 2\sin(8000\pi t) + 4\sin(11000\pi t)$, for all t, is sampled uniformly and reconstructed using an ideal PAM.

a. Sketch the spectrum for the sequence $x(n) = x_a(nT)$ when $1/T = 10$ kHz and $1/T = 7$ kHz, respectively. (5 p)

b. Determine the reconstructed signals for the two cases in **a.** above. Motivate the answer by sketching the spectra of the reconstructed signals. (5 p)

6 A sequence $x(n)$ is given as $x(n) = 2\sin(0.125\pi n) + 4\cos(0.75\pi n + 0.5\pi)$ for all n.

a. Compute the complex Fourier series coefficients in the complex Fourier series expansion of $x(n)$. (5 p)

b. A finite-length sequence $x_1(n)$ of length 16 is formed according to $x_1(n) = x(n), n = 0, 1, \ldots, 15$. Compute $X_1(k)$ for all values of k ($k = 0, 1, \ldots, 15$) where $X_1(k)$ is a 16-point DFT for $x_1(n)$. (5 p)

7 A system for interpolation of the sampling frequency is shown in Figure 1.

a. Determine the interpolation factor. (2 p)

b. Determine the values of the edges $\omega_c T_3$ and $\omega_s T_3$ so that the information in $x(n)$ is preserved and imaging/aliasing is avoided. Also determine the gain constant G so that $y(m)$ has the same energy as $x(n)$, which means that both sequences are sampled versions of the same underlying continuous-time signal but with different sampling frequencies. Motivate the answer by sketching the spectrum for each of the sequences $v_1(p)$, $v_2(p)$, and $y(m)$. (8 p)

Relation between the Fourier transforms for the upsampler:

$$V_1(e^{j\omega T_3}) = X(e^{j\omega T_1}), \quad T_1 = 5T_3$$

Relation between the Fourier transforms for the downsampler:

$$Y(e^{j\omega T_2}) = \frac{1}{4}\sum_{k=0}^{3} V_2(e^{j(\omega T_3 - 2\pi k/4)}), \quad T_2 = 4T_3.$$

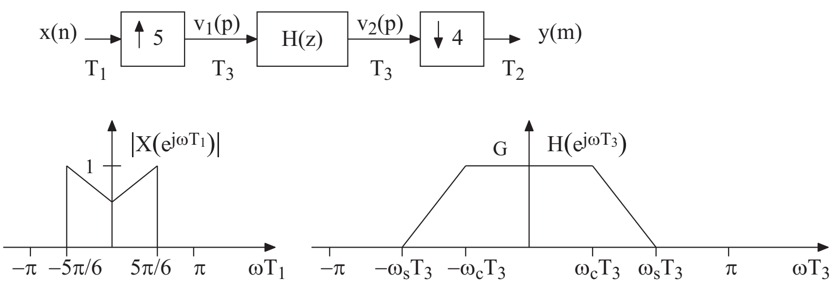

Figure 1: