LINKÖPING UNIVERSITY

Exam in TSEI50 Linear Systems

Exam code:	TEN1
Date:	2016-10-28 **Time:** 8–12
Place:	R34, R36, R44
Examiner:	Håkan Johansson
Department:	ISY
Allowed aids:	Pocket calculator Söderkvist: Formler och Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller och Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler och Lexikon Mathematical tables
Number of tasks:	7
Grading:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem. **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Solutions:	Will be published no later than three working days after the exam at http://www.commsys.isy.liu.se/en/student/kurser/TSEI50/
Result:	Available by 2016-11-11

1 **a.** Explain what steady-state (stationary) and transient responses are. (2 p)

b. Given the impulse response $h(n)$ of an LTI system, how can we determine if it is a stable system or not? (2 p)

c. What does the sampling theorem say? (2 p)

d. The transfer function of an LTI system is given as $H(z) = z^{-1} + z$. Is it a causal system? (2 p)

e. A filter (LTI system) has the transfer function $H(z) = 0.5 - 0.5z^{-1}$. Is it a lowpass or highpass filter? (2 p)

2 A causal discrete-time LTI system is realized according to Fig. 1 with $a_0 = a_2 = 0.125$, $a_1 = 0.25$, and $b_1 = -0.5$.

a. Determine the system's difference equation. (2 p)

b. Determine the system's transfer function $H(z)$ and its zeros and poles. Is it a stable system? (4 p)

c. Determine the system's impulse response $h(n)$. (4 p)

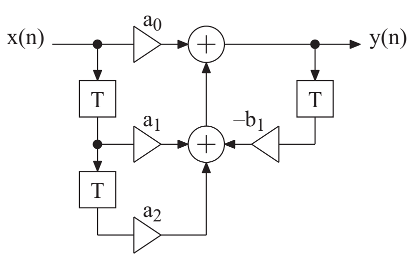

Figure 1:

3 Determine the output sequence $y(n)$ of a causal discrete-time LTI system with the impulse response $h(n) = 0.5^n u(n)$ when the input sequence is $x(n) = u(n) + \sin(0.5\pi n)$, where $u(n)$ is the unit step sequence. (10 p)

4 A causal discrete-time LTI system has the following transfer function:

$$H(z) = G\frac{(z - re^{j\alpha})(z - re^{-j\alpha})}{z^2}$$

a. Determine the values of r and α so that the output sequence becomes $y(n) = 0$ for all n when the input sequence is $x(n) = 2\sin(0.25\pi n)$ for all n. (6 p)

b. Determine the value of G so that the energy of the impulse response is unity, i.e., $\sum_n h^2(n) = 1$. (4 p)

5 A continuous-time signal $x_a(t) = \sin(1000\pi t) + 2\cos(2000\pi t) + 3\sin(2600\pi t)$, for all t, is sampled uniformly and then reconstructed using an ideal PAM, generating $x_r(t)$.

a. Sketch the spectrum for the sequence $x(n) = x_a(nT)$ when $f_s = 1/T = 2500$ Hz and $f_s = 1/T = 1600$ Hz, respectively. (6 p)

b. Determine the reconstructed signal $x_r(t)$ for each of the two cases in **a.** above. Is any of the reconstructed signals equal to the original signal $x_a(t)$? Motivate the answer by sketching the spectra of the reconstructed signals. (4 p)

6 A sequence $x(n)$ is given as $x(n) = 1 + \cos(0.875\pi n + 0.25\pi)$ for all n.

a. Show that $x(n)$ is periodic with period $N = 16$. (2 p)

b. A finite-length sequence $x_1(n)$ of length 16 is formed according to $x_1(n) = x(n), n = 0, 1, \ldots, 15$. Compute $X_1(k)$, $|X_1(k)|$, and $\arg\{X_1(k)\}$, for all values of k ($k = 0, 1, \ldots, 15$) where $X_1(k)$ is a 16-point DFT for $x_1(n)$. (8 p)

7 The sequence $x(n)$ has a real spectrum as seen in Fig. 2(a) and is a sampled version of a continuous-time signal $x_a(t)$ according to $x(n) = x_a(nT)$ for some T.

a. Assume that a new sequence is formed as $y(m) = x_a(mT_1)$, $T_1 = T/5$. That is, $y(m)$ is also a sampled version of $x_a(t)$, but the sampling frequency is here five times higher. Sketch the spectrum of $y(m)$, i.e., sketch $Y(e^{j\omega T_1})$. (4 p)

b. Assume that $y(m)$ is obtained from $x(n)$ through the system in Fig. 2(b) using upsampling by five followed by a filter (LTI system) with the transfer function $H(z)$. Determine the gain constant G, and the values of the edges $\omega_c T_1$ and $\omega_s T_1$ so that the desired $y(m)$ in **a.** above is obtained. Motivate the answer by sketching the spectra involved in the system of Fig. 2(b). (6 p)

Relation between the Fourier transforms for upsampling by L:

$$X_1(e^{j\omega T_1}) = X(e^{j\omega T}), \quad T_1 = T/L$$

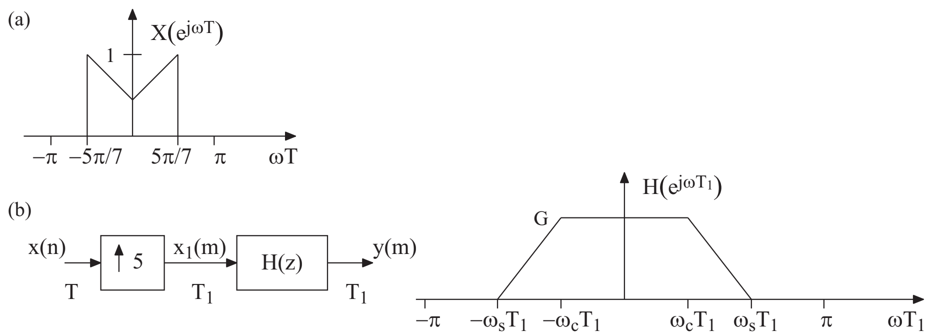

Figure 2: