Tekniska Högskolan i Linköping
Institutionen för Systemteknik, Kommunikationssystem

Exam in TSEI50/TEN1, Linear Systems

Time:	2015-10-29, 14-18
Place:	G32
Examiner:	Håkan Johansson
Aid:	Pocket calculator Söderkvist: Formler & Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller & Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler & Lexikon Mathematical tables
Instructions:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem! **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Results:	Available by 2015-11-12

1 a) Show that the transfer function of a causal linear and time-invariant (LTI) system with a finite-length impulse response (FIR) has all of its poles at $z = 0$. (2 p)

b) Under what conditions will the discrete Fourier transform (DFT) coincide with a Fourier series expansion (except for a scaling constant)? (2 p)

c) The transfer function of an LTI system is given as $H(z) = 1 + z$. Is it a causal or noncausal system? (2 p)

d) Draw a signal-flow graph (realization) that corresponds to the following difference equation: $y(n) = 0.9y(n-3) + 0.5x(n) + 0.4x(n-1)$, where $x(n)$ and $y(n)$ are the input and output sequences, respectively. (2 p)

e) Downsampling by two means that $y(n) = x(2n)$ where $x(n)$ and $y(n)$ are the input and output sequences, respectively. Choose a simple input $x(n)$ and show that the downsampling corresponds to a time-varying system. (2 p)

2) A casual discrete-time LTI infinite-length impulse response (IIR) system is described by the following transfer function:

$$H(z) = \frac{1}{1 + 0.4z^{-1}} + \frac{2.5z^{-1}}{1 - 0.5z^{-1}}$$

a) Determine the poles and zeros of the system. Is it a stable system? (3 p)

b) Determine the region of convergence (ROC) of $H(z)$. (2 p)

c) Determine the system's difference equation that gives the relation between the input sequence $x(n)$ and output sequence $y(n)$. (2 p)

d) Determine the impulse response of the system. (3 p)

3) A causal discrete-time LTI system has the following impulse response:

$h(n) = (-0.1)^n u(n)$ where $u(n)$ is the unit step sequence.

Determine the output sequence $y(n)$ from the system when the input sequence is (with $\delta(n)$ being the unit impulse sequence):

$x(n) = 2\delta(n) + \delta(n-1) + 0.4^{n-2}u(n-2)$ (10 p)

4) A casual discrete-time LTI FIR system is described by the following transfer function:

$$H(z) = G(1 - 2\cos(\omega_0 T)z^{-1} + z^{-2})$$

where G and $\omega_0 T$ are real-valued constants to be determined. Assume that the input sequence is $x(n) = 2\sin(0.2\pi n) + 3\sin(0.8\pi n)$. Determine the values of G and $\omega_0 T$ so that the output sequence becomes $y(n) = \sin(0.2\pi n + \Phi)$. The value of Φ can be ignored, i.e., it does not have to be determined. (10 p)

5) A continuous-time signal $x_a(t)$ is given as

$$x_a(t) = \cos(2\pi \times 50000t) + 2\cos(2\pi \times 80000t) + \sin(2\pi \times 100000t) \text{, for all } t.$$

The signal is sampled uniformly with a sampling frequency of $1/T$, and reconstructed using an ideal PAM.

a) Sketch the Fourier transforms (spectra) for the sequences $x_1(n) = x_a(nT)$ for $1/T = 180$ kHz, and $x_2(n) = x_a(nT)$ for $1/T = 120$ kHz. Indicate level constants (gain) and edges (angles) in the figures so that the Fourier transforms are completely specified. (6 p)

b) Determine the reconstructed signals, $x_{r1}(t)$ and $x_{r2}(t)$, for the two cases in a). Motivate the answer by sketching the spectra of $x_{r1}(t)$ and $x_{r2}(t)$. (4 p)

6) A sequence $x(n)$ is given as $x(n) = 2 + \cos(0.625\pi n + 0.2)$ for all n. A finite-length sequence $x_1(n)$ of length 32 is formed according to $x_1(n) = x(n)$, $n = 0, 1, \ldots, 31$. Compute $|X_1(k)|$ and $\arg\{X_1(k)\}$ for all values of k ($k = 0, 1, ..., 31$) where $X_1(k)$ is a 32-point DFT for $x_1(n)$. (10 p)

7) The sequence $x(n)$ has a spectrum as seen in Fig. 1(a) and is a sampled version of a continuous-time signal $x_a(t)$ according to $x(n) = x_a(nT)$ for some T.

a) Assume that a new sequence is formed as $y(m) = x_a(mT_1)$, $T_1 = T/4$. That is, $y(m)$ is also a sampled version of $x_a(t)$, but the sampling frequency is here four times higher. Sketch the spectrum of $y(m)$, i.e., sketch $Y(e^{j\omega T_1})$. (4 p)

b) Assume that $y(m)$ is obtained from $x(n)$ through the system in Fig. 1(b) using an upsampler followed by a filter (LTI system) with transfer function $H(z)$. Determine the gain constant G, and the values of the edges $\omega_c T_1$ and, $\omega_s T_1$ so that the desired $y(m)$ is obtained. Motivate the answer by sketching the spectra and filter frequency responses involved. (6 p)

Relation between the Fourier transforms for upsampling by L:

$$X_1(e^{j\omega T_1}) = X(e^{j\omega T}), \quad T = LT_1$$

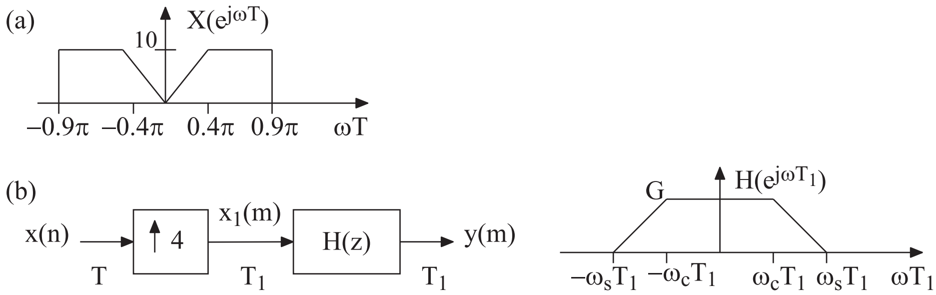

Figure 1.