Tekniska Högskolan i Linköping
Institutionen för Systemteknik, Elektroniksystem

Exam in TSEI50/TEN1, Linear Systems

Time:	2014-10-31, 8-12
Place:	P34, P42
Examiner:	Håkan Johansson
Aid:	Pocket calculator Söderkvist: Formler & Tabeller Ingelstam, Rönngren, Sjöberg: Tefyma Ekbom: Tabeller & Formler NT Nordling: Physics Handbook for Science and Engineering Strid: Formler & Lexikon Mathematical tables
Instructions:	Maximum 70 points, 30 points required to pass the exam. **Note** that a **motivation/solution** is required to get the maximal number of points for a problem! **Note** that 10, 8, 6, 4, or 2 points obtained at the **seminars** means that you do not have to solve Problem 1, 1(a)-(d), 1(a)-(c), 1(a)-(b), or 1(a), respectively.
Results:	Available by 2014-11-14

1 a) For which types of sequences can we use the Fourier transform and Fourier series, respectively? (2 p)

b) Explain what a linear and time-invariant system is. (2 p)

c) Given the transfer function H(z) of an LTI system, how can we determine whether it is stable or not? Does it depend on whether the system is causal or not? (2 p)

d) What do we mean by stationary and transient parts of a sequence? (2 p)

e) Draw a signal-flow graph (realization) that corresponds to the following difference equation: y(n)=0.5y(n-1)+x(n)-x(n-2). (2 p)

2) A causal discrete-time LTI system has an impulse response in parallel form as $h(n) = h_1(n) + h_2(n)$ with $h_1(n) = 0.2^n u(n)$ and $h_2(n) = 0.7^n u(n)$ where $u(n)$ is the unit step sequence.

a) Determine the system's transfer function $H(z)$. (3 p)

b) Determine the system's region of convergence (ROC). (2 p)

c) Determine the zeros and poles of $H(z)$. Is the system stable? (3 p)

d) Determine the system's difference equation, that is, the relation between the system's input $x(n)$ and output $y(n)$. (2 p)

3) A causal discrete-time LTI system is given in cascade form according to Fig. 1 below. Determine the overall system's transfer function $H(z)=Y(z)/X(z)$ as well as its poles and zeros for each of the two cases below, where $\delta(n)$ denotes the unit impulse sequence, and $u(n)$ denotes the unit step sequence.

a) $h_1(n) = \delta(n) + \delta(n-1)$, $h_2(n) = \delta(n) + 2\delta(n-1)$ (5 p)

b) $h_1(n) = 0.5^n u(n)$, $h_2(n) = 0.4^n u(n)$ (5 p)

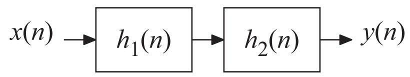

Figure 1.

4) A causal discrete-time LTI system has the following transfer function:

$$H(z) = a_0 + a_1 z^{-1} + a_0 z^{-2}$$

Assume that the input sequence is $x(n) = \sin(0.23\pi n) + \sin(0.77\pi n)$ for all n.

Determine the values of a_0 and a_1 so that the output sequence becomes $y(n) = 1.5\sin(0.77\pi n + \phi)$ regardless of the value of ϕ (the value of ϕ does not have to be determined). (10p)

5) A continuous-time signal $x_a(t)$ is given according to $x_a(t) = \cos(\omega_0 t) + \sin(\omega_1 t)$ where $\omega_0 = 2\pi \times 12000$ rad/s and $\omega_1 = 2\pi \times 15000$ rad/s. Three sequences, $x_1(n)$, $x_2(n)$, and $x_3(n)$, are formed by sampling $x_a(t)$ with the sampling frequencies $f_{s1} = 40$ kHz, $f_{s2} = 25$ kHz, and $f_{s3} = 20$ kHz, respectively.

a) Sketch the Fourier transforms for the three different sequences, i.e., sketch $X_1(e^{j\omega T_1})$, $X_2(e^{j\omega T_2})$, and $X_3(e^{j\omega T_3})$. Indicate level constants (gain) and edges (angles) in the figures so that the Fourier transforms are completely specified. (6 p)

b) Assume that three continuous-time signals, $x_{r1}(t)$, $x_{r2}(t)$, and $x_{r3}(t)$, are formed through ideal PAMs, i.e., $P_i(j\omega)$, $i = 1, 2, 3$, are:

$$P_i(j\omega) = \begin{cases} \dfrac{1}{f_{si}}, & |\omega| \le \pi f_{si} \\ 0, & \text{otherwise} \end{cases}$$

Sketch the Fourier transforms for the three different signals, i.e., sketch $X_{r1}(j\omega)$, $X_{r2}(j\omega)$, and $X_{r3}(j\omega)$. Which of the three reconstructed signals $x_{r1}(t)$, $x_{r2}(t)$, and $x_{r3}(t)$ will in this case be equal to the original signal $x_a(t)$? (4 p)

6) Assume that a finite-duration sequence $x(n)$ where $x(n) = 0$ for $n > 7$ and $n < 0$ has a real spectrum according to Fig. 2 below. Assume further that an 8-point DFT, $X_1(k)$, is computed for $x_1(n) = x(n)$, $n = 0, 1, ..., 7$. Determine $|X_1(k)|$ and $\arg\{X_1(k)\}$ for all values of k ($k = 0, 1, ..., 7$). (10 p)

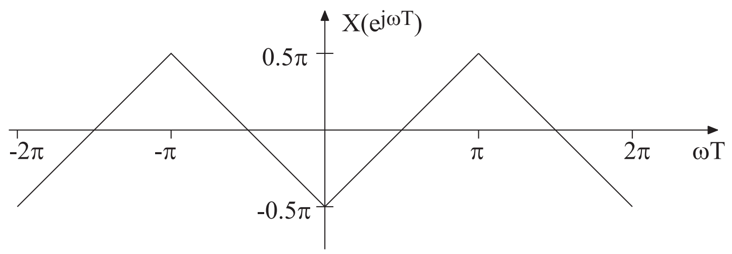

Figure 2.

7) A system for decimation of the sampling frequency by 4/3 is shown in Fig. 3 below. Determine the values of $\omega_c T$ and $\omega_s T$ so that the information in $x(n)$ is retained and imaging/aliasing is avoided. Also indicate the gain constant G so that $y(m)$ has the same energy as $x(n)$, which implies that both sequences correspond to sampled versions of the same underlying continuous-time signal but with different sampling frequencies. Motivate the answer by sketching the spectrum for the sequences $v_1(p)$, $v_2(p)$, and $y(m)$. (10 p)

Hint 1: Relation between the Fourier transforms for the upsampler:

$$V_1(e^{j\omega T_3}) = X(e^{j\omega T_1}), \quad T_1 = 3T_3$$

Hint 2: Relation between the Fourier transforms for the downsampler:

$$Y(e^{j\omega T_2}) = \frac{1}{4}\sum_{k=0}^{3} V_2(e^{j(\omega T_3 - 2\pi k/4)}), \quad T_2 = 4T_3$$

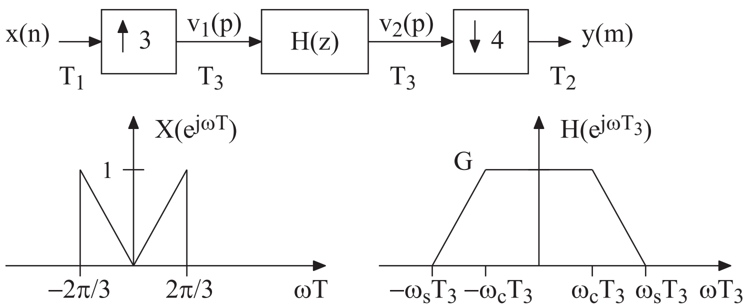

Figure 3.