LECTURE 1, Linear Systems, TSEI50

Content of the Course

Basic theory and analysis methods for discrete-time (DT) signals and systems

Sampling, quantization, and reconstruction (CT/DT interface)

Analysis Tools

Similar to continuous-time (CT) case (**required pre-knowledge!**)

Time-domain

- Sequences of numbers (discrete-time signals)
- Difference equations, impulse response, step response, convolution

Transform domain

- Fourier series, Fourier transform, z-transform
- Frequency response, transfer function, poles and zeros

Relations between CT and DT signals when sampling and reconstructing

Linear and time-invariant systems (LTI systems) in focus

Lectures

Short overview (slides), can be downloaded

White board

On the White Board

Continuous-time signals and systems

Discrete-time signals and systems

Sampling and reconstruction

Analysis tools and notations

(First lecture is an introduction-like presentation. All concepts will be explained and exemplified in more detail during the course.)

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Sequences (discrete-time signals)

Different categories of sequences

- Affect analysis and analysis methods

Basic sequences

- Important for the analysis of discrete-time systems

Operations on sequences

- Few basic operations required for LTI-systems

Discrete-time systems

Mapping of an input sequence onto an output sequence

Classification of systems

- Linear/nonlinear
- Time-invariant/time-varying
- Stable/unstable
- Causal/noncausal

On the White Board

Different categories of sequences

- Finite-duration sequences
- Infinite-duration sequences (right-sided, left-sided, two-sided)
- Periodic sequences
- Bounded sequences
- Absolutely summable and square-summable sequences
- (Even and odd sequences, energy and power sequences)

Basic sequences

- Unit impulse and unit step sequences
- Sinusoidal and exponential sequences

Operations on sequences

- Addition, multiplication, shift (delay), and reversal

Discrete-time systems

- Linearity
- Time invariance
- Stability
- Causality

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Computation of the output sequence

Convolution

Can be used for all LTI systems

1) Graphically, for illustration and understanding

2) Analytically

Solution to linear difference equation
(practical LTI systems normally represented in this way)

$$\sum_{k=0}^{M} b_k y(n-k) = \sum_{k=0}^{N} a_k x(n-k)$$

1) Particular and homogenous solution

2) Iterative solution - practical implementation

Transform-domain methods

1) Fourier transform

2) z-transform

On the White Board

- Convolution sum
  - Some general aspects
  - Graphical example
  - Analytical example
- Properties of the convolution sum
  - Cascade connection
  - Parallel connection

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Iterative solution of linear difference equations

Difference equation: $$\sum_{k=0}^{M} b_k y(n-k) = \sum_{k=0}^{N} a_k x(n-k)$$

Design of LTI systems

Synthesis

Given a specification: determine M, N, a_k, and b_k

Realization/structure/algorithm

Signal-flow graphs, block diagrams

Implementation

Mapping of algorithm to executable (pseudo)code

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Iterative solution for causal LTI systems

Realization

Implementation

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LTI Systems

Implemented in the time-domain

Analysis in the time-domain or transform domain

Time-domain analysis

Convolution

Difference equation - iterative solution

Stability - impulse response absolute summable

Causality - impulse response zero for $n < 0$

Transform-domain analysis

Convolution in time domain - multiplication in transform domain

- Simplifies analysis and computations

Fourier transform (Fourier series for periodic signals)

- Frequency properties of signals and systems

z-transform

- Generalization of the Fourier transform
- Stability analysis

On the White Board

- Fourier series
  - Discrete amplitude and phase spectrum
- Fourier transform
  - Continuous amplitude and phase spectrum
  - Convergence conditions
  - Properties (linearity, shift, convolution, Parseval's relation)

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z-transform

Generalization of the Fourier transform

Can be used for a broader class of sequences

Frequency response

Fourier transform of the impulse response of a system

Describes the system’s frequency properties

Essential when designing filters

Transfer function

z-transform of the impulse response of a system

Generalization of the frequency response

Describes the system’s properties like stability

On the White Board

- z-transform
- Region of convergence
- Rational transforms - poles and zeros
- Inverse transformation methods
- Properties (linearity, shift, convolution)
- Frequency response
  - Some general aspects
  - Derivation of the frequency response from the difference equation

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Transfer function

z-transform of impulse response h(n) <--> H(z)

1) Simplifies solution of difference equations

Convolution in time domain <--> multiplication in transform domain

2) Stability/causality check

3) Fourier transform special case of z-transform <-->

Frequency response special case of transfer function <-->

Relation between poles/zeros and magnitude/phase response

Useful for the understanding of pole/zero locations and frequency response behavior of the system

On the White Board

Transfer function

Solution of difference equation

Stability

Causality

Magnitude and phase response from pole/zero diagram

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Sampling and reconstruction

Original CT signal $x_a(t)$

Sampled DT signal (sequence): $x(n) = x_a(nT)$

Reconstructed CT signal $x_r(t)$

Sampling period T, sampling frequency $f_s = 1/T$

Time-domain operations but **the analysis can be done in the frequency domain using the signals' Fourier transforms!**

Selection of sampling frequency

How to select f_s so that $x_a(t)$ can be reconstructed from $x(n)$?

Sampling theorem says $f_s > 2f_0$, when $x_a(t)$ bandlimited to f_0

Reconstruction

How to perform reconstruction so that $x_r(t) = x_a(t)$ when the sampling theorem is fulfilled? - ideal reconstruction via PAM

Practical reconstruction: D/A converter and analog filter

Distortion

What will the error $e(t) = x_r(t) - x_a(t)$ be when the sampling theorem is not fulfilled?

Undersampling - aliasing distortion

Anti-aliasing filter - reduces aliasing distortion

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Sampling

Poisson's summation formula

Reconstruction

Pulse amplitude modulation (PAM)

Ideal reconstruction

Practical reconstruction: D/A converter and analog filter

Distortion

Error when undersampling

Anti-aliasing filter

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Discrete Fourier transform (DFT)

Fourier transform continuous function, ωT continuous variable

In practice computed for a finite number of values (points) of ωT

The DFT is used for this purpose

Defined for finite-duration sequences

$$x(n), \quad n = 0, 1, \ldots, N-1$$

N-point DFT: $X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$

Inverse DFT: $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \quad n = 0, 1, \ldots, N-1$

Two cases

1) Finite-duration sequences

Correct (exact) computation of the Fourier transform for $\omega T = \frac{2\pi k}{N}$

2) Infinite-duration sequences

a) Generally only approximate computation of the Fourier transform

b) Periodic sequences - DFT is the same as a Fourier series expansion except for a different scaling constant => correct spectrum comput.

On the White Board

- DFT for sequences of finite-duration
- DFT for sequences of finite-duration - increased resolution
- DFT for sequences of infinite-duration
  - Windowing
  - General case
  - Periodic sequences
  - Coherent sampling
- Fast Fourier transform (FFT)
  - Efficient computation of the DFT

LECTURE 10, Linear Systems, TSEI50

Stochastic (random) processes

Used as models for random-like signals

Computation of expected average error power, SNR, etc.

In the course: simplified description (not the whole theory)

White noise

Samples in the sequence e(n) are independent

Samples in the sequence e(n) have the same statistical properties

The mean value of e(n) is zero

Applications

Thermal noise in electrical circuits

Quantization noise in digital systems

Quantization

Nonlinear operation

Linear model used for computations

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Quantization and its linear model

Quantization errors in A/D conversion

Computation of average power when modeling the error as uniformly distributed white-noise

Filtering of white noise

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Multirate systems

Systems with several different sampling frequencies

1) Reconstruction followed by resampling

2) Digital sampling rate conversion - preferable!

Interpolation by an integer factor of L

Increases the sampling rate

Uses LTI system (filter) and upsampler

Decimation by an integer factor of M

Decreases the sampling rate

Uses LTI system (filter) and downsamplers

Application examples

Interconnection between systems with different sampling frequencies. Ex. CD 44.1 kHz, DAT 48 kHz

Oversampled A/D and D/A converters

Relaxes requirements on analog filters and A/D converters

Reduced arithmetic complexity

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Interpolation by an integer factor of L

Decimation by an integer factor of M

Oversampled A/D converters

LECTURE 12, Linear Systems, TSEI50

Summary

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Analysis Tools

Time-domain

Sequences of numbers (discrete-time signals)

Difference equation, impulse response, step response, convolution

Transform domain

Fourier series, Fourier transform, z-transform

Frequency response, transfer function, poles and zeros

Relations between CT and DT signals when sampling and reconstructing

Linear and time-invariant systems (LTI systems) in focus

Applications

Filtering

Interpolation and decimation

(Principles, design considered in other courses)

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Sequences (chapter 2)

Basic operations

Basic sequences

Categories of sequences

Discrete-time systems (chapter 3)

Linear difference equations

Properties

Realization/structure/algorithm - block diagram, signal-flow graph

Time-domain analysis (chapter 3)

Computation of output sequence

1) Iteration of the difference equation

2) Convolution between input and impulse response

Stability

Causality

Frequency analysis (chapters 4, 6, 8)

Fourier transform

Transform, inverse transform, properties,

Fourier series

Discrete Fourier transform (DFT)

Frequency response of a system

z-transform analysis (chapters 5, 6)

- Transform, inverse transform, properties
- Transfer function of a system
- Poles and zeros
- Relations between poles/zeros and magnitude/phase responses
- Solution of difference equations
- Stability
- Causality
- Steady state and transient responses

Sampling and reconstruction (chapter 7)

- Poisson's summation formula
- Sampling theorem
- Pulse amplitude modulation (PAM)
- Ideal reconstruction
- Practical reconstruction (D/A converter and analog filter)

Stochastic processes (parts of chapters 9, 10, 11)

- White noise
- Linear model of quantization
- Quantization noise analysis (SNR)
- Filtering of white noise

Multirate systems (chapter 12: 12.2, 12.3, 12.6)

- Interpolation
- Decimation
- Oversampled A/D converters