# Fundamentals of Wireless Communications
# Additional Homework

Erik G. Larsson

October 14, 2008

P1. Discuss, under what circumstances (what type of systems, what type of physical conditions) will receive antenna diversity help the most? Are there cases where it will help only very little or not at all?

P2. Consider $n$-branch diversity reception with selection combining: a receiver that picks out the best branch,

$$\hat{k} = \text{argmax}|h_k|$$

and then bases the decision on $y_{\hat{k}}$ only. What diversity order can this receiver achieve?

P3. Consider $n$-branch diversity combining with channel $\boldsymbol{h}$ and received vector

$$\boldsymbol{y} = \boldsymbol{h}s + \boldsymbol{e}$$

in the standard manner. ($\boldsymbol{h}$ known at the receiver.)

Suppose the noise is white $\boldsymbol{e} \sim N(\boldsymbol{0}, \sigma^2\boldsymbol{I})$ but the fading is correlated: $\boldsymbol{h} \sim N(\boldsymbol{0}, \boldsymbol{Q})$ where $\boldsymbol{Q}$ is a known, positive semidefinite matrix (not necessarily nonsingular). Determine the optimal receiver, and derive its diversity order. Give an example of a situation in practice when this model is relevant.

P4. Consider the same problem as in P3 but assume that the fading is uncorrelated, $\boldsymbol{h} \sim N(\boldsymbol{0}, \boldsymbol{I})$ but the noise is colored: $\boldsymbol{e} \sim N(\boldsymbol{0}, \boldsymbol{\Psi})$ where $\boldsymbol{\Psi}$ is known, and positive definite. Determine the optimal receiver, and its diversity order. Give an example of a situation in practice when this model is relevant.

P5. For receive and transmit diversity in wireless links, discuss how far apart the antennas should be located. What is the consequence (if any) of having the antennas too close or too far apart?

Then suppose you want to use an antenna array not primarily for communications but instead in order to locate (using direction-finding) a radio transmitter. Discuss spacing between the antennas would be appropriate in this scenario. What is the consequence (if any) of having the antennas too close or too far apart?

P6. The following space-time block code is used on an $n_t = 2, n_r = 1$ system:

$$\boldsymbol{X}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{X}_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \boldsymbol{X}_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{1}$$

The channel is flat Rayleigh fading, and the nosie is white Gaussian.

Which pair of matrices are most likely to be mistaken for each other at the receiver? Is the code a good one? What is its diversity order?

P7. The Alamouti code is implemented in a system, but that during the implementation, the complex conjugates are "forgotten" and the code matrix

$$\boldsymbol{X} = \begin{bmatrix} s_1 & s_2 \\ s_2 & -s_1 \end{bmatrix} \tag{2}$$

is transmitted instead.

Discuss the implications, in terms of structure of the optimal detector, and diversity order.

P8. Read the lecture note "MIMO detection methods – how they work". Select the method you like the most (but not zero-forcing, and not reference [4]) and read about it in some depth by following the literature references and/or appropriate literature search. Be prepared to give a 5-10 minute presentation of the method (on board or using OH).

P9. Let $H$ be a positive definite matrix. Prove that

$$[H^{-1}]_{k,k} \geq \frac{1}{H_{k,k}}$$

Explain the implications of this result on the zero-forcing receiver (decorrelator).

P10. Consider the model $y = Hx + e$ where $e$ is white and Gaussian with i.i.d. $CN(0, \sigma^2)$ elements.

a) Give an expression for the pairwise error-probability for ML detection, i.e., for the receiver which takes

$$\hat{x} = \text{argmin}\|y - Hx\|$$

(express the result in terms of the Gaussian Q-function)

b) Derive the pairwise error-probability for ZF detection, i.e., for the receiver that computes $z = (H^H H)^{-1} H^H y$ and then takes $x_i$ as the point in the signal constellation which is closest to $z_i$.

P11. Consider a $n_t = 2, n_r = 1$ system, fast i.i.d. Rayleigh fading.

a) Give an expression for the ergodic capacity, assuming the transmitter sends uncorrelated streams with equal power

b) Suppose the Alamouti code is used. Give an expression for the ergodic capacity.

c) Comment on the relation between the results in a) and b).

P12. Repeat P11, but for $n_t = 2, n_r = 2$.