Practical beamforming lecture, Reading and exercises

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1 Reading

A good introduction to this topic is ”A vector-perturbation technique for near-capacity multiantenna multiuser communication-part II: perturbation”, by Hochwald et al., ”Linear precoding via conic optimization for fixed MIMO receivers” by Wiesel et al., ”Transmitter optimization for the multi-antenna downlink with per-antenna power constraints” by Yu and Lan, and ”Convex optimization-based beamforming”, by Gershman et al.

2 Exercises

- **H1:** Does Slater's condition hold for the linear precoder power minimization problem

$$\begin{aligned} \underset{\boldsymbol{A}}{\text{minimize}} \quad & \sum_{i=1}^{K} \|\boldsymbol{a}_i\|^2 \\ \text{subject to} \quad & \frac{|\boldsymbol{h}_k^r \boldsymbol{a}_k|^2}{\sum_{j \neq k} |\boldsymbol{h}_k^r \boldsymbol{a}_j|^2 + \sigma^2} \geq \gamma_k, k = 1, ..., K, \end{aligned} \tag{1}$$

  in the special case of $H_{i,j} = 1$, $i = 1, 2$, $j = 1, ..., M$, $\sigma^2 = 1$, $\gamma_k = 2$, $k = 1, 2$?

- **H2:** for the H1 problem, show that an optimal parametrization of $\boldsymbol{A}$ is given by

$$\boldsymbol{a}_k = c_k \left(\sum_j \mu_j (\boldsymbol{h}_j^r)^{\mathrm{H}} \boldsymbol{h}_j^r + \boldsymbol{I} \right)^{-1} (\boldsymbol{h}_k^r)^{\mathrm{H}}. \tag{2}$$

  Can you remove dependence on μ_k in your parametrization?

- **H3:** Prove that you can parametrize the optimal linear detector vectors of the so-called virtual uplink problem

$$\begin{aligned} \underset{\{q_k, \boldsymbol{u}_k\}}{\text{minimize}} \quad & \sum_{i=1}^{K} \sigma^2 q_k \\ \text{subject to} \quad & \frac{q_k |\boldsymbol{h}_k^r \boldsymbol{u}_k|^2}{\sum_{j \neq k} q_j |\boldsymbol{h}_j^r \boldsymbol{u}_k|^2 + 1} \geq \gamma_k, k = 1, ..., K, \end{aligned} \tag{3}$$

  as

$$\boldsymbol{u}_k = c'_k \left(\sum_{j \neq k} q_j (\boldsymbol{h}_j^r)^{\mathrm{H}} \boldsymbol{h}_j^r + \boldsymbol{I} \right)^{-1} (\boldsymbol{h}_k^r)^{\mathrm{H}}. \tag{4}$$

- **H4:** Show that the optimal $\sum_j q_j$ in (3) is equal to the optimal $\sum_j \mu_j$ in the SDP problem in the lecture.

  Hints: You can use that $\boldsymbol{Z} \geq \boldsymbol{g}\boldsymbol{g}^{\mathrm{H}}$ if and only if $\boldsymbol{g}^{\mathrm{H}} \boldsymbol{Z}^{-1} \boldsymbol{g} \leq 1$; and that there is a unique solution that satisfies the SINR constraints in (3) with equality for the optimal choice of $\{\boldsymbol{u}_k\}$ in H3.

- **H5:** Implement a vector perturbation precoder with a linear precoder of your choice. Plot sum rate as a function of total power, with and without perturbation. How much do you gain by using the perturbation? Compare to the sum rate capacity given by Erik. Mail your plot to Daniel before the class/bring it on a usb, and be prepared to present it.