Lecture Note
Some Notes on Asymptotic Equivalence

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Several of the papers that we read in the class make extensive use of asymptotic approximations. Here we highlight some key points, by means of examples.

- Consider

  $$x_n \sim N(0, Q)$$

  and let

  $$\hat{Q} = \frac{1}{N}\sum_{n=1}^{N} x_n x_n^H$$

  Each term in the sum has variance $O(1)$ and the terms are i.i.d.. Hence each element of $\hat{Q}$ has variance $O(1/N)$ so

  $$\hat{Q} = Q + O(1/\sqrt{N})$$

  Now apply this to the array model:

  $$y_n = Ax_n + e_n$$

  and let

  $$R = E[y_n y_n^H] = APA^H + \sigma I$$

  For the SCM estimate of R, we have

  $$\hat{R} = \frac{1}{N}\sum_{n=1}^{N} y_n y_n^H = R + O(1/\sqrt{N})$$

  The consequence is that we can replace R by $\hat{R}$ whenever an error of $O(1/\sqrt{N})$ is tolerable. This is often the case in the analysis in the papers we are reading.
- Consider the formula for asymptotic variance,

  $$\text{var}(\hat{\theta}) \approx \frac{E[f_n'(\theta)^2|_{\theta=\theta_0}]}{f''(\theta_0)^2}$$

  Typically, var$(\hat{\theta})$ is $O(1/N)$ where N is the number of collected data samples. This means that $f_n'(\theta) = O(1/\sqrt{N})$.
  Suppose now that

  $$f_n'(\theta) = O(1/\sqrt{N}) + H.O.T.$$

  where the H.O.T. goes to zero faster than $O(1/\sqrt{N})$. For example, it may be the product of two factors, each of which tends to zero as $O(1/\sqrt{N})$. Then, the H.O.T. may be safely neglected in the analysis for large N - the asymptotic variance will not change.